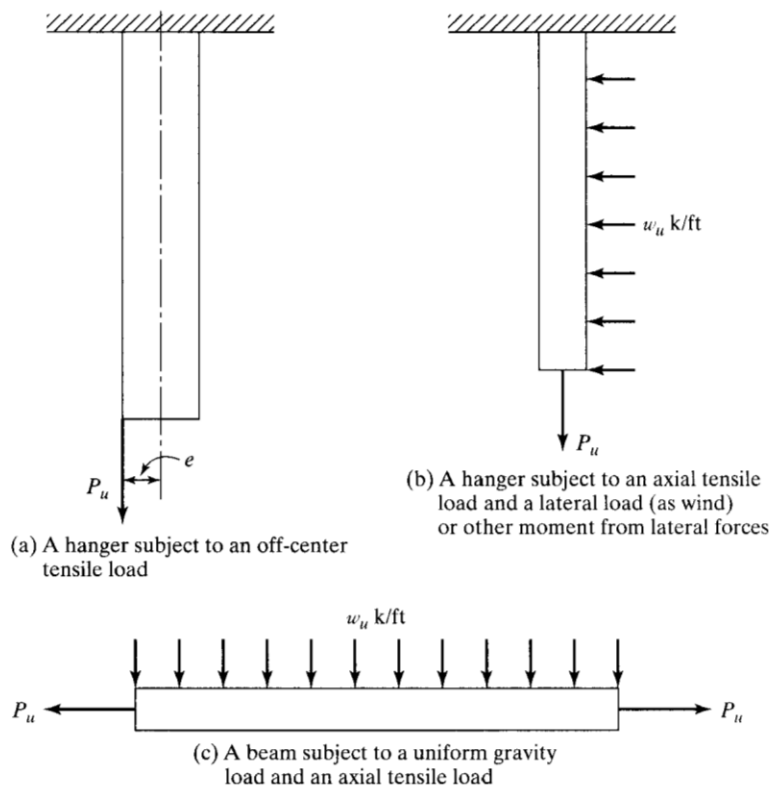
Chapter 9: Bending and Axial Force

❖ <u>Members subjected to bending and axial tension:</u>

A few types of members subjected to both bending and axial tension are shown in the figure below. In section H1 of the AISC specification, the interaction equations that follows are given for symmetric shapes subjected to bending and axial tensile forces.



(a) A hanger subject to an off-center tensile load

(b) A hanger subject to an axial tensile load and a lateral load (as wind) or other moment from lateral forces

(c) A beam subject to a uniform gravity load and an axial tensile load

$$For\ \frac{P_r}{P_c} \geq 0.2, \qquad \frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \quad (AISC\ Equation\ H1-1a)$$

$$For\ \frac{P_r}{P_c} < 0.2, \qquad \frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \quad (AISC\ Equation\ H1-1b)$$

In which:

P_r = required axial tensile strength, P_u kips

P_c = design axial tensile strength $(\varphi_t P_n)$ kips

M_r = required flexural strength, M_u kft

M_c = design flexural strength $(\varphi_b M_n)$ kft

❖ Example 1:

A 50 ksi W12 × 40 tension member with no holes is subjected to the axial loads $P_D = 25$ k and $P_L = 30$ k, as well as the bending moments $M_{Dy} = 10$ ft-k and $M_{Ly} = 25$ ft-k. Is the member satisfactory if $L_b < L_p$?

Using a W12 × 40 ($A = 11.7\text{ in}^2$)

LRFD	ASD
$P_r = P_u = (1.2)(25\text{ k}) + (1.6)(30\text{ k}) = 78\text{ k}$	$P_r = P_a = 25\text{ k} + 30\text{ k} = 55\text{ k}$
$M_{ry} = M_{uy} = (1.2)(10\text{ ft-k}) + (1.6)(25\text{ ft-k}) = 52\text{ ft-k}$	$M_{ry} = M_{ay} = 10\text{ ft-k} + 25\text{ ft-k} = 35\text{ ft-k}$
$P_c = \phi P_n = \phi_t F_y A_g = (0.9)(50\text{ ksi})(11.7\text{ in}^2) = 526.5\text{ k}$	$P_c = \dfrac{P_n}{\Omega_c} = \dfrac{F_y A_g}{\Omega_t} = \dfrac{(50\text{ ksi})(11.7\text{ in}^2)}{1.67} = 350.3\text{ k}$
$M_{cy} = \phi_b M_{py} = 63.0\text{ ft-k}$ (AISC Table 3-4)	$M_{cy} = \dfrac{M_{cy}}{\Omega_b} = 41.9\text{ ft-k}$ (AISC Table 3-4)
$\dfrac{P_r}{P_c} = \dfrac{78\text{ k}}{526.5\text{ k}} = 0.148 < 0.2$	$\dfrac{P_r}{P_c} = \dfrac{55\text{ k}}{350.3\text{ k}} = 0.157 < 0.2$
∴ **Must use AISC Eq. H1-1b**	∴ **Must use AISC Eq. H1-1b**
$\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$	$\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$
$\dfrac{78}{(2)(526.5)} + \left(0 + \dfrac{52}{63}\right)$	$\dfrac{55}{(2)(350.3)} + \left(0 + \dfrac{35}{41.9}\right)$
$= 0.899 < 1.0$ **OK**	$= 0.914 < 1.0$ **OK**

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❖ Example 2:

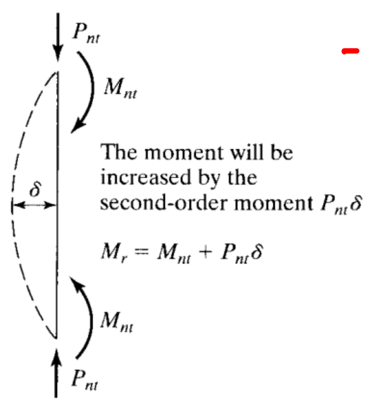
A W10 × 30 tensile member with no holes, consisting of 50 ksi steel and with $L_b = 12.0$ ft, is subjected to the axial service loads $P_D = 30$ k and $P_L = 50$ k and to the service moments $M_{Dx} = 20$ ft-k and $M_{Lx} = 40$ ft-k. If $C_b = 1.0$, is the member satisfactory?

Using a W10 × 30 ($A = 8.84$ in^2, $L_p = 4.84$ ft and $L_r = 16.1$ ft, $\phi_b M_{px} = 137$ ft-k, BF for LRFD = 4.61 from AISC Table 3-2)

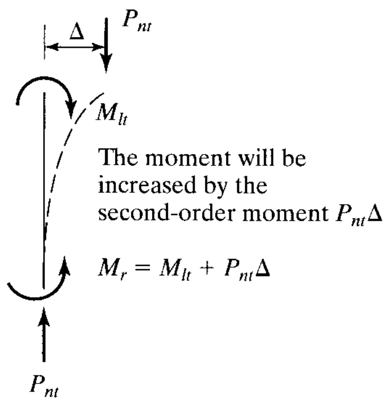
LRFD	ASD
$P_r = P_u = (1.2)(30\text{k}) + (1.6)(50\text{k}) = 116\text{ k}$	$P_r = P_a = 30\text{ k} + 50\text{ k} = 80\text{ k}$
$M_{rx} = M_{ux} = (1.2)(20\text{ ft-k}) + (1.6)(40\text{ ft-k})$	$M_{rx} = M_{ax} = 20\text{ ft-k} + 40\text{ ft-k}$
$= 88\text{ ft-k}$	$= 60\text{ ft-k}$
$M_{ry} = 0$	
$P_c = \phi P_n = \phi_t F_y A_g = (0.9)(50\text{ ksi})(8.84\text{ in}^2)$	$P_c = \frac{P_n}{\Omega_c} = \frac{F_y A_g}{\Omega_c} = \frac{(50\text{ ksi})(8.84\text{ in}^2)}{1.67}$
$= 397.8\text{ k}$	$= 264.7\text{ k}$
$M_{cx} = \phi_b M_{nx} = C_b[\phi_b M_{px} - BF(L_b - L_p)]$	$M_{cx} = \frac{M_{nx}}{\Omega_b} = C_b\left[\frac{M_{px}}{\Omega_b} - BF(L_b - L_p)\right]$
$= 1.0[137 - 4.61(12.0 - 4.84)]$	$= 1.0[91.3 - (3.08)(12 - 4.84)]$
$= 104.0\text{ ft-k}$	$= 69.2\text{ ft-k}$
$\frac{P_r}{P_c} = \frac{116}{397.8} = 0.292 > 0.2$	$\frac{P_r}{P_c} = \frac{80}{264.7} = 0.302 > 0.2$
∴ **Must use AISC Eq. H1-1a**	∴ **Must use AISC Eq. H1-1a**
$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0$	$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{rx}}{M_{cy}}\right) \le 1.0$
$\frac{116}{397.8} + \frac{8}{9}\left(\frac{88}{104.0} + 0\right)$	$\frac{80}{264.7} + \frac{8}{9}\left(\frac{60}{69.2} + 0\right)$
$= 1.044 > 1.0$ **N.G.**	$= 1.073 > 1.0$ **N.G.**

❖ First and Second Order Moment for Members Subjected to Axial Compression and Bending:

When a beam column is subjected to moment along its unsupported length, it will be displaced laterally in the plane of bending. The result will be an increase or secondary moment equal to the axial compression load times the lateral displacement or eccentricity. In the figure below, we can see that the member moment is increased by an amount (P_{nt} δ), where P_{nt} is the axial compression load determined by a first order analysis. This moment will cause additional lateral deflection, which will in turn cause a larger column moment, which will cause a larger lateral deflection, and so on until equilibrium is reached. M_r is the required moment strength of the member. M_{nt} is the first order moment, assuming no lateral translation of the frame.



If a frame is subjected to sidesway when the ends of the column can move laterally with respect to each other, additional secondary moments will result. In the figure below, the secondary moment produced due to sidesway is equal to $P_{nt}\,\Delta$. The moment Mr is assumed by the AISC specification to equal M_{lt} (which is the moment due to the lateral loads) plus the moment due to $P_{nt}\,\Delta$.



The required total flexural strength of a member must at least equal the sum of the first order and second order moments. Several methods are available for determining the required strength. The AISC specification chapter C states that we can either make a second order analysis to determine the maximum required strength or use a first order analysis or amplify the moments obtained with some amplification factors called B_1 and B_2.

- <u>Approximate second order analysis:</u>

You can find this method in appendix 8 of the AISC specification. Using this method we will make two first order analyses one an analysis where the frame is assumed to be braced so that it cannot sway. We will call this moment M_{nt} and will multiply them by a magnification factor B_1 to account for the Pδ effect. When we will analyze the frame again, allowing it to sway.

We will call these moments M_{lt} and will multiply them by a magnification factor B_2 to account for the $P\Delta$ effect.

The final moment in a member will equal,

$$M_r = B_1 M_{nt} + B_2 M_{lt} \quad (AISC\ equation\ C2-1a)$$

The final axial strength P_r must equal,

$$P_r = P_{nt} + B_2 P_{lt} \quad (AISC\ equation\ C2-1b)$$

- <u>Magnification Factors:</u>

The magnification factors are B_1 and B_2. With B_1, the analyst attempts to estimate the $P_{nt}\delta$ effect for a column, whether the frame is braced or unbraced against sidesway. With B_2, the analyst attempts to estimate the $P_{lt}\Delta$ effect in unbraced frames.

The horizontal deflection of a multistory building due to wind or seismic load is called drift (Δ). Drift is measured with drift index (Δ_H/L), where Δ_H is the first order lateral inter-story deflection and L is the story height. For the comfort of the occupants of the building, the index is usually limited at working loads to a value between 0.0015 and 0.0030, and at factored loads to about 0.0040.

The expression of B_1 was derived for a member braced against sidesway. It will be used only to magnify the M_{nt} moments (those moment computed assuming that there is no lateral translation of the frame).

$$B_1 = \frac{C_m}{1 - \alpha \frac{P_r}{P_{e1}}} \geq 1.0 \; (\, AISC\ Equation\ C2 - 2)$$

In this expression C_m is a term that is defined in the next section, α is a factor equal to 1 for the LRFD method; P_r is the required axial strength of the member, $and\ P_{e1}$ is the member Euler buckling strength calculated on the basis of zero sidesway.

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} \quad (\, AISC\ Equation\ C2 - 5)$$

One is permitted to use the first order estimate of P_r (that is, $P_r = P_{nt} + P_{lt}$) when calculating magnification factor B_1. Also, K_1 is the effective length factor in the plane of bending, determined based on the assumption of no lateral translation, and should be equal to 1.0 unless analysis justifies a smaller value.

In a similar fashion P_{e2} is the elastic critical buckling resistance for the story in question, determined by a sidesway buckling analysis. For this analysis, K_2L is the effective length in the plane of bending, based on the sidesway buckling analysis. For this case, the sidesway buckling resistance may be calculated with the following expression, in which Σ is used to include the entire column on that level or story.

$$\sum P_{e2} = \sum \frac{\pi^2 EI}{(K_2 L)^2} \quad (AISC\ equation\ C2-6a)$$

Furthermore, the AISC permits the use of the following alternative expression for calculating $\sum P_{e2}$

$$\sum P_{e2} = R_m \frac{\sum HL}{\Delta_H} \quad (AISC\ equation\ C2-6b)$$

Rm = 1 for braced frame system and 0.85 for moment frame system.

$\sum H$ = story shear produced by the lateral loads used to compute Δ_H, Kips

L = story length, in

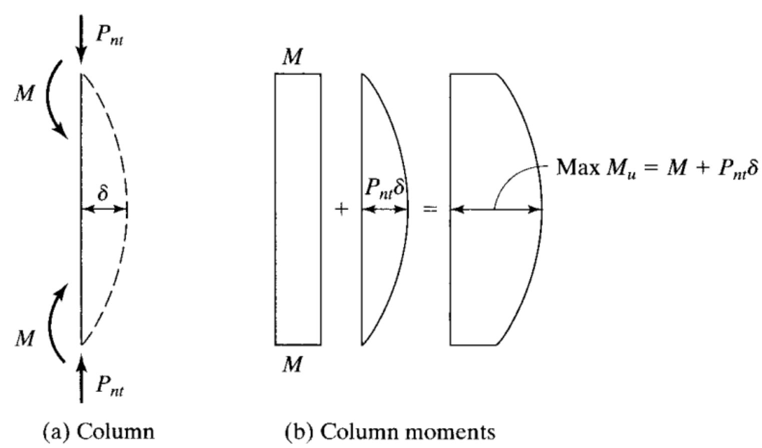
Δ_H = First order interstory drift due to the lateral loads

The value shown for $\sum P_{nt}$ and $\sum P_{e2}$ are for all of the columns on the floor in question. This is considered to be necessary because the B2 term is used to magnify column moments for sidesway. For sidesway to occur in a particular column, it is necessary for all the columns on the floor to sway simultaneously. The $\sum H$ value used in the first of the B2 expression represents the sum of the lateral loads acting above the floor being considered.

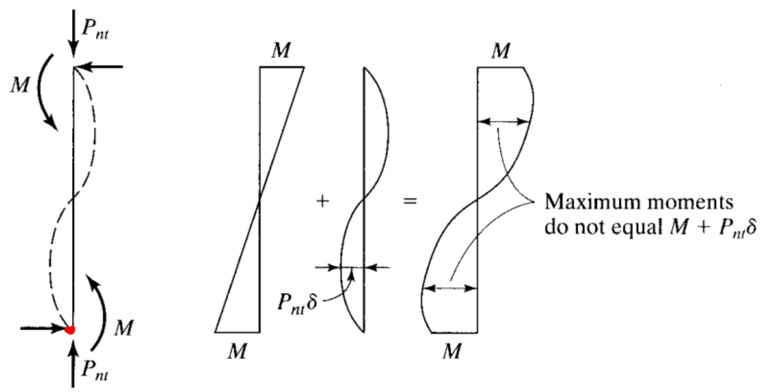
$$B_2 = \frac{1}{1 - \alpha \frac{\sum P_{nt}}{\sum P_{e2}}}$$

❖ Moment modification or C_m factors:

In the expression for B_1, a term C_m called the modification factor was included. The magnification factor B_1 was developed for the largest possible lateral displacement. On many occasions the displacement is not that large, and B_1 over magnifies the column moment. As a result, the moment may need to be reduced or modified with the C_m factor.



In the figure above, we have column bent in single curvature, with equal end moments such that the column bends laterally by an amount δ at mid depth. The maximum total moment occurs in the column clearly will equal M plus the increased moment $P_{nt}\delta$. As a result no modification is required and C_m = 1.0. An entirely different situation is considered in the figure below, where the end moments tend to bend the member in reveres curvature. The initial maximum moment occurs at one of the ends, and we should not increase by the value of $P_{nt}\delta$ because we will be overdoing the moment magnification. The purpose of the modification factor is to modify or reduce the magnified moment.



(a) Column (b) Column moments

Modification factor is based on the rotational restraint at the member ends and on the moment gradients in the member. The AISC specification C1 includes two categories of C_m.

In category 1, the members are prevented from joint translation or sidesway, and they are not subject to transverse loading between their ends. For such a member, the modification factor is based on an elastic first order analysis.

$$C_m = 0.6 - 0.4\frac{M_1}{M_2} \quad (AISC\ equation\ C2-4)$$

In this expression $\frac{M_1}{M_2}$ is the ratio of the smaller moment to the largest moment at the ends of the unbraced length in the plane of bending under consideration. The ratio is negative if the moments cause the member to bend in single curvature, and positive if they bend the member in reversed or double curvature.

Category 2 applies to members that are subjected to transverse loadings between the joints in the plane of loading. The AISC specification states that the value of Cm for this situation may be determined by rotational analysis or by setting it conservatively equal to 1.0. The value of Cm of category 2 may be determined for various end conditions and loads by the values given in Table C-C2.1.

$P_u = P_r$ = is the required column axial load

P_{e1} = is the elastic buckling load for a braced column.

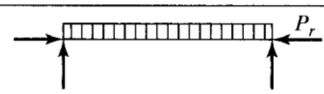
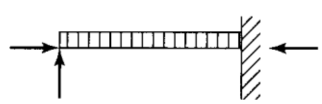
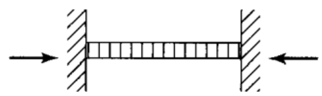
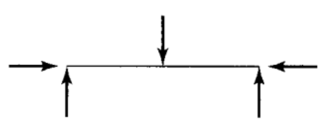
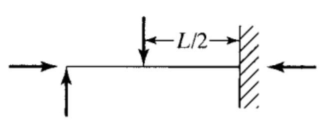
$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} \quad (\,AISC\ Equation\ C2-5)$$

- <u>Beam column in braced frame:</u>

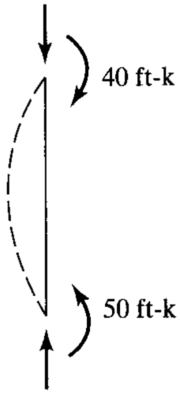
The same equations are used for member subjected to axial compression and bending as were used for member subjected to axial tension and bending. Pu is referring to compression force rather than tension force.

To analyze beam column or member subjected to both bending and axial compression, we need to make both first and second order moment analysis to obtain the bending moment. The first order moment is usually obtained by making an elastic analysis and consists for the moment M_{nt} (due to lateral loads – due to lateral translation)

Theoretically, if both the loads and frame are symmetrical M_{lt} will be zero. Similarly, if the frame is braced M_{lt} will be zero.

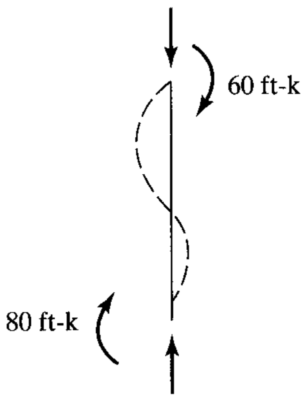
Case	ψ	C_m
	0	1.0
	−0.4	$1 - 0.4\frac{\alpha P_r}{P_{e1}}$
	−0.4	$1 - 0.4\frac{\alpha P_r}{P_{e1}}$
	−0.2	$1 - 0.2\frac{\alpha P_r}{P_{e1}}$
	−0.3	$1 - 0.3\frac{\alpha P_r}{P_{e1}}$
	−0.2	$1 - 0.2\frac{\alpha P_r}{P_{e1}}$

Source: Commentary on the Specification, Appendix 8–Table C–A–8.1, p16.1–525. June 22, 2010. "Copyright © American Institute of Steel Construction. Reprinted with permission. All rights reserved."



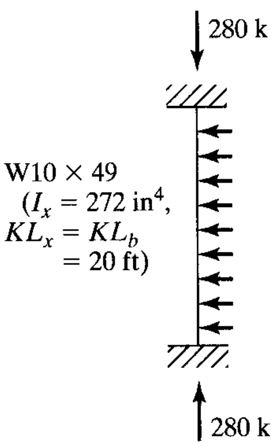
(a) No sidesway and no transverse loading.
Moments bend member in single curvature.

$$C_m = 0.6 - (0.4)\left(-\frac{40}{50}\right) = 0.92$$



(b) No sidesway and no transverse loading.
Moments bend member in reverse curvature.

$$C_m = 0.6 - 0.4\left(+\frac{60}{80}\right) = 0.30$$

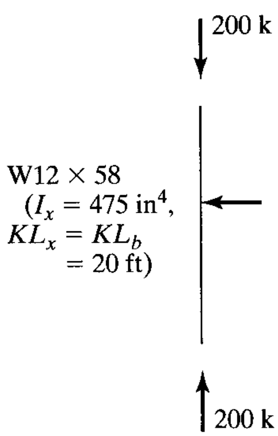


(c) Member has restrained ends and transverse loading and is bent about *x* axis.
C_m can be determined from Table 11.1 (AISC Table C-A-8.1) as follows:

$$\alpha P_r = 280 \text{ k}$$

$$P_{e1} = \frac{\pi^2 EI}{(KL)_x^2} = \frac{(\pi^2)(29 \times 10^3)(272)}{(12 \times 20)^2} = 1351 \text{ k}$$

$$C_m = 1 - 0.4\left(+\frac{280}{1351}\right) = 0.92$$



(d) Member has unrestrained ends and transverse loading and is bent about *x* axis.
C_m can be determined from Table 11.1 (AISC Table C-A-8.1).

$$\alpha P_r = 200 \text{ k}$$

$$P_{e1} = \frac{(\pi^2)(29 \times 10^3)(475)}{(12 \times 20)^2} = 2360 \text{ k}$$

$$C_m = 1 - 0.2\left(+\frac{200}{2360}\right) = 0.98$$

❖ Example 3:

A 12-ft W12 × 96 (50 ksi steel) is used as a beam-column in a braced frame. It is bent in single curvature with equal and opposite end moments and is not subjected to intermediate transverse loads. Is the section satisfactory if $P_D = 175$ k, $P_L = 300$ k, and first-order $M_{Dx} = 60$ ft-k and $M_{Lx} = 60$ ft-k?

Solution. Using a W12 × 96 ($A = 28.2$ in^2, $I_x = 833$ in^4, $\phi_b M_{px} = 551$ ft-k, $L_p = 10.9$ ft, $L_r = 46.7$ ft, $BF = 5.78$ k for LRFD).

LRFD

$$P_{nt} = P_u = (1.2)(175) + (1.6)(300) = 690 \text{ k}$$

$$M_{ntx} = M_{ux} = (1.2)(60) + (1.6)(60) = 168 \text{ ft-k}$$

For a braced frame, let $K = 1.0$

$$\therefore (KL)_x = (KL)_y = (1.0)(12) = 12 \text{ ft}$$

$$P_c = \phi_c P_n = 1080 \text{ k (AISC Table 4-1)}$$

$$P_r = P_{nt} + \beta_2 P_{lt} = 690 + 0 = 690 \text{ k}$$

$$\frac{P_r}{P_c} = \frac{690}{1080} = 0.639 > 0.2$$

$\therefore$ Must use AISC Eq. H1-1a

$$C_{mx} = 0.6 - 0.4\frac{M_1}{M_2}$$

$$C_{mx} = 0.6 - 0.4\left(-\frac{168}{168}\right) = 1.0$$

LRFD

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_1L_x)^2} = \frac{(\pi^2)(29{,}000)(833)}{(1.0 \times 12 \times 12)^2}$$

$$= 11{,}498 \text{ k}$$

$$B_{1x} = \frac{C_{mx}}{1 - \frac{\alpha P_r}{P_{e1x}}} = \frac{1.0}{1 - \frac{(1.0)(690)}{11{,}498}} = 1.064$$

$$M_{rx} = B_{1x} M_{ntx} = (1.064)(168) = 178.8 \text{ ft-k}$$

Since $L_b = 12 \text{ ft} > L_p = 10.9 \text{ ft} < L_r = 46.6 \text{ ft}$

$\therefore$ **Zone 2**

$$\phi_b M_{px} = 1.0[551 - (5.78)(12 - 10.9)] = 544.6 \text{ ft-k}$$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right)$$

$$= \frac{690}{1080} + \frac{8}{9}\left(\frac{178.8}{544.6} + 0\right) = 0.931 < 1.0 \text{ OK}$$

$\therefore$ **Section is satisfactory.**

We can use table 6-1 and the following simplified equations to solve (example 3).

$$For\ pP_r \geq 0.2, \qquad pP_r + b_x M_{rx} + b_y M_{ry} \leq 1.0\ (Equation\ 6-1)$$

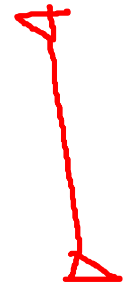
$$For\ pP_r < 0.2, \qquad \frac{1}{2} pP_r + \frac{9}{8}\left(b_x M_{rx} + b_y M_{ry}\right) \leq 1.0\ (Equation\ 6-2)$$

❖ <u>Example 4:</u>

Repeat Example 11-3, using the AISC simplified method of Part 6 of the Manual and the values for K, L, P_r and M_{rx} determined in that earlier example.

LRFD	ASD
$P_{e1x} = \dfrac{\pi^2 EI_x}{(K_1L_x)^2} = \dfrac{(\pi^2)(29{,}000)(833)}{(1.0 \times 12 \times 12)^2}$	$P_{e1x} = \dfrac{\pi^2 EI_x}{(K_1L_x)^2} = \dfrac{(\pi^2)(29{,}000)(833)}{(1.0 \times 12 \times 12)^2}$
$= 11{,}498$ k	$= 11{,}498$ k
$B_{1x} = \dfrac{C_{mx}}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \dfrac{1.0}{1 - \dfrac{(1.0)(690)}{11{,}498}} = 1.064$	$B_{1x} = \dfrac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \dfrac{1.0}{1 - \dfrac{(1.6)(475)}{11{,}498}} = 1.071$
$M_{rx} = B_{1x}M_{ntx} = (1.064)(168) = 178.8$ ft-k	$M_{rx} = (1.071)(120) = 128.5$ ft-k
Since $L_b = 12$ ft $> L_p = 10.9$ ft $< L_r = 46.6$ ft	Since $L_b = 12$ ft $> L_p = 10.9$ ft $< L_r = 46.6$ ft
∴ **Zone 2**	∴ **Zone 2**
$\phi_b M_{px} = 1.0[551 - (5.78)(12 - 10.9)] = 544.6$ ft-k	$\dfrac{M_{px}}{\Omega_b} = 1.0[367 - 3.85\,(12 - 10.9)] = 362.7$ ft-k
$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right)$	$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) = \dfrac{475}{720} + \dfrac{8}{9}\left(\dfrac{128.5}{362.7} + 0\right)$
$= \dfrac{690}{1080} + \dfrac{8}{9}\left(\dfrac{178.8}{544.6} + 0\right) = 0.931 < 1.0$ OK	$= 0.975 < 1.0$ OK
∴ **Section is satisfactory.**	∴ **Section is satisfactory.**

❖ Example 5:



A 14-ft W14 × 120 (50 ksi steel) is used as a beam-column in a braced frame. It is bent in single curvature with equal and opposite moments. Its ends are rotationally restrained, and it is not subjected to intermediate transverse loads. Is the section satisfactory if $P_D = 70$ k, and $P_L = 100$ k and if it has the first-order moments $M_{Dx} = 60$ ft-k, $M_{Lx} = 80$ ft-k, $M_{Dy} = 40$ ft-k, and $M_{Ly} = 60$ ft-k?

Solution. Using a W14 × 120 ($A = 35.3\ \text{in}^2$, $I_x = 1380\ \text{in}^4$, $I_y = 495\ \text{in}^4$, $Z_x = 212\ \text{in}^3$, $Z_y = 102\ \text{in}^3$, $L_p = 13.2$ ft, $L_r = 51.9$ ft, BF for LRFD $= 7.65$ k).

LRFD

$$P_{nt} = P_u = (1.2)(70) + (1.6)(100) = 244\ \text{k}$$

$$M_{ntx} = M_{ux} = (1.2)(60) + (1.6)(80) = 200\ \text{ft-k}$$

$$M_{nty} = M_{uy} = (1.2)(40) + (1.6)(60) = 144\ \text{ft-k}$$

For a braced frame $K = 1.0$

$$KL = (1.0)(14) = 14\ \text{ft}$$

$$P_c = \phi_c P_n = 1370\ \text{k (AISC Table 4-1)}$$

$$P_r = P_{nt} + \beta_2 P_{lt} = 244 + 0 = 244\ \text{k}$$

$$\frac{P_r}{P_c} = \frac{244}{1370} = 0.178 < 0.2$$

∴ Must use AISC Equation H1-1b

$$C_{mx} = 0.6 - 0.4\left(-\frac{200}{200}\right) = 1.0$$

$$P_{e1x} = \frac{(\pi^2)(29{,}000)(1380)}{(1.0 \times 12 \times 14)^2} = 13{,}995\ \text{k}$$

$$B_{1x} = \frac{1.0}{1 - \dfrac{(1.0)(244)}{13{,}995}} = 1.018$$

$$M_{rx} = (1.018)(200) = 203.6\ \text{ft-k}$$

$$C_{my} = 0.6 - 0.4\left(-\frac{144}{144}\right) = 1.0$$

$$P_{e1y} = \frac{(\pi^2)(29{,}000)(495)}{(1.0 \times 12 \times 14)^2} = 5020\ \text{k}$$

$$B_{1y} = \frac{1.0}{1 - \dfrac{(1.0)(244)}{5020}} = 1.051$$

$$M_{ry} = (1.051)(144) = 151.3\ \text{ft-k}$$

From AISC Table 6-1, for $KL = 14$ ft and $L_b = 14$ ft

$$p = 0.730 \times 10^{-3},\ b_x = 1.13 \times 10^{-3}$$
$$b_y = 2.32 \times 10^{-3}$$

LRFD

$$\frac{1}{2}p\,P_r + \frac{9}{8}(b_x M_{rx} + b_y M_{ry}) \le 1.0$$

$$= \frac{1}{2}(0.730 \times 10^{-3})(244)$$

$$+ \frac{9}{8}(1.13 \times 10^{-3})(203.6)$$

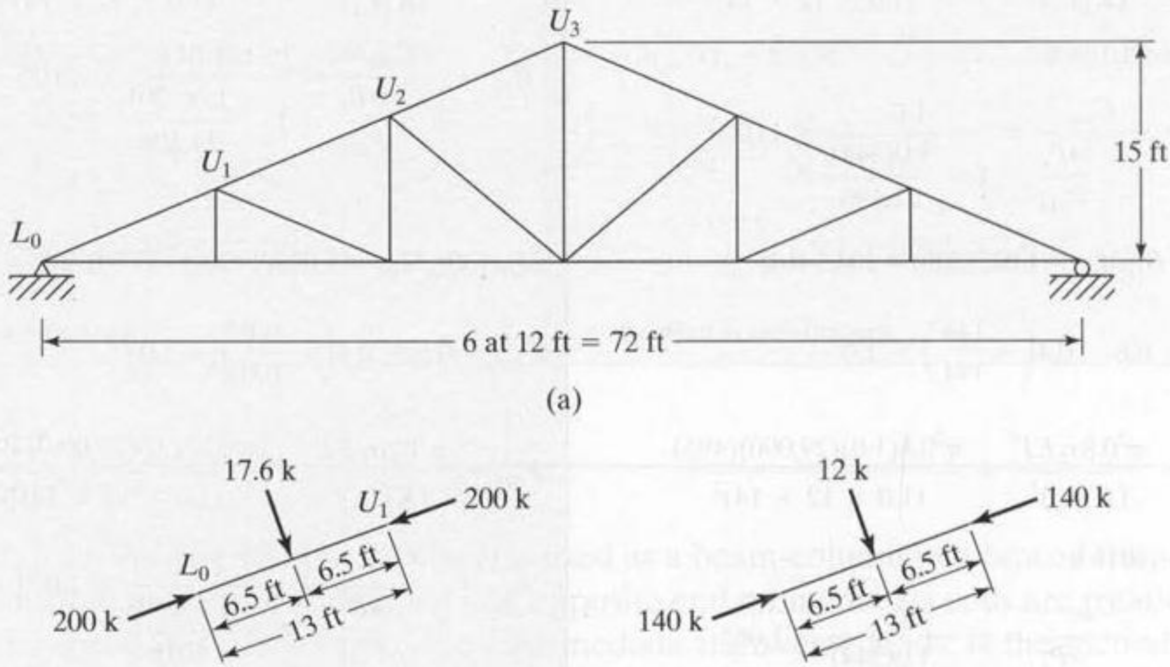
$$+ \frac{9}{8}(2.32 \times 10^{-3})(151.3)$$

$$= 0.743 \le 1.0 \quad \text{OK}$$

Section is satisfactory but perhaps overdesigned.

❖ Example 6:

For the truss shown in Fig. 11.7(a), a W8 × 35 is used as a continuous top chord member from joint L_0 to joint U_3. If the member consists of 50 ksi steel, does it have sufficient strength to resist the loads shown in parts (b) and (c) of the figure? The factored or LRFD loads are shown in part (b), while the service or ASD loads are shown in part (c). The 17.6 k and 12 k loads represent the reaction from a purlin. The compression flange of the W8 is braced only at the ends about the x-x axis, $L_x = 13$ ft, and at the ends and the concentrated load about the y-y axis, $L_y = 6.5$ ft and $L_b = 6.5$ ft.



(a)

(b) Factored loads (LRFD)

(c) Service loads (ASD)

Using a W8 × 35 ($A = 10.3\text{ in}^2$, $I_x = 127\text{ in}^4$, $r_x = 3.51$ in, $r_y = 2.03$ in, $L_P = 7.17$ ft, $\phi_b M_{Px} = 130$ ft-k, $\frac{M_{Px}}{\Omega_b} = 86.6$ ft-k, $r_x/r_y = 1.73$).

LRFD

$P_{nt} = P_u$ from figure $= 200\text{ k} = P_r$

Conservatively assume $K_x = K_y = 1.0$. In truth, the K-factor is somewhere between $K = 1.0$ (pinned–pinned end condition) and $K = 0.8$ (pinned–fixed end condition) for segment L_oU_i

$$\left(\frac{KL}{r}\right)_x = \frac{(1.0)(12 \times 13)}{3.51} = 44.44 \leftarrow$$

$$\left(\frac{KL}{r}\right)_y = \frac{(1.0)(12 \times 6.5)}{2.03} = 38.42$$

From AISC Table 4-22, $F_y = 50$ ksi

$$\phi_c F_{cr} = 38.97\text{ ksi}$$

$$\phi_c P_n = (38.97)(10.3) = 401.4\text{ k} = P_c$$

$$\frac{P_r}{P_c} = \frac{200}{401.4} = 0.498 > 0.2$$

$\therefore$ Must use AISC Eq. H1-1a

Computing P_{e1x} and C_{mx}

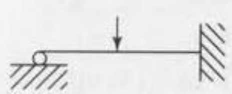
$$P_{e1x} = \frac{(\pi^2)(29{,}000)(127)}{(1.0 \times 12 \times 13)^2} = 1494\text{ k}$$

From Table 11.1

For

$$C_{mx} = 1 - 0.2\left(\frac{1.0\,(200)}{1494}\right) = 0.973$$

For

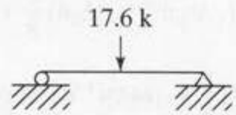


$$C_{mx} = 1 - 0.3\left(\frac{1.0\,(200)}{1494}\right) = 0.960$$

Avg $C_{mx} = 0.967$

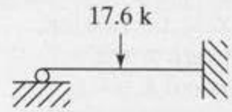
Computing M_{ux}

For



$$M_{ux} = \frac{PL}{4} = \frac{(17.6)(13)}{4} = 57.2\text{ ft-k}$$

For



$$M_{ux} = \frac{3\,PL}{16} = \frac{(3)(17.6)(13)}{16} = 42.9\text{ ft-k}$$

Avg $M_{ux} = 50.05\text{ ft-k} = M_{rx}$

$$B_{1x} = \frac{0.967}{1 - \dfrac{(1)(200)}{1494}} = 1.116$$

$M_r = (1.116)(50.05) = 55.86$ ft-k

Since $L_b = 6.5\text{ ft} < L_p = 7.17$ ft

$\therefore$ Zone ①

$\phi_b M_{nx} = 130\text{ ft-k} = M_{cx}$

Using Equation H1-1a

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0$$

$$\frac{200}{401.4} + \frac{8}{9}\left(\frac{55.86}{130} + 0\right) \le 1.0$$

$0.880 \le 1.0$ Section OK

From AISC Table 6-1

$(KL)_y = 6.5$ ft

$$(KL)_{yEQUIV} = \frac{(KL)_x}{r_x/r_y} = \frac{13}{1.73} = 7.51\text{ ft} \leftarrow$$

$p = 2.50 \times 10^{-3}$, for $KL = 7.51$ ft

$b_x = 6.83 \times 10^{-3}$, for $L_b = 6.5$ ft

$p\,P_r + b_x\,M_{rx} + b_y\,M_{ry} \le 1.0$

$= (2.50 \times 10^{-3})(200) + (6.83 \times 10^{-3})(55.86) + 0$

$= 0.882 \le 1.0$ Section OK

Section is Satisfactory.

❖ Design of Beam Column Braced or Unbraced:

The design of beam column involves a trial and error procedure. A trail section is selected by a procedure and then checked with the appropriate interaction equation. If the section does not satisfy the equation, or if it is too much on the safe side (overdesigned), another section is selected and the interaction equation is applied again.

A common method used for selecting sections to resist both moment and axial loads is the equivalent axial load or effective axial load method. In this method the axial load P_u and the bending moments M_{ux}, M_{uy} are replaced with a fictitious concentric load P_{ueq}, equivalent to approximately to the actual axial load plus the moment effect.

Equations are used to convert the bending moment into an equivalent axial load $P_{\bar{u}}$, which is added to the design axial load P_u. The total of $P_u + P_{\bar{u}}$ is equivalent or effective axial load P_{equ}, and it is used to enter the concentric column tables of part 4 of the AISC manual.

Then use table 4-1
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Page 721
Assume B1 and B2 =1
take u = 2

$$P_{equ} = P_u + M_{ux}m + M_{uy}mu$$

Given table

To apply this expression, a value of m is taken from the first approximation section of table 11-3, and u is assumed equal to 2. In applying the equation, the moments must be used in kft. The equation is solved for P_{equ}. After that a column is selected from the concentrically loaded column tables. Then the equation of P_{equ} is solved again with a revised value of m from the subsequent approximation part of the table, and the value of u is kept equal to 2.

TABLE 11.3 Preliminary Beam–Column Design F_y = 36 ksi, F_y = 50 ksi

Values of *m*														
F_y	36 ksi							50 ksi						
KL(ft)	10	12	14	16	18	20	22 and over	10	12	14	16	18	20	22 and over
1st Approximation														
All Shapes	2.0	1.9	1.8	1.7	1.6	1.5	1.3	1.9	1.8	1.7	1.6	1.4	1.3	1.2
Subsequent Approximation														
W4	3.1	2.3	1.7	1.4	1.1	1.0	0.8	2.4	1.8	1.4	1.1	1.0	0.9	0.8
W5	3.2	2.7	2.1	1.7	1.4	1.2	1.0	2.8	2.2	1.7	1.4	1.1	1.0	0.9
W6	2.8	2.5	2.1	1.8	1.5	1.3	1.1	2.5	2.2	1.8	1.5	1.3	1.2	1.1
W8	2.5	2.3	2.2	2.0	1.8	1.6	1.4	2.4	2.2	2.0	1.7	1.5	1.3	1.2
W10	2.1	2.0	1.9	1.8	1.7	1.6	1.4	2.0	1.9	1.8	1.7	1.5	1.4	1.3
W12	1.7	1.7	1.6	1.5	1.5	1.4	1.3	1.7	1.6	1.5	1.5	1.4	1.3	1.2
W14	1.5	1.5	1.4	1.4	1.3	1.3	1.2	1.5	1.4	1.4	1.3	1.3	1.2	1.2

Source: This table is from a paper in AISC *Engineering Journal* by Uang, Wattar, and Leet (1990).

❖ Example 7:

Select a trial W 12 section for both LRFD and ASD for the following data: $F_y = 50$ ksi, $(KL)_x = (KL)_y = 12$ ft, $P_{nt} = 690$ k and $M_{ntx} = 168$ ft-k for LRFD, and $P_{nt} = 475$ k

LRFD	ASD
Assume B_1 and $B_2 = 1.0$	Assume B_1 and $B_2 = 1.0$
$\therefore\ P_r = P_u = P_{nt} + B_2(P_{lt})$	$\therefore\ P_r = P_a = P_{nt} + B_2(P_{lt})$
$P_u = 690 + 0 = 690$ k	$P_a = 475 + 0 = 475$ k
and, $M_{rx} = M_{ux} = B_1(M_{ntx}) + B_2(M_{ltx})$	and, $M_{rx} = M_{ax} = B_1(M_{ntx}) + B_2(M_{ltx})$
$M_{ux} = 1.0(168) + 0 = 168$ ft-k	$M_{ax} = 1.0(120) + 0 = 120$ ft-k
$P_{ueq} = P_u + M_{ux}\,m + M_{uy}\,mu$	$P_{aeq} = P_a + M_{ax}m + M_{ay}mu$
From "1st Approximation" part of Table 11.3	From "1st Approximation" part of Table 11.3
$m = 1.8$ for $KL = 12$ ft, $F_y = 50$ ksi	$m = 1.8$ for $KL = 12$ ft, $F_y = 50$ ksi
$u = 2.0$ (assumed)	$u = 2.0$ (assumed)
$P_{ueq} = 690 + 168(1.8) + 0 = 992.4$ k	$P_{aeq} = 475 + 120(1.8) + 0 = 691.0$ k
1st trial section: W12 × 96 ($\Phi_c P_n = 1080$ k) from AISC Table 4-1	1st trial section: W12 × 96 ($P_n/\Omega_c = 720$ k) from AISC Table 4-1
From "Subsequent Approximation" part of Table 11.3, W12's	From "Subsequent Approximation" part of Table 11.3, W12's
$m = 1.6$	$m = 1.6$
$P_{ueq} = 690 + 168(1.6) + 0 = 958.8$ k	$P_{aeq} = 475 + 120(1.6) + 0 = 667.0$ k
Try W12 × 87, ($\Phi_c P_n = 981$ k > 958.8 k)	**Try W12 × 96,** ($P_n/\Omega_c = 720$ k > 667.0 k)

Note: These are trial sizes. B_1 and B_2, which were assumed, must be calculated and these W12 sections checked with the appropriate interaction equations.

❖ Example 8:

Select a trial W section for both LRFD and ASD for an unbraced frame and the following data: $F_y = 50$ ksi, $(KL)_x = (KL)_y = 10$ ft.

For LRFD: $P_{nt} = 175$ k and $P_{lt} = 115$ k, $M_{ntx} = 102$ ft-k and $M_{ltx} = 68$ ft-k, $M_{nty} = 84$ ft-k and $M_{lty} = 56$ ft-k

For ASD: $P_{nt} = 117$ k and $P_{lt} = 78$ k, $M_{ntx} = 72$ ft-k and $M_{ltx} = 48$ ft-k, $M_{nty} = 60$ ft-k and $M_{lty} = 40$ ft-k

Solution

LRFD	ASD
Assume B_{1x}, B_{1y}, B_{2x} and $B_{2y} = 1.0$	Assume B_{1x}, B_{1y}, B_{2x} and $B_{2y} = 1.0$
$\therefore\ P_r = P_u = P_{nt} + B_2(P_{lt})$	$\therefore\ P_r = P_a = P_{nt} + B_2(P_{lt})$
$P_u = 175 + 1.0(115) = 290$ k	$P_a = 117 + 1.0(78) = 195$ k
and, $M_{rx} = M_{ux} = B_{1x}(M_{ntx}) + B_{2x}(M_{ltx})$	and, $M_{rx} = M_{ax} = B_{1x}(M_{ntx}) + B_{2x}(M_{ltx})$
$M_{ux} = 1.0(102) + 1.0(68) = 170$ ft-k	$M_{ax} = 1.0(72) + 1.0(48) = 120$ ft-k
and, $M_{ry} = M_{uy} = B_{1y}(M_{nty}) + B_{2x}(M_{lty})$	and, $M_{ry} = M_{ay} = B_{1y}(M_{nty}) + B_{2x}(M_{lty})$
$M_{uy} = 1.0(84) + 1.0(56) = 140$ ft-k	$M_{ay} = 1.0(60) + 1.0(40) = 100$ ft-k
$P_{ueq} = P_u + M_{ux}m + M_{uy}mu$	$P_{aeq} = P_a + M_{ax}\,m + M_{ay}mu$
From "1st Approximation" part of Table 11.3	From "1st Approximation" part of Table 11.3
$m = 1.9$ for $KL = 10$ ft, $F_y = 50$ ksi	$m = 1.9$ for $KL = 10$ ft, $F_y = 50$ ksi
$u = 2.0$ (assumed)	$u = 2.0$ (assumed)
$P_{ueq} = 290 + 170(1.9) + 140(1.9)(2.0) = 1145$ k	$P_{aeq} = 195 + 120(1.9) + 100(1.9)(2.0) = 803$ k
1st trial section from Table 4.1:	1st trial section from Table 4.1:
W14 → W14 × 99 ($\Phi_c P_n = 1210$ k)	W14 → W14 × 99 ($P_n/\Omega_c = 807$ k)
W12 → W12 × 106 ($\Phi_c P_n = 1260$ k)	W12 → W12 × 106 ($P_n/\Omega_c = 838$ k)
W10 → W10 × 112 ($\Phi_c P_n = 1280$ k)	W10 → W10 × 112 ($P_n/\Omega_c = 851$ k)
Suppose we decide to use a W14 section:	Suppose we decide to use a W14 section:
From "Subsequent Approximation" part of Table 11.3, W14's	From "Subsequent Approximation" part of Table 11.3, W14's
$m = 1.5$	$m = 1.5$
$P_{ueq} = 290 + 170(1.5) + 140(1.5)(2.0) = 965$ k	$P_{aeq} = 195 + 120(1.5) + 100(1.5)(2.0) = 675$ k
Try W14 × 90, ($\Phi_c P_n = 1100$ k $>$ 965 k)	**Try W14 × 90,** ($P_n/\Omega_c = 735$ k $>$ 675 k)

Note: These are trial sizes. B_{1x}, B_{1y}, B_{2x} and B_{2y}, which were assumed, must be calculated and these W14 sections checked with the appropriate interaction equations.

❖ Example 9:

Select the lightest W12 section for both LRFD and ASD for the following data: $F_y = 50$ ksi, $(KL)_x = (KL)_y = 12$ ft, $P_{nt} = 250$ k, $M_{ntx} = 180$ ft-k and $M_{nty} = 70$ ft-k for LRFD, and $P_{nt} = 175$ k, $M_{ntx} = 125$ ft-k and $M_{nty} = 45$ ft-k for ASD. $C_b = 1.0$, $C_{mx} = C_{my} = 0.85$.

LRFD

Assume $B_{1x} = B_{1y} = 1.0$, B_2 not required

$\therefore\ P_r = P_u = P_{nt} + B_2(P_{lt})$

$P_u = 250 + 0 = 250$ k

and, $M_{rx} = M_{ux} = B_1(M_{ntx}) + B_2(M_{ltx})$

$M_{ux} = 1.0(180) + 0 = 180$ ft-k

and, $M_{ry} = M_{uy} = B_1(M_{nty}) + B_2(M_{lty})$

$M_{uy} = 1.0(70) + 0 = 70$ ft-k

$P_{ueq} = P_u + M_{ux}m + M_{uy}mu$

From "Subsequent Approximation" part of Table 11.3, W12's

$m = 1.6$

$u = 2.0$ (assumed)

$P_{ueq} = 250 + 180(1.6) + 70(1.6)(2.0) = 762$ k

Try W12 × 72, $(\Phi_c P_n = 806\text{ k} > 762\text{ k})$ **from Table 4.1**

From Table 6.1 for $KL = 12$ ft and $L_b = 12$ ft

$p = 1.24 \times 10^{-3}$, $b_x = 2.23 \times 10^{-3}$, $b_y = 4.82 \times 10^{-3}$

$P_r/\Phi_c P_n = 250/806 = 0.310 > 0.2$
Use modified Equation H1-1a.

$1.24 \times 10^{-3}(250) + 2.23 \times 10^{-3}(180) + 4.82 \times 10^{-3}(70) = 1.049 > 1.0$ N.G.

Try W12×79, $(\Phi_c P_n = 887\text{ k} > 762\text{ k})$ from Table 4.1

From Table 6.1 for $KL = 12$ ft and $L_b = 12$ ft

$p = 1.13 \times 10^{-3}$, $b_x = 2.02 \times 10^{-3}$, $b_y = 4.37 \times 10^{-3}$

$1.13 \times 10^{-3}(250) + 2.02 \times 10^{-3}(180) + 4.37 \times 10^{-3}(70) = 0.952 < 1.0$ OK

LRFD

Check $B_{1x} = B_{1y} = 1.0$

$$P_{e1x} = \frac{\pi^2 EI^*}{(K_1 L)^2} = \frac{\pi^2(29{,}000)(662)}{(1.0 \times 12 \times 12)^2} = 9138\text{ k}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \frac{0.85}{1 - \dfrac{1.0(250)}{9138}} = 0.87 < 1;$$

$B_{1x} = 1.0$, OK

$$P_{e1y} = \frac{\pi^2 EI^*}{(K_1 L)^2} = \frac{\pi^2(29{,}000)(216)}{(1.0 \times 12 \times 12)^2} = 2981\text{ k}$$

$$B_{1y} = \frac{C_{my}}{1 - \dfrac{\alpha P_r}{P_{e1y}}} = \frac{0.85}{1 - \dfrac{1.0(250)}{2981}} = 0.93 < 1;$$

$B_{1y} = 1.0$, OK

With $B_{1x} = B_{1y} = 1.0$, section is sufficient based on previous check using modified Equation H1-1a.

Will perform additional check using Equation H1-1a:

For W12 × 79, $\Phi M_{px} = 446$ ft-k, $L_p = 10.8$ ft, $L_r = 39.9$ ft

$BF = 5.67$, $L_b = 12$ ft, Zone 2, $C_b = 1.0$, $\Phi M_{py} = 204$ ft-k

$\Phi M_{nx} = C_b\,[\Phi M_{px} - BF(L_b - L_p)]\ \Phi M_{px}$

$\Phi M_{nx} = 1.0\,[446 - 5.67(12 - 10.8)] = 439.2$ ft-k

$\Phi M_{ny} = \Phi M_{py} = 204$ ft-k

Equation H1-1a:

$$\frac{250}{887} + \frac{8}{9}\left(\frac{180}{439.2} + \frac{70}{204}\right) = 0.951 < 1.0 \quad \text{OK}$$

Use W12 × 79, LRFD.

Chapter 7: Analysis and design of Beams for Moments

❖ Introduction to flexural memebr (beams):

i. Types of Beams:

Beams are usually said to be members that support transverse loads. They are probably thought of as used in horizontal positions and subjected to gravity or vertical loads. Among the many types of beams are joists, lintels, spandrels, and floor beams.

- Joists are the closely spaced beams supporting the floors and roofs of buildings.
- Lintels are the beams over openings in masonry walls, such as windows and doors.
- Spandrel beams support the exterior walls of buildings and perhaps part of the floor and hallway loads.
- Stringers are the beams in bridge floors running parallel to the roadway
- Floor beams are the larger beams in many bridge floors, which are perpendicular to the roadway of the bridge.

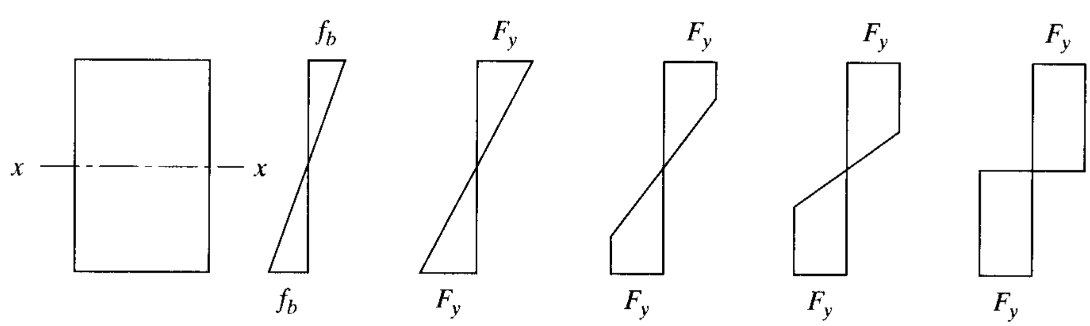
ii. Section Used as Beams:

The W shapes will normally prove to be the most economical beam sections, and they have largely replaced channels and S sections for beam usage. Channels are sometimes used for beams subjected to light loads and in places where clearances available required narrow flanges. They have very little resistance to lateral forces and need to be braced. The W shapes have more steel concentrated in their flanges than do S beams and thus have larger moment of inertia and resisting moment for the same weights. Another common type of beam section is the bar joist. This type of section, which is used to support floor and roof slabs, is actually a light shop-fabricated parallel chord truss. It is particularly economical for long spans and light loads.

iii. Bending Stresses:

For an introduction to bending stresses, the rectangular beam and stress diagram in the figure below are considered. If the beam is subjected to some bending moment, the stress at any point may be computed with the usual flexural formula:

$$f_b = \frac{Mc}{I}$$



This expression is applicable only when the maximum computed stress in the beam is below the elastic limit. The value (I/c) is a constant for a particular section and is known as the section modulus (S). The flexural formula may then be written as follows:

$$f_b = \frac{Mc}{I} = \frac{M}{S}$$

Initially when the moment is applied to the beam, the stress will very linearly from the neutral axis to the extreme fibers. If the moment is increased, there will continue to be a linear variation of stress until the yield stress is reached in the outmost fibers. The yield moment of a cross section is defined as the moment that will produce the yield stress in the outmost fiber of the section.

If the moment in a ductile steel beam is increased beyond the yield moment, the outmost fibers that had previously been stressed to their yield stress will continue to have the same stress, but will yield, and the duty of providing the necessary additional resisting moment will fall on the fibers nearer to the neutral axis. This process will continue, with more and more parts of the beam cross section stressed to the yield stress, until finally a full plastic distribution is approached. When the stress distribution has reached this stage, a plastic hinge is said to have formed, because no additional moment can be resisted at the section. Any additional moment applied at the section will cause the beam to rotate.

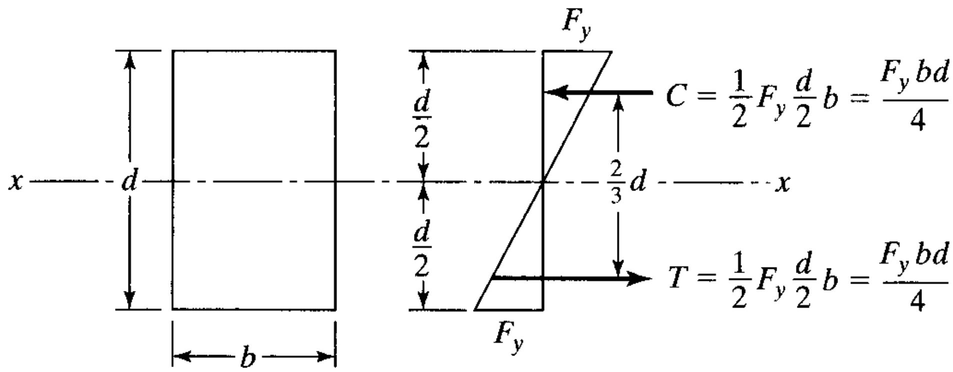
The plastic moment is the moment that will produce full plasticity in a member cross section and create a plastic hinge. The ratio of the plastic moment M_p to the yield moment M_y is called the shape factor. The shape factor equals to 1.5 for rectangular sections and varies from about 1.10 to 1.20 for standard rolled-beam sections.

iv. Elastic Design:

Until recent years, almost all steel beams were designed on the basis of elastic theory. The maximum load that a structure could support was assumed to equal the load that first causes a stress somewhere in the structure equal to the yield stress of the material. The members were designed so that computed bending stresses for service loads did not exceed the yield stress divided by a safety factor. Engineering structures have been designed for many decades by this method, with satisfactory results. The design profession, however, has long been aware that ductile material members do not fail until a great deal of yielding occurs after the yield stress is first reached. This mean members have greater margins of safety against collapse than the elastic theory would seem to indicate.

v. The Plastic Modulus:

The yield moment M_y equals the yield stress times the elastic modulus. The elastic modulus I/c or $bd^2/6$ for a rectangular section, and the yield moment equals $F_y\ bd^2/6$. This value can be obtained by considering the resisting internal couple shown in the figure below:

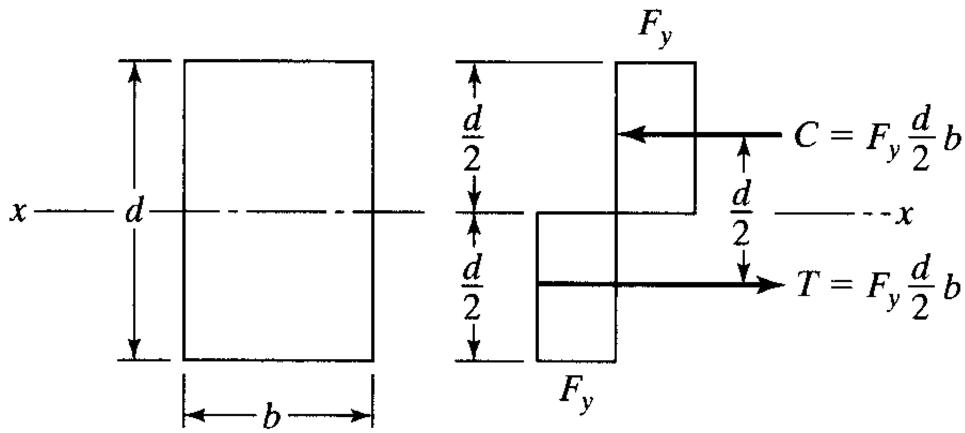


The resisting moment equals T or C times the lever arm between them:

$$M_y = \frac{F_y bd}{4} * \frac{d}{2} = \frac{F_y bd^2}{6}$$

The elastic modulus can again be seen to equal $bd^2/6$ for a rectangular beam. The resisting moment at full plastic can be determined in a similar manner. The result is called plastic moment M_p. it is also the nominal moment of the section M_n. this plastic or nominal moment equals T or C times the lever arm between them.

$$M_p = M_p = \frac{F_y bd}{2} * \frac{d}{2} = \frac{F_y bd^2}{4}$$



The plastic moment is said to equal the yield stress times the plastic section modulus. From the forgoing expression for a rectangular section, the plastic section modulus Z can be seen to equal $bd^2/4$. The shape factor, which equal $M_p/M_y = F_yZ/F_yS$, or $Z/S = 1.5$ for rectangular section.

Note: the total internal compression must equal the total internal tension. In the plastic condition, the areas above and below the plastic neutral axis must be equal.

❖ Example 1:

Determine M_y, M_n, and Z for the steel tee beam shown in the figure below. Also, calculate the shape factor and the nominal load w_n that can be place on the beam for a 12 ft simple span, $F_y = 50$ ksi.

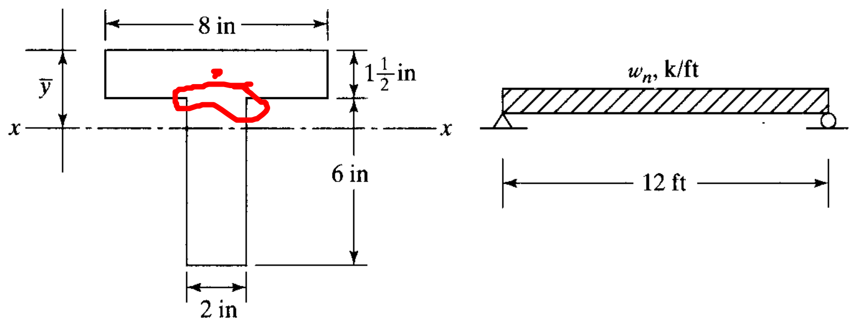


FIGURE 8.5

Solution. Elastic calculations:

$$A = (8\text{ in})\left(1\frac{1}{2}\text{ in}\right) + (6\text{ in})(2\text{ in}) = 24\text{ in}^2$$

$$\bar{y} = \frac{(12\text{ in})(0.75\text{ in}) + (12\text{ in})(4.5\text{ in})}{24\text{ in}^2} = 2.625\text{ in from top of flange}$$

$$I = \frac{1}{12}(8\text{ in})(1.5\text{ in})^3 + (8\text{ in})(1.5\text{ in})(1.875\text{ in})^2 + \frac{1}{12}(2\text{ in})(6\text{ in})^3 + (2\text{ in})(6\text{ in})(1.875\text{ in})^2 = 122.6\text{ in}^4$$

$$S = \frac{I}{c} = \frac{122.6\text{ in}^4}{4.875\text{ in}} = 25.1\text{ in}^3$$

$$M_y = F_y S = \frac{(50\text{ ksi})(25.1\text{ in}^3)}{12\text{ in/ft}} = 104.6\text{ ft-k}$$

Plastic calculations (plastic neutral axis is at base of flange):

$$Z = (12\text{ in}^2)(0.75\text{ in}) + (12\text{ in}^2)(3\text{ in}) = 45\text{ in}^3$$

$$M_n = M_p = F_y Z = \frac{(50\text{ ksi})(45\text{ in}^3)}{12\text{ in/ft}} = 187.5\text{ ft-k}$$

$$\text{Shape factor} = \frac{M_p}{M_y}\ \text{or}\ \frac{Z}{S} = \frac{45\text{ in}^3}{25.1\text{ in}^3} = 1.79$$

$$M_n = \frac{w_n L^2}{8}$$

$$\therefore \quad w_n = \frac{(8)(187.5 \text{ ft-k})}{(12 \text{ ft})^2} = 10.4 \text{ k/ft}$$

The value of the plastic section moduli for the standard steel beam sections are tabulated in table 3-2 of the AISC manual, W shape Selection by Z_x.

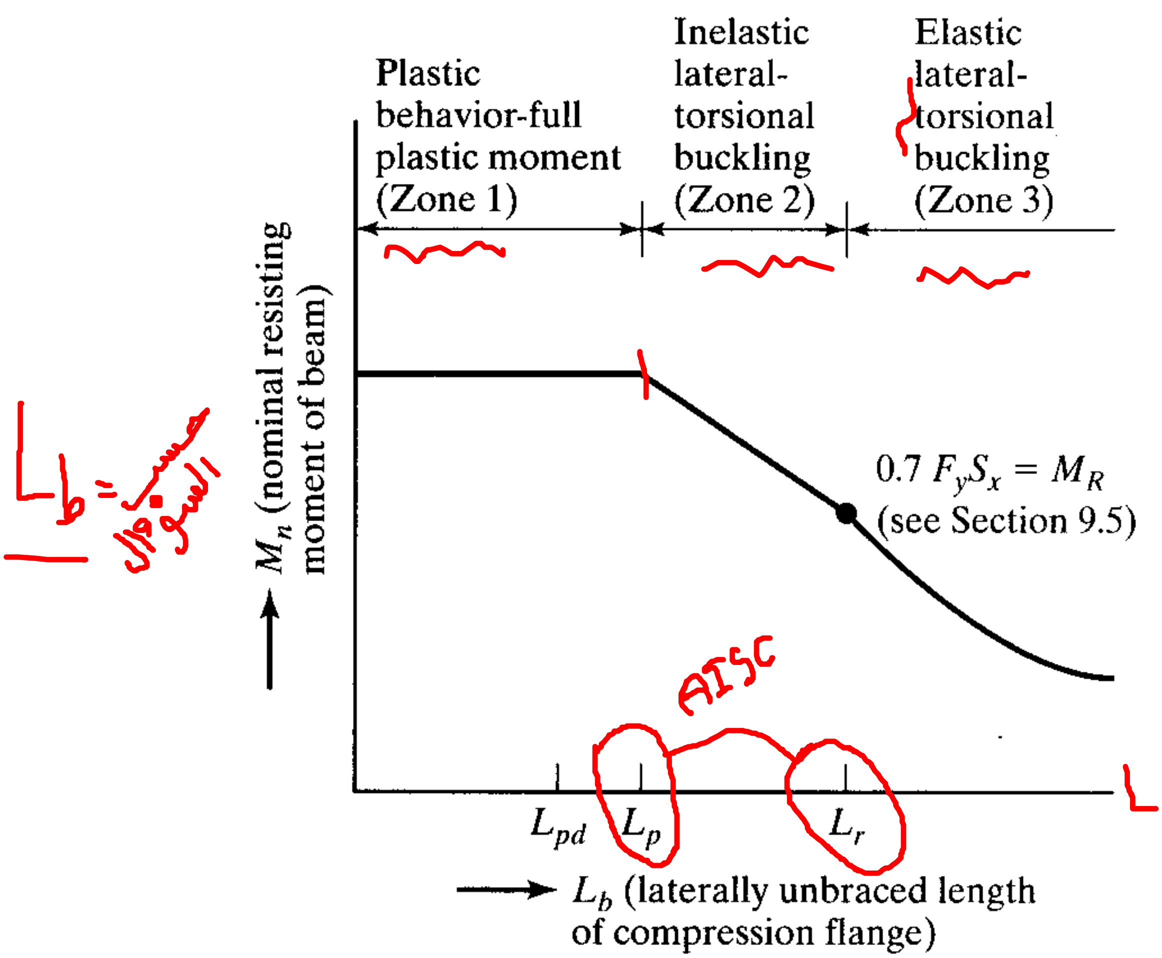
❖ Design of beam for moment:

If gravity load is applied to along simple supported beam the beam will bend downward, and its upper part will be placed in compression and will act as a compression member. The cross section of this column will consist of the portion of the beam cross section above the neutral axis. For the usual beam, the column will have a much smaller moment of inertia about its y or vertical axis than about its x axis. If nothing is done to brace it perpendicular to the y axis, it will buckle laterally at a much smaller load that would otherwise have been required to produce a vertical failure.

Lateral buckling will not occur if the compression flange of the member is braced laterally or if twisting of the beam is prevented at frequent intervals. The buckling moment of a series of compact ductile steel beam with different lateral bracing will be discussed in this chapter. We will look at beams as follows:

1. First the beams will be assumed to have continuous lateral bracing for their compression flange.
2. Next the beams will be assumed to be braced at short intervals.
3. Then the beams will be assumed to be braced laterally at large intervals.

A typical curve showing the buckling moments of one these beams with varying unbraced lengths is presented.



❖ Plastic Behavior (Zone 1)

- If we were to take a compact beam whose compression flange is continuously braced laterally, we would find that we could load it until its full plastic moment M_p is reached at some point or points.
- If we take one of these compact beams and provide closely spaced lateral spacing for its compression flange, we will find that we can still load it until the plastic moment is achieved if the spacing between the bracing does not exceed a certain value, called L_p. Most beam fall in Zone 1.

❖ Inelastic buckling (Zone 2)

- If we now increase the spacing between the lateral bracing, the section may be loaded until some, but not all, of the compression fibers are stressed to F_y. That means, in this zone we can bend the member until the yield strain is reached in some, but not all, of its compression elements before buckling occurs. This is referring to as inelastic buckling.
- As we increase the unbraced length, we will find that the moment the section resist will decrease, until finally it will buckle before the yield stress is reached anywhere in the cross section. The maximum

unbraced length (L_r) at which we can still reach F_y at one point is the end of the inelastic range.

❖ Elastic buckling (Zone 3)

- If the unbraced length is greater than L_r, the section will buckle elastically before the yield stress is reached anywhere. As the unbraced length is further increased, the buckling moment becomes smaller and smaller.

❖ Yielding behavior – full plastic moment, Zone 1

If the unbraced length L_b of a compression flange of compact I or C shaped section does not exceed L_p (if elastic analysis being used) or L_{pd} (if plastic analysis being used), then the member's bending strength about its major axis may be determined as follows:

$$M_n = M_p = F_y Z \quad (LRFD\ Equation\ F2-1)$$

$$\varphi_b M_n = \varphi_b M_p = \varphi_b F_y Z \quad (\varphi_b = 0.9)$$

When an elastic analysis approach is used to establish member force, L_b may not exceed the value L_p to follow if M_n is equal F_yZ.

$$L_p = 1.76\, r_y \sqrt{\frac{E}{F_y}} \quad (AISC\ Equation\ F2-5)$$

When a plastic analysis is used to establish member forces, L_b (which is defined as the lateral unbraced length of the compression flange at plastic hinge locations associated with failure mechanisms) may not exceed the value L_{pd} to follow if M_n is to equal F_yZ.

$$L_{pd} = \left[0.12 + 0.076\left(\frac{M1}{M2}\right)\right]\left(\sqrt{\frac{E}{F_y}}\right) r_y \ (AISC\ Appendix\ Equation\ A\_1\_5\ )$$

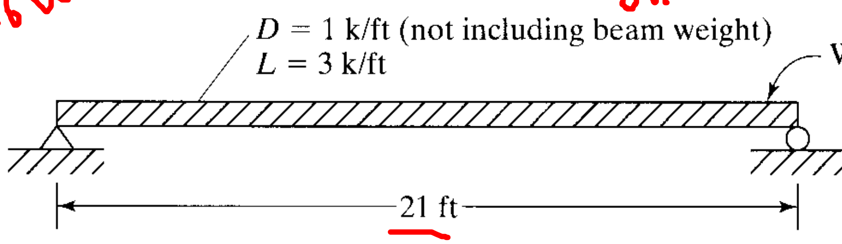
In this expression, M1 is the smaller moment at the end of the unbraced length of the beam and M2 is the larger moment at the end of the unbraced length, and the ratio M1/M2 is positive when the moments cause the member to be bent in double curvature and negative if they bend it in single curvature.

- Design of beams, Zone 1

Included in the items those needs to be considered in beam design are moments, shears, deflections, lateral bracing for the compression flanges, and others. Beam will probably be selected that provide sufficient design moment capacity $Ø_bM_n$ and then checked to see if any of the other items are critical. The factored moment will be computed, and a section having that much design moment capacity will be initially selected from the AISC manual, part 3 table 3-2. From this table, steel shapes having sufficient plastic moduli to resist certain moments can quickly be selected.

❖ Example 2:

If the compact and laterally braced section shown in the figure below sufficiently strong to support the given loads if F_y = 50 ksi? Check the beam with the LRFD method.



1+0.044

$$D.L = 1\frac{k}{ft}$$

$$L.L = 3\frac{k}{ft}$$

$$L_b = 0$$

$$W_u = 1.2\,D.L + 1.6\,L.L = 1.2 * 1 + 1.6 * 3 = 6\;k/ft$$

$$M_u = \frac{W_u * L^2}{8} = \frac{6 * 21^2}{8} = 330.75\;kft$$ العزم المسلط

$M_u \leq \varphi_b M_n = \varphi_b F_y Z$ =0.9*50*95.4=4239k-in

Table 3-2=358 k-ft

Select W21 to carry the applied load

$$\therefore Z_{req} = \frac{M_u}{\varphi_b F_y} = \frac{330.75 * 12}{0.9 * 50} = 88.2\ in^3$$

From table 3-2: Try W 21*44 (Z_x = 95.4 in^3 > 88.2 in^3)

$$W_u = 1.2\ D.L + 1.6\ L.L = 1.2 * (1 + 0.044) + 1.6 * 3 = 6.0528\ k/ft$$

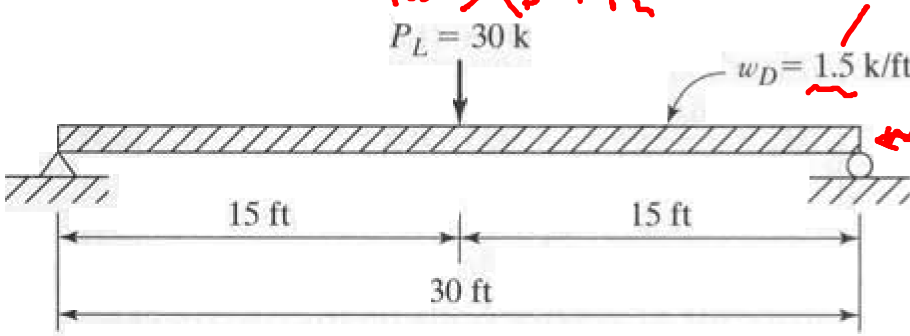
$$M_u = \frac{W_u * L^2}{8} = \frac{6.0528 * 21^2}{8} = 333.6606\ kft$$

$$Z_{req} = \frac{M_u}{\varphi_b F_y} = \frac{333.6606 * 12}{0.9 * 50} = 88.97616\ in^3 < 95.4\ in^3$$

$$\therefore Use\ W\ 21 * 44$$

❖ Example 3:

Select a beam section by using the LRFD method for the span and loading in the figure below, assuming full lateral support is provided for the compression flange by the floor slab above and F_y = 50 ksi.



$$D.L = 1.5\ \frac{k}{ft}$$

$$L.L = 30\ k$$

$$L_b = 0$$

$$W_u = 1.2\ D.L = 1.2 * 1.5 = 1.8\frac{k}{ft}$$

$$P_u = 1.6 * L.L = 1.6 * 30 = 48\ k$$

$$M_u = \frac{W_u * L^2}{8} + \frac{P_u * L}{4} = \frac{1.8 * 30^2}{8} + \frac{48 * 30}{4} = 562.5\ kft$$

$$M_u \le \varphi_b M_n = \varphi_b F_y Z$$

$$\therefore Z_{req} = \frac{M_u}{\varphi_b F_y} = \frac{562.5 * 12}{0.9 * 50} = 150\ in^3$$

From table 3-2: Try W 24*62 ($Z_x = 153\ in^3 > 150\ in^3$)

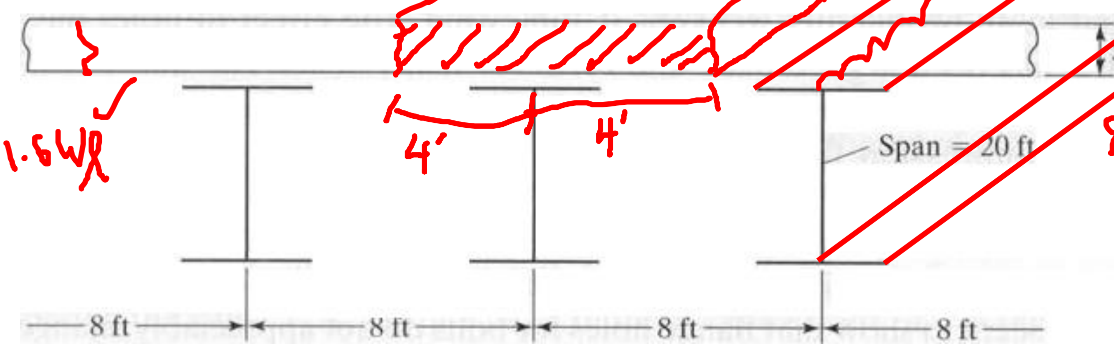
$$W_u = 1.2\ D.L = 1.2 * (1.5 + 0.062) = 1.8744\ \frac{k}{ft}$$

$$M_u = \frac{W_u * L^2}{8} + \frac{P_u * L}{4} = \frac{1.8744 * 30^2}{8} + \frac{48 * 30}{4} = 570.87\ kft$$

$$Z_{req} = \frac{M_u}{\varphi_b F_y} = \frac{570.87 * 12}{0.9 * 50} = 152.232\ in^3 < 153\ in^3\ \therefore Use\ W\ 24 * 62$$

❖ Example 4:

The 5 in reinforced concrete slab shown in the figure below is to be supported with steel W sections 8 ft on centers. The beam which will span 20 ft, are assumed to be simple supported. If the concrete slab is designed to support a live load of 100 psf determine the lightest steel sections required to support the slab by the LRFD procedure. It is assumed that the compression flange of the beam will be fully supported laterally by the concrete slab. The concrete weights 150 Ib/ft^3. F_y = 50 ksi.



$$Slab\ wt = \frac{5}{12} * 150 * 8 = 500\frac{Ib}{ft} = D.L$$

$$L.L = 8 * 100 = 800\frac{Ib}{ft}$$

$$L_b = 0$$

$$W_u = 1.2\ D.L + 1.6\ L.L = 1.2 * 500 + 1.6 * 800 = \frac{1880Ib}{ft} = 1.88\ k/ft$$

$$M_u = \frac{W_u * L^2}{8} = \frac{1.88 * 20^2}{8} = 94\ kft$$

$$M_u \le \varphi_b M_n = \varphi_b F_y Z$$

$$\therefore Z_{req} = \frac{M_u}{\varphi_b F_y} = \frac{94 * 12}{0.9 * 50} = 25.0667\ in^3$$

From table 3-2, Try W 10*22 ($Z_x = 26\ in^3 > 25.0667\ in^3$)

$$W_u = 1.2\ D.L + 1.6\ L.L = 1.2 * (500 + 22) + 1.6 * 800 = 1906.4 \frac{Ib}{ft}$$

$$= 1.9064\ k/ft$$

$$M_u = \frac{W_u * L^2}{8} = \frac{1.9064 * 20^2}{8} = 95.32\ kft$$

$$Z_{req} = \frac{M_u}{\varphi_b F_y} = \frac{95.32 * 12}{0.9 * 50} = 25.4187\ in^3 < 26\ in^3$$

$$\therefore Use\ W\ 10 * 22$$

❖ <u>Holes in beams:</u>

It is often necessary to have holes in steel beams. They are required for the installation of bolts and sometimes for pipes, ducts, etc. If at all possible, these types of holes should be completely avoided. When necessary, they should be placed through the web if the shear is small and through the flange if the moment is small and the shear is large.

If we have bolts holes in the compression flange only and they are filled with bolts, we do not need to consider any corrections.

The flexural strength of beams with holes in their tension flanges are predicted by comparing the value (F_yA_{fg}) with (F_uA_{fn}). In these expressions, A_{fg} is the gross area of the tension flange while A_{fn} is the net tension flange area after the holes are subtracted. In the expressions given herein for computing M_n, there is term Y_t, which is called the hole reduction coefficient. It is value is taken as 1.0 if $F_y/F_u \leq 0.8$. For cases when the ratio of F_y/F_u is > 0.8, Y_t is taken as 1.1.

a. If $F_uA_{fn} \geq Y_t F_yA_{fg}$, the limit state of tension rupture does not apply and there is no reduction in M_n because of the holes.

b. If $F_uA_{fn} < Y_t F_yA_{fg}$, the nominal flexural strength of the member at the holes is to be determined by the flowing expression, in which S_x is the section modulus of the member:

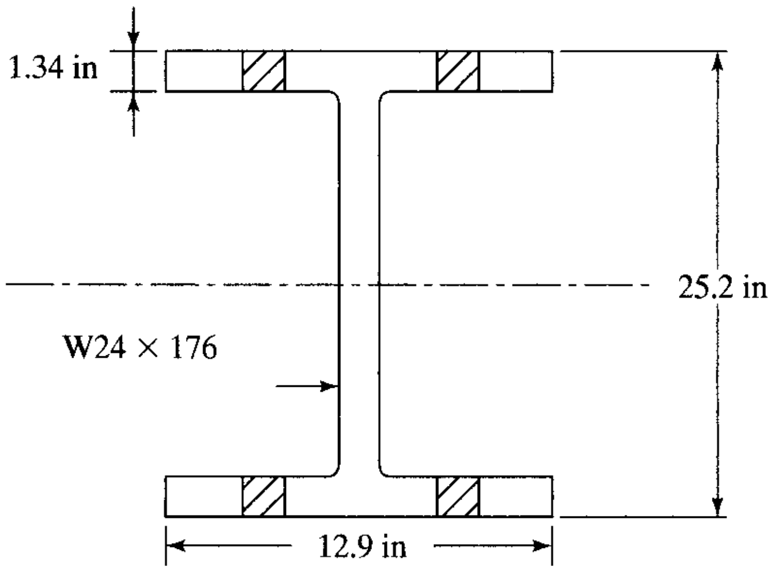
$$M_n = \frac{F_uA_{fn}}{A_{fg}} * S_x \quad (AISC\ Equation\ F13-1)$$

❖ <u>Example 5:</u>

Determine $\emptyset_bM_n$ for the W24x176 (F_y = 50 ksi, F_u = 65 ksi) beam, shown in the figure below, for the following situations:

a. Using the AISC Specification and assume two lines of 1-in bolts in standard holes in each flange.

b. Using the AISC Specification and assume four lines of 1-in bolts in standard holes in each flange.



Using W24x176 (bf = 12.9 in, t_f = 1.34 in, S_x = in^3)

a. $$A_{fg} = b_f t_f = 12.9 * 1.34 = 17.286\ in^2$$

$$A_{fn} = 17.286 - 2 * \left(1\frac{1}{16} + \frac{1}{16}\right) * 1.34 = 14.271\ in^2$$

$$F_u A_{fn} = 65 * 14.271 = 927.615\ k$$

$$\frac{F_y}{F_u} = \frac{50}{65} = 0.77 < 0.8\ \therefore Y_t = 1.0$$

$$Y_t F_y A_{fg} = 1 * 50 * 17.286 = 864.3\ k$$

$$F_u A_{fn} = 927.615\ k > Y_t F_y A_{fg} = 864.3\ k$$

$$\therefore Tensile\ rupture\ does\ not\ apply\ and\ \varphi_b M_{px}$$

$$= 1920\ kft\ (from\ AISC\ Table\ 3-2)$$

b.

$$A_{fn} = 17.286 - 4 * \left(1\frac{1}{16} + \frac{1}{16}\right) * 1.34 = 11.256\ in^2$$

$$\frac{F_y}{F_u} = \frac{50}{65} = 0.77 < 0.8\ \therefore Y_t = 1.0$$

$$F_u A_{fn} = 65 * 11.256 = 731.64\ k$$

$$Y_t F_y A_{fg} = 1 * 50 * 17.286 = 864.3\ k$$

$$F_u A_{fn} = 927.615\ k < Y_t F_y A_{fg} = 864.3\ k$$

$$\therefore Tensile\ rupture\ expression\ does\ apply$$

$$M_n = \frac{F_u A_{fn}}{A_{fg}} * S_x = \frac{65 * 11.256 * 450}{17.286} = 19046.51163\ k\ in$$

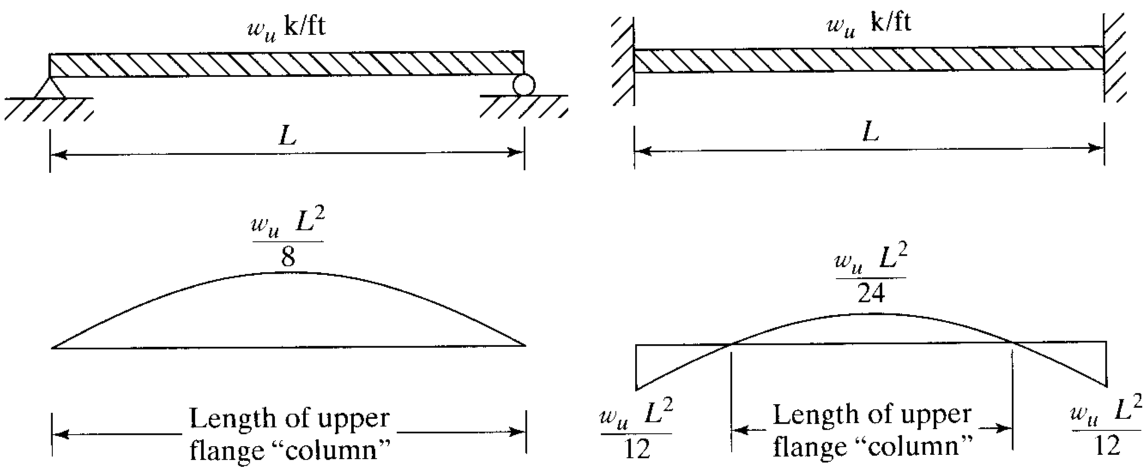
$$= \frac{19046.51163\ k\ in}{12} = 1587.2093\ k\ ft$$

$$\varphi_b M_n = 0.9 * 1587.2093 = 1428.4884\ k\ ft$$

❖ Bending coefficients:

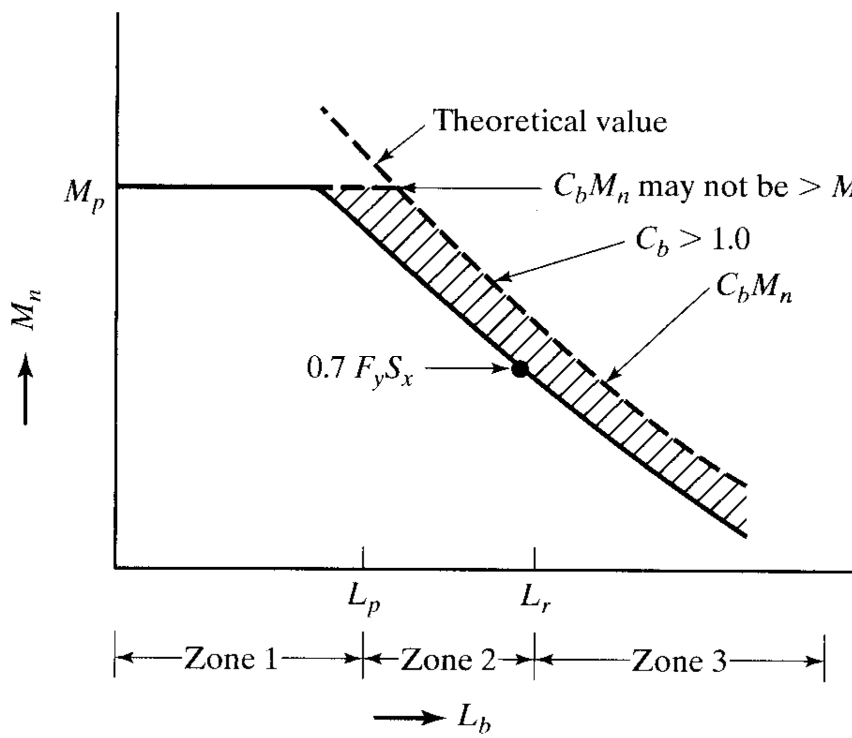
In the formulas for inelastic and elastic buckling, we will use a term C_b, called the lateral-torsional buckling modification factor for nonuniform moment diagrams, when both ends of the unsupported segment are braced.

As we can see that the moment in the unbraced beam of part (a) of the figure below causes a worse compression flange situation than does the moment in the unbraced beam of part (b). For one reason, the upper flange of the beam in part (a) is in compression for its entire length, while in (b) the length of the upper flange that is in compression is much less.



(a) Single curvature (b) Reverse curvature

For the simply supported beam of part (a) of the figure, $C_b = 1.14$, while for the beam of part (b), $C_b = 2.38$. The basic moment capacity equations for Zone 2 and 3 were developed for laterally unbraced beam subject to single curvature, with C_b = 1.0. Frequently, beams are not bent in single curvature, with the result that they can resist more moment. To handle this situation, the AISC Specification provides moment or C_b coefficient larger than 1.0 that are to be multiplied by the computed M_n values. The value of M_n Multiplied by C_b may not be larger than the plastic M_n of Zone 1, which is equal to F_yZ



The value of C_b for singly symmetric members in single curvature and all doubly symmetric members is determined from the expression to follow in which M_{max} is the largest moment in an unbraced segment of a beam, while M_A, M_B, and M_C are, respectively, the moments at the ¼ point, ½ point, and ¾ point in the segment.

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \quad (AISC\ Equation\ F1-1)$$

C_b is equal 1.0 for cantilevers or overhangs where the free end is unbraced. Check table 3-1 for C_b values.

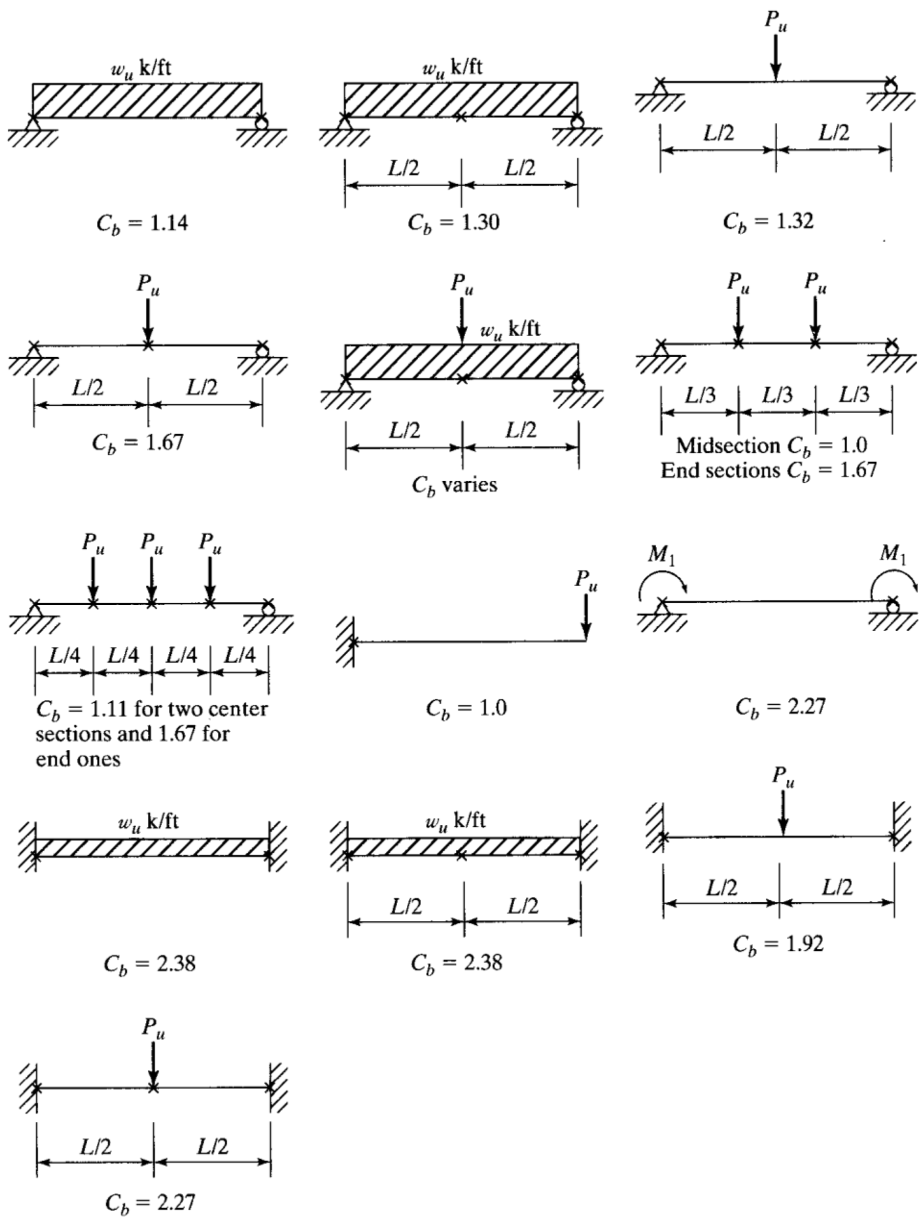


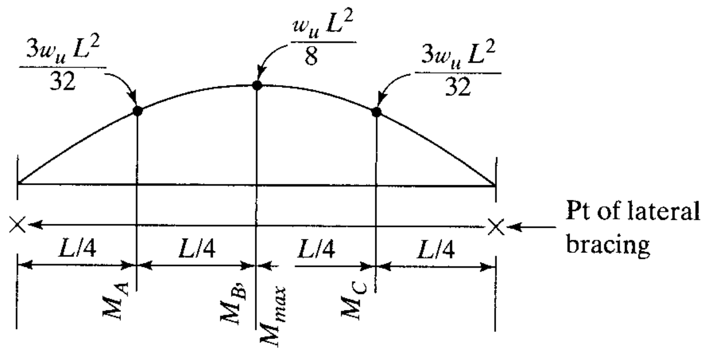
FIGURE 9.10

Sample C_b values for doubly symmetric members. (The X marks represent points of lateral bracing of the compression flange.)

Example 6:

Determine C_b for the beam shown in the figure part (a) and (b). Assume the beam is a doubly symmetric member.

a.



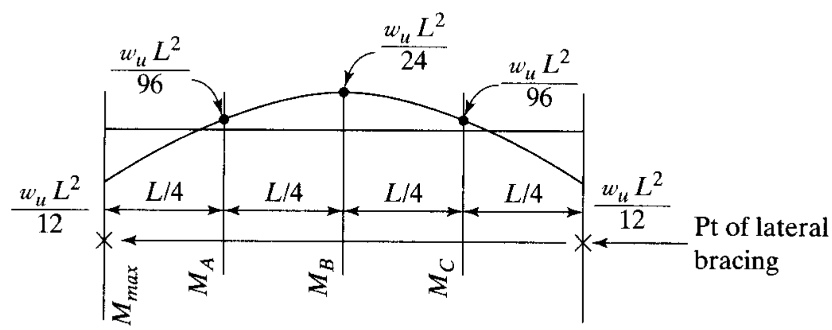
$$USE: \frac{w_u L^2}{8} = \frac{1}{8}$$

$$\frac{3w_u L^2}{32} = \frac{3}{32}$$

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3M_A + 4M_B + 3M_C}$$

$$C_b = \frac{12.5\left(\frac{1}{8}\right)}{2.5\left(\frac{1}{8}\right) + 3\left(\frac{3}{32}\right) + 4\left(\frac{1}{8}\right) + 3\left(\frac{3}{32}\right)} = 1.14$$

b.



$$USE: \frac{w_u L^2}{12} = \frac{1}{12}$$

$$\frac{w_u L^2}{96} = \frac{1}{96}$$

$$\frac{w_u L^2}{24} = \frac{1}{24}$$

$$C_b = \frac{12.5\left(\frac{1}{12}\right)}{2.5\left(\frac{1}{12}\right) + 3\left(\frac{1}{96}\right) + 4\left(\frac{1}{24}\right) + 3\left(\frac{1}{96}\right)} = 2.38$$

- Moment Capacity Zone 2:

When constant moment occurs along the unbraced length, or as the unbraced length of the compression flange of a beam or the distance between points of torsional bracing is increased beyond L_p, the moment capacity of the section will become smaller and smaller. Finally, at an unbraced length L_r, the section will buckle elastically as soon as the yield stress is reached. The nominal moment strength for unbraced length between L_p and L_r is calculated with the equation to follow:

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p \;(AISC\ Equation\ F2-2)$$

L_r is a function of several of the section's properties, such as its cross sectional area, yield stress, and torsional properties. The formula needed for the calculation of L_r is given in the AISC Specification (F1) and its values are given in table 3-2.

For the cases when the unbraced length falls between L_p and L_r, the nominal moment strength will fall approximately on a straight line between $M_{nx} = F_yZ_x$ at L_p, and $0.7F_yS_x$ at L_r. For intermediate values of the unbraced length between L_p and L_r, we may interpolate between the end values that fall on a straight line. Should C_b be larger than 1.0, the nominal moment strength will be larger, but not larger that $M_p = F_y Z_x$

$$\varphi_b M_{nx} = C_b\left[\varphi_b M_{px} - BF\left(L_b - L_p\right)\right] \leq \varphi_b M_{px}$$

❖ Example 7:

Determine the LRFD design moment capacity of a W24x62 with F_y = 50 ksi, L_b = 8 ft and C_b = 1.0.

Using a W24x62 (from AISC table 3-2: $\O_b M_{px}$ = 574 kft, $\O_b M_{rx}$ = 344 kft, L_b = 4.87 ft, L_r = 14.4 ft, BF for LRFD = 24.1 k)

$$L_p < L_b \lesssim L_r$$

$$4.87 < 8 \lesssim 14.4 \therefore falls\ in\ Zone\ 2$$

$$\varphi_b M_{nx} = C_b\left[\varphi_b M_{px} - BF\left(L_b - L_p\right)\right] \leq \varphi_b M_{px}$$

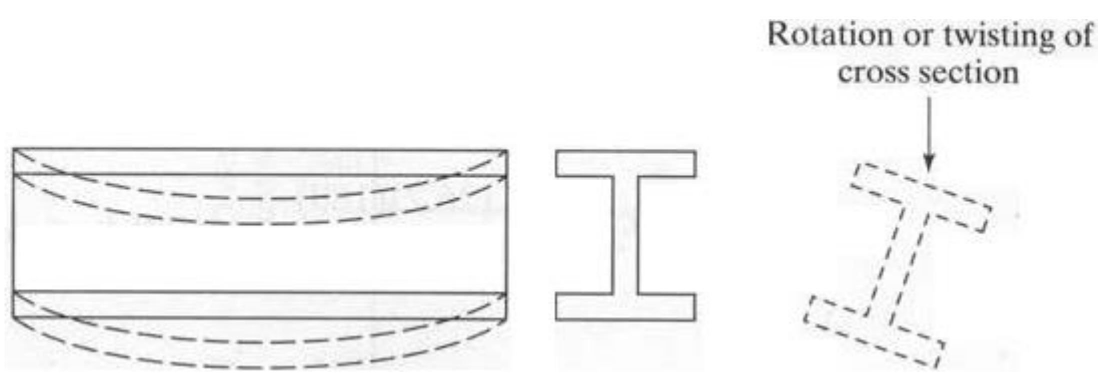
$$\varphi_b M_{nx} = 1.0[574 - 24.1(8 - 4.87)] \leq 574$$

$$499\ kft \leq 574\ kft$$

$$\therefore \varphi_b M_{nx} = 499\ kft$$

❖ Elastic Buckling Zone 3:

When the unbraced length of a beam is greater than L_r, the beam will fall in Zone 3. Such a member may fall due to the buckling of the compression portion of the cross section laterally about the weaker axis, with twist of the entire section about the beam longitudinal axis between the points of lateral bracing. The beam will bend initially about the stronger axis until a certain critical moment M_{cr} is reach. At that time, it will buckle laterally about its weaker axis. As it bend laterally, the tension in the other flange try to keep the beam straight. As a result, the buckling of a beam will be a combination of lateral bending and twisting of the beam cross section.



If the unbraced length of a compression flange of a beam section or the distance between points that prevent twisting of the entire cross section is greater than L_r, the section will buckle elastically before the yield stress is reached anywhere in the section. if the section F2-2 of the AISC

Specification, the buckling stress for doubly symmetric I shape members is calculated with the following expression:

$$M_n = F_{cr}\, S_x \leq M_p\ (AISC\ Equation\ F2-3)$$

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x\, h_o}\left(\frac{L_b}{r_{ts}}\right)^2}\ (AISC\ Equation\ F2-4)$$

In this calculation,

r_{ts} = effective radius of gyration, in (provided is AISC table 1-1)

J = torsional constant, in^4 (AISC Table 1-1)

c = 1.0 for doubly symmetric I shape.

h_o = distance between flange centroid, in (AISC Table 1-1)

❖ <u>Example 8:</u>

Using AISC Equation F2-4 determine the value of $\varphi_b M_{nx}$ for a W 18x97 with F_y = 50ksi and an unbraced length L_b = 38 ft. assume that C_b = 1.0.

Using W18x97 (L_r = 30.4 ft, r_{ts} = 3.08 in, j = 5.86 in^4, c = 1.0 for doubly symmetric I section, S_x = 188 in^3, h_o = 17.7 in, Z_x = 211 in^3)

$$note: L_b = 38\ ft > L_r = 30.4\ ft (from\ table\ 3-2)$$ → 23

$section\ is\ in\ Zone\ 3$

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x\, h_o} \left(\frac{L_b}{r_{ts}}\right)^2}$$

$$F_{cr} = \frac{1.0 * \pi^2 * 29000}{\left(\frac{12 * 38}{3.08}\right)^2} \sqrt{1 + 0.078 \frac{5.86 * 1.0}{188 * 17.7} \left(\frac{12 * 38}{3.08}\right)^2} = 26.2\ ksi$$

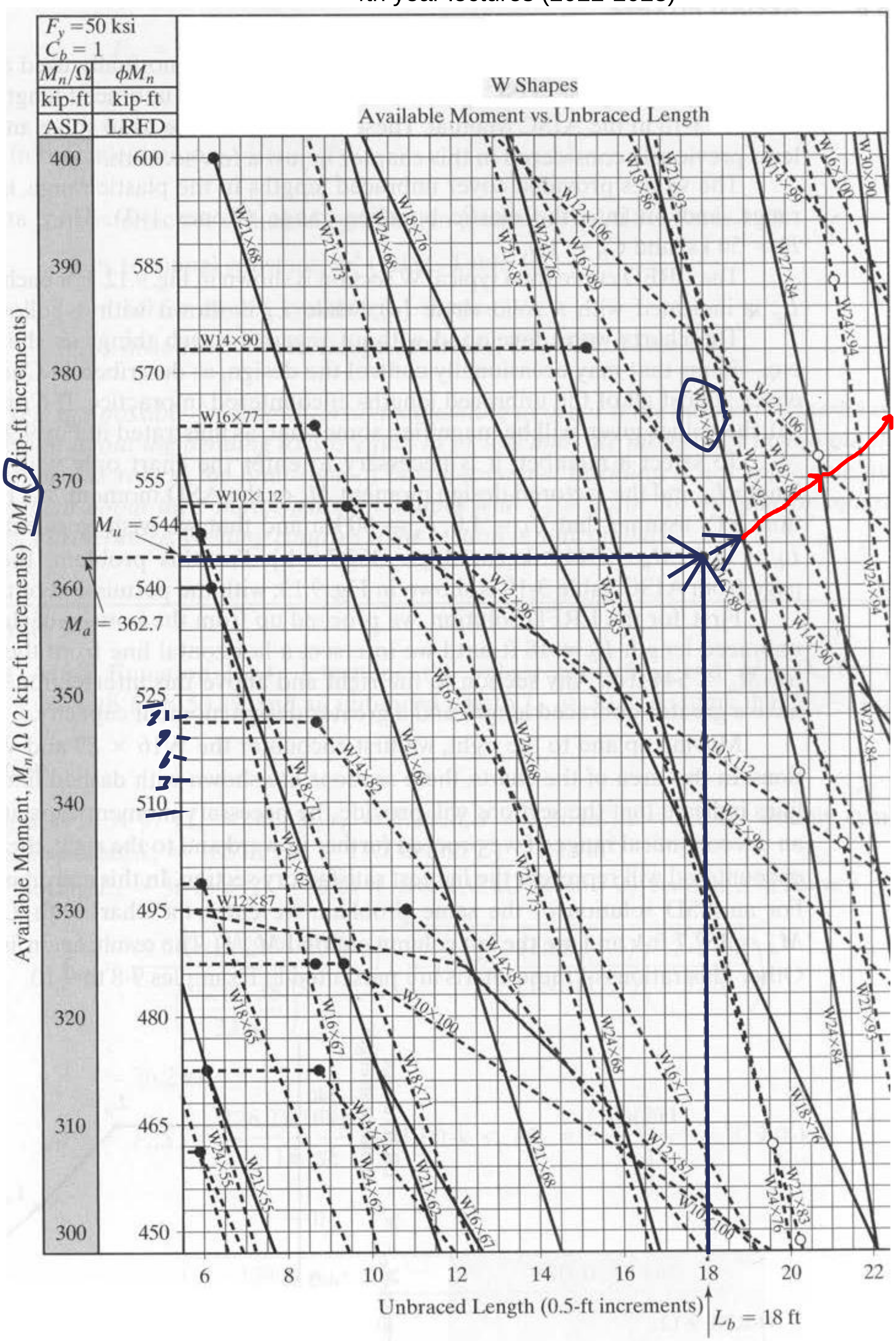
$$M_{nx} = F_{cr}\, S_x \leq M_p = F_y\, Z_x$$

$$M_{nx} = \frac{26.2 * 188}{12} = 410\ kft \leq M_p = \frac{50 * 211}{12} = 879\ kft$$

$$\varphi_b M_{nx} = 0.9 * 410 = 369\ kft$$

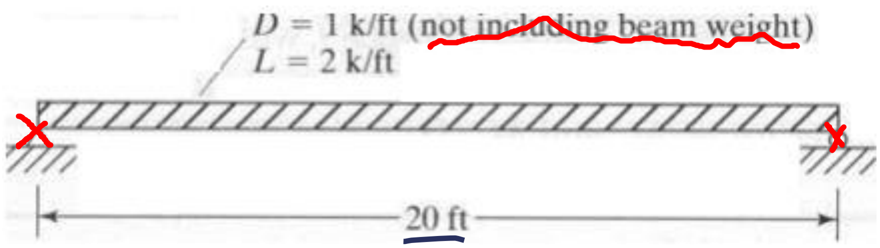
- <u>Design charts:</u>

The values of $\varphi_b M_n$ for sections normally used as beams have been computed by the AISC, plotted for a wide range of unbraced lengths, and shown as table 3-10 in the AISC manual. The values provided cover unbraced lengths in the plastic range, in the inelastic range, and on into the elastic buckling range (Zone 1-3). They are plotted for F_y = 50 ksi and C_b = 1.0. If the value of C_b is greater than 1.0, the value given will be magnified.


F_y = 50 ksi
C_b = 1
M_n/Ω kip-ft ASD
ϕM_n kip-ft LRFD
W Shapes
Available Moment vs. Unbraced Length
Available Moment, M_n/Ω (2 kip-ft increments) ϕM_n (3 kip-ft increments)
M_u = 544
M_a = 362.7
Unbraced Length (0.5-ft increments)
L_b = 18 ft
W14×90
W16×77
W10×112
W12×87
W21×68
W21×73
W24×68
W18×76
W21×83
W24×76
W12×106
W16×89
W18×86
W21×93
W14×99
W16×100
W30×90
W27×84
W24×94
W24×84
W12×96
W14×82
W18×71
W21×62
W16×67
W18×65
W14×74
W24×62
W24×55
W21×55
W10×100
W21×93
W27×84

- Example 9:

Using 50 ksi steel, select the lightest available section for the beam shown in the figure below, which has lateral bracing provided for its compression flange, only at its ends. Assume that Cb = 1.0. Use the LRFD method.



$$W_u = 1.2\,D.L + 1.6\,L.L = 1.2 * 1 + 1.6 * 2 = 4.4\ k/ft$$

$$M_u = \frac{W_u * L^2}{8} = \frac{4.4 * 20^2}{8} = 220\ kft$$

Enter AISC Table 3-10 with L_b = 20 ft and M_u = 220 kft.

Try W12x53

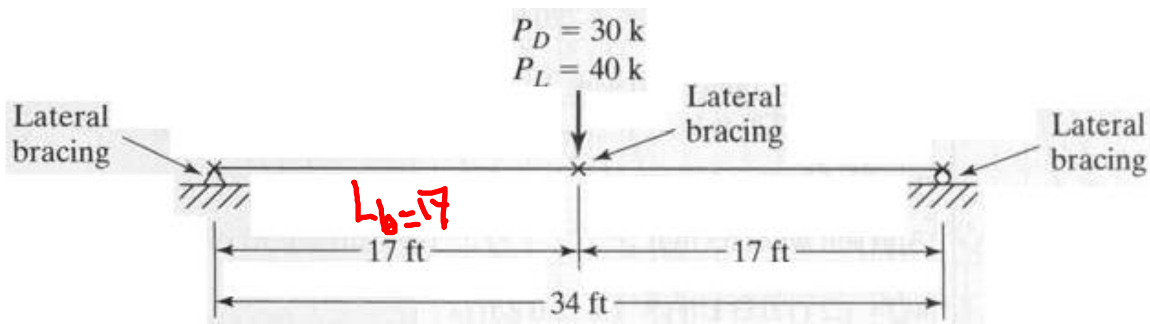
$$W_u = 1.2\,D.L + 1.6\,L.L = 1.2 * 1.053 + 1.6 * 2 = 4.46\ k/ft$$

$$M_u = \frac{W_u * L^2}{8} = \frac{4.46 * 20^2}{8} = 223\ kft$$

$$Re - enter\ table\ 3-10\ \therefore Use\ W12x53\ (\varphi_b M_n = 230.5 \geq M_u = 223\ kft\ OK$$

❖ Example 10:

Using 50 ksi steel and LRFD method, select the lightest available section for the situation shown in the figure below. Bracing is provided only at the ends and center line of the member, and L_b = 17 ft.



$$P_u = 1.2\ P_D + 1.6\ P_L = 1.2 * 30 + 1.6 * 40 = 100\ k$$

$$M_u = \frac{P_u * L}{4} = \frac{100 * 34}{4} = 850\ kft$$

From table 3-1: bending coefficient (C_b = 1.67)

$$M_u\ effective = \frac{850}{1.67} = 508.982\ kft = 509\ kft$$

Enter AISC Table 3-10 with L_b = 17 ft and M_u effective= 509 kft.

Try W24x76 ($\varphi_b M_p from\ table\ 3-2 = 750 kft < M_u = 850\ kft\ \therefore N.G$

Try W27x84 ($\varphi_b M_p from\ table\ 3-2 = 915 kft$

$$W_u = 1.2\ D.L = 1.2 * 0.084 = 0.101\ \frac{k}{ft}$$

$$M_u = \frac{W_u * L^2}{8} + \frac{P_u * L}{4} = \frac{0.101 * 34^2}{8} + \frac{100 * 34}{4} = 865\ kft$$

$$\varphi_b M_n = C_b \left[\varphi_b M_p - \left(\varphi_b M_p - \varphi_b 0.7 F_y S_x \right) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq \varphi_b M_p$$

$$\varphi_b M_n = 1.67 \left[915 - (915 - 559) \left(\frac{17 - 7.31}{20.8 - 7.31} \right) \right] \leq 915$$

$$\varphi_b M_n = 1101\ kft \leq 915 kft$$

∴ $\phi M_n = 915 > 865$ ∴ ok

$$OR: \varphi_b M_{nx} = C_b \left[\varphi_b M_{px} - BF \left(L_b - L_p \right) \right] \leq \varphi_b M_{px}$$

$$\varphi_b M_{nx} = 1.67 [915 - 26.4(17 - 7.31)] \leq 915$$

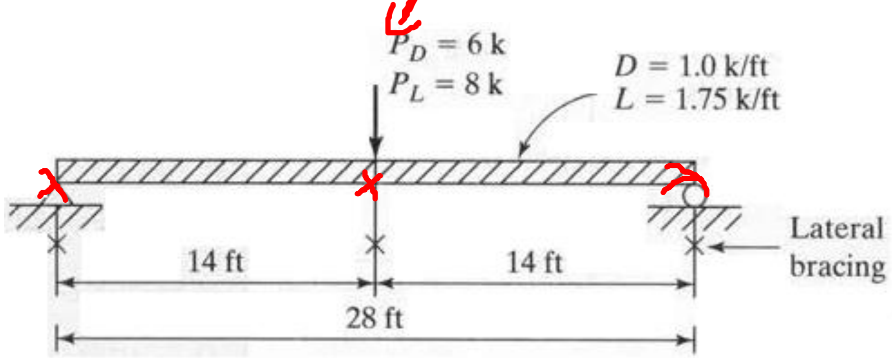
$$\varphi_b M_{nx} = 1100\ kft \leq 915 kft$$

$$\therefore\ \varphi_b M_{nx} = 915 kft$$

$$\therefore Use\ W27x84\ (\varphi_b M_n = 915 \geq M_u = 865\ kft\ OK)$$

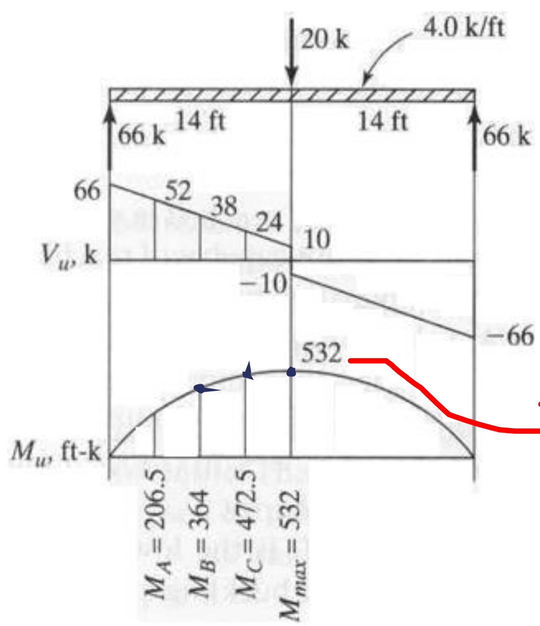
- <u>Example 11:</u>

Using 50 ksi steel and LRFD method select the lightest available section for the situation shown in the figure below. Bracing is provided only at the ends and at midspan.



$$W_u = 1.2\ D.L + 1.6 * L.L = 1.2 * 1 + 1.6 * 1.75 = 4\frac{k}{ft}$$

$$P_u = 1.2 * D.l + 1.6 * L.L = 1.2 * 6 + 1.6 * 8 = 20\ k$$



$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

$$C_b = \frac{12.5 * 532}{2.5 * 532 + 3 * 206.5 + 4 * 364 + 3 * 472.5} = 1.38$$

$$M_u\, effective = \frac{532}{1.38} = 385.507 = 386\, kft$$

Enter AISC Table 3-10 with L_b = 14 ft and M_u effective= 386 kft.

Try W21x62 ($\varphi_b M_p from\ table\ 3-2 = 540kft > M_u = 532\ kft\ \therefore ok$

$$W_u = 1.2\, D.L + 1.6 * L.L = 1.2 * (1 + 0.062) + 1.6 * 1.75 = 4.0744 \frac{k}{ft}$$

$$P_u = 1.2 * D.l + 1.6 * L.L = 1.2 * 6 + 1.6 * 8 = 20\, k$$

$$M_u = \frac{W_u * L^2}{8} + \frac{P_u * L}{4} = \frac{4.0744 * 28^2}{8} + \frac{20 * 28}{4} = 539.2912\, kft$$

$$\varphi_b M_n = C_b \left[\varphi_b M_p - (\varphi_b M_p - \varphi_b 0.7 F_y S_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le \varphi_b M_p$$

$$\varphi_b M_n = 1.38\left[540 - (540 - 333)\left(\frac{14 - 6.25}{18.1 - 6.25}\right)\right] \le 540$$

$$\varphi_b M_n = 558.3759\, kft \le 540kft$$

$$OR: \varphi_b M_{nx} = C_b\left[\varphi_b M_{px} - BF(L_b - L_p)\right] \le \varphi_b M_{px}$$

$$\varphi_b M_{nx} = 1.38[540 - 17.5(14 - 6.25)] \leq 540$$

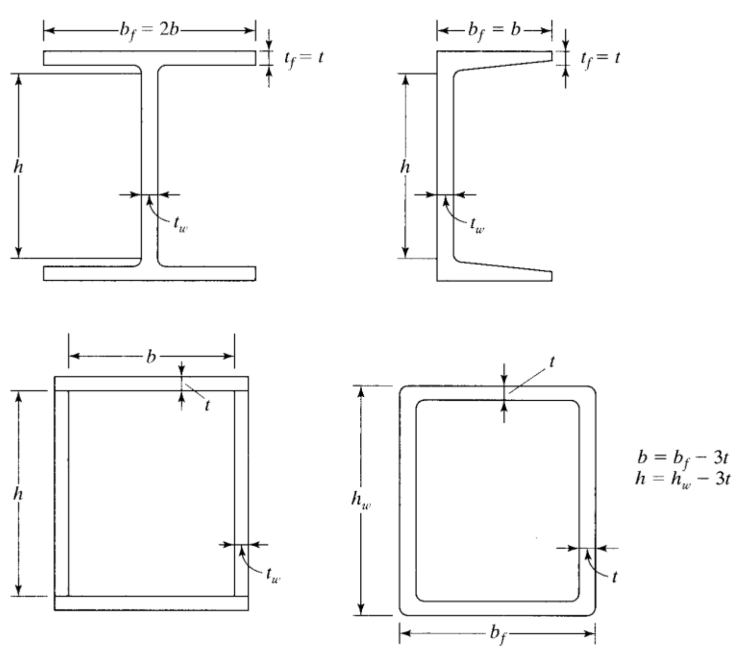
$$\varphi_b M_{nx} = 558.0375\, kft \leq 540 kft$$

$$\therefore\ \varphi_b M_{nx} = 540 kft$$

$$\therefore Use\ W21x62\ (\varphi_b M_n = 540\, kft \geq M_u = 539.29\, kft\ OK)$$

❖ Noncompact Sections:

A compact section is a section that has a sufficiently stock profile so that it is capable of developing a fully plastic stress distribution before buckling locally (web or flange). For a section to be compact, the width thickness ratio of the flange of W or other I sections must not exceed a (b/t) value ($\lambda_p = 0.38\sqrt{E/F_y}$), similarly, the web in flexural compression must not exceed an (h/t_w) value ($\lambda_p = 3.76\sqrt{E/F_y}$). The values of b, h, t, and t_w are shown in the figure below.



A noncompact section is one for which the yield stress can be reached in some, but not all, of its compression element before buckling occurs. It is not capable of reaching a fully plastic stress distribution. The noncompact section are those that have width thickness ratio greater that (λ_p), but not greater than (λ_r) (from table B4.1b of AISC specification). For the noncompact range, the width thickness ratio of the flanges of W shapes must not exceed (λ_r = $1\sqrt{E/F_y}$), while those for the web must not exceed (λ_r = $5.7\sqrt{E/F_y}$).

If we have section with noncompact flange, when $\lambda_p < \lambda \leq \lambda_r$ the value of M_n is given by the equation to follow, in which $k_c = 4/\sqrt{\frac{h}{t_w}} \geq 0.35 \leq 0.76$:

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)\right] \quad (AISC\ Equation\ F3-1)$$

Almost all of standard W, M, S, and C shapes listed in the AISC Manual are compact, and none of them fall into the slender classification. All of these shapes have compact webs, but few of them have noncompact flanges. You need to be careful when you work with built up sections as they may fall into noncompact or slender classification. For built up section with slender flanges (that is $\lambda > \lambda_r$)

$$M_n = \frac{0.9\,E\,k_c S_x}{\lambda^2}\ (AISC\ Equation\ F3-1)$$

TABLE 9.1 Width-to-Thickness Ratios: Compression Elements in Members Subject to Flexure

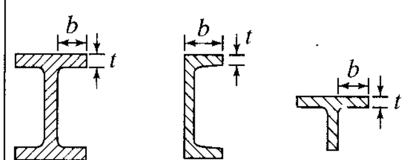
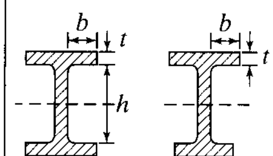
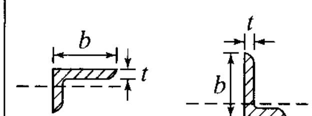
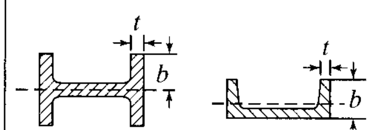
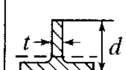
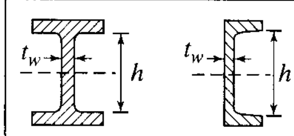
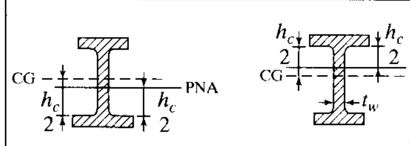
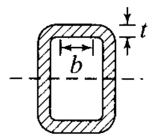
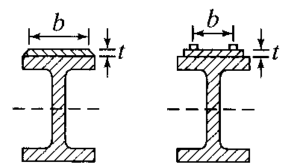
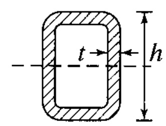
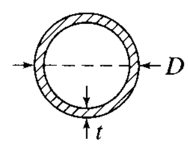
	Case	Description of Element	Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratios: λ_r compact / noncompact)	Limiting Width-to-Thickness Ratios: λ_r noncompact / slender)	Example
Unstiffened Elements	10	Flanges of rolled I-shaped sections, channels, and tees	b/t	$0.38\sqrt{\frac{E}{F_y}}$	$1.0\sqrt{\frac{E}{F_y}}$	
Unstiffened Elements	11	Flanges of doubly and singly symmetric I-shaped built-up sections	b/t	$0.38\sqrt{\frac{E}{F_y}}$	$0.95\sqrt{\frac{K_c E}{F_L}}$ [a][b]	
Unstiffened Elements	12	Legs of single angles	b/t	$0.54\sqrt{\frac{E}{F_y}}$	$0.91\sqrt{\frac{E}{F_y}}$	
Unstiffened Elements	13	Flanges of all I-shaped sections and channels in flexure about the weak axis	b/t	$0.38\sqrt{\frac{E}{F_y}}$	$1.0\sqrt{\frac{E}{F_y}}$	
Unstiffened Elements	14	Stems of tees	d/t	$0.84\sqrt{\frac{E}{F_y}}$	$1.03\sqrt{\frac{E}{F_y}}$	
Stiffened Elements	15	Webs of doubly-symmetric I-shaped sections and channels	h/t_w	$3.76\sqrt{\frac{E}{F_y}}$	$5.70\sqrt{\frac{E}{F_y}}$	
Stiffened Elements	16	Webs of singly-symmetric I-shaped sections	h_c/t_w	$\frac{\frac{h_c}{h_p}\sqrt{\frac{E}{F_y}}}{\left(0.54\frac{M_p}{M_y}-0.09\right)^2} \le \lambda_r$ [c]	$5.70\sqrt{\frac{E}{F_y}}$	

TABLE 9.1 (Continued)

	Case	Description of Element	Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratios λ_r compact/ noncompact)	λ_r noncompact/ slender)	Example
Stiffened Elements	17	Flanges of rectangular HSS and boxes of uniform thickness	b/t	$1.12\sqrt{\frac{E}{F_y}}$	$1.40\sqrt{\frac{E}{F_y}}$	
	18	Flange cover plates and diaphragm plates between lines of fasteners or welds	b/t	$1.12\sqrt{\frac{E}{F_y}}$	$1.40\sqrt{\frac{E}{F_y}}$	
	19	Webs of rectangular HSS and boxes	h/t	$2.42\sqrt{\frac{E}{F_y}}$	$5.70\sqrt{\frac{E}{F_y}}$	
	20	Round HSS	D/t	$0.07\frac{E}{F_y}$	$0.31\frac{E}{F_y}$	

[a] $K_c = \frac{4}{\sqrt{h/t_w}}$ but shall not be taken less than 0.35 nor greater than 0.76 for calculation purposes.

[b] $F_L = 0.7F_y$ for major axis bending of compact and noncompact web built-up I-shaped members with $S_{xy}/S_{xc} \geq 0.7$, $F_L = F_y S_{xy}/S_{xc} > 0.5F_y$ for major-axis bending of compact and noncompact web built-up I-shaped members with $S_{xy}/S_{xc} < 0.7$.

[c] M_y is the moment at yielding of the extreme fiber. M_p = plastic bending moment, kip-in. (N-mm)

E = modulus of elasticity of steel = 29,000 ksi (200 000 MPa)

F_y = specified minimum yield stress, ksi (MPa)

❖ <u>Example 12:</u>

Determine the LRFD flexural design stress for the 50 ksi W12x65 section which has full lateral bracing.

Solution

Using a W12 × 65 (b_f = 12.00 in, t_f = 0.605 in, S_x = 87.9 in^3, Z_x = 96.8 in^3)

Is the flange noncompact?

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29 \times 10^3}{50}} = 9.15$$

$$\lambda = \frac{b_f}{2t_f} = \frac{12.00}{(2)(0.605)} = 9.92$$

$$\lambda_r = 1.0\sqrt{\frac{E}{F_y}} = 1.0\sqrt{\frac{29 \times 10^3}{50}} = 24.08$$

$$\lambda_p = 9.15 < \lambda = 9.92 < \lambda_r = 24.08$$

∴ The flange is noncompact.

Calculate the nominal flexural stress.

$$M_p = F_y Z = (50)(96.8) = 4840 \text{ in-k}$$

$$M_n = \left[M_p - (M_p - 0.7F_y S_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)\right] \quad \text{(AISC Eq F3-1)}$$

$$M_n = \left[4840 - (4840 - 0.7 \times 50 \times 87.9)\left(\frac{9.92 - 9.15}{24.08 - 9.15}\right)\right]$$

$$= 4749 \text{ in-k} = 395.7 \text{ ft-k}$$

Determine $\phi_b M_n$

LRFD $\phi_b = 0.9$
$\phi_b M_n = (0.9)(395.7) = 356$ ft-k

Note: These values correspond to the values given in AISC Table 3-2.

Eccentric Connections

ECCENTRIC BOLTED CONNECTIONS: SHEAR ONLY

The column bracket connection shown in Figure 2 is an example of a bolted connection subjected to eccentric shear.

Elastic Analysis

In Figure 3a, the fastener shear areas and the load are shown separate from the column and bracket plate. The eccentric load P can be replaced with the same load acting at the centroid plus the couple, $M = Pe$, where e is the eccentricity. If this

FIGURE 2

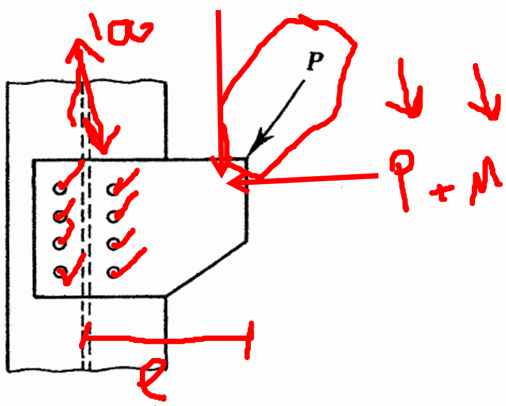
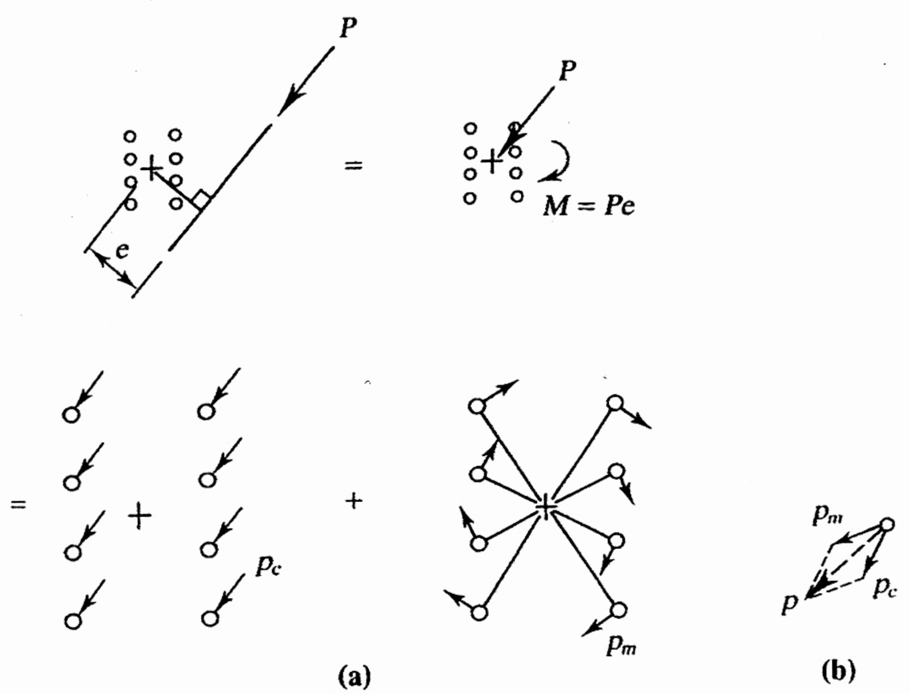


FIGURE 3



replacement is made, the load will be concentric, and each fastener can be assumed to resist an equal share of the load, given by $p_c = P/n$, where n is the number of fasteners. The fastener forces resulting from the couple can be found by considering the shearing stress in the fasteners to be the result of torsion of a cross section made up of the cross-sectional areas of the fasteners. If such an assumption is made, the shearing stress in each fastener can be found from the torsion formula

$$f_v = \frac{Md}{J} \qquad 1$$

where

d = distance from the centroid of the area to the point where the stress is being computed
J = polar moment of inertia of the area about the centroid

and the stress f_v is perpendicular to d. Although the torsion formula is applicable only to right circular cylinders, its use here is conservative, yielding stresses that are somewhat larger than the actual stresses.

If the parallel-axis theorem is used and the polar moment of inertia of each circular area about its own centroid is neglected, J for the total area can be approximated as

$$J = \sum Ad^2 = A \sum d^2 \qquad 2$$

provided all fasteners have the same area, A. Equation 8.1 can then be written as

$$f_v = \frac{Md}{A \sum d^2}$$

and the shear force in each fastener caused by the couple is

$$p_m = Af_v = A\frac{Md}{A \sum d^2} = \frac{Md}{\sum d^2}$$

The two components of shear force thus determined can be added vectorially to obtain the resultant force, p, as shown in Figure 3b, where the lower right-hand fastener is used as an example. When the largest resultant is determined, the fastener size is selected so as to resist this force. The critical fastener cannot always be found by inspection, and several force calculations may be necessary.

It is generally more convenient to work with rectangular components of forces. For each fastener, the horizontal and vertical components of force resulting from direct shear are

$$p_{cx} = \frac{P_x}{n} \text{ and } p_{cy} = \frac{P_y}{n}$$

where P_x and P_y are the x- and y-components of the total connection load, P, as shown in Figure 4. The horizontal and vertical components caused by the eccentricity can be found as follows. In terms of the x- and y-coordinates of the centers of the fastener areas,

$$\sum d^2 = \sum(x^2 + y^2)$$

where the origin of the coordinate system is at the centroid of the total fastener shear area. The x-component of p_m is

$$p_{mx} = \frac{y}{d} p_m = \frac{y}{d} \frac{Md}{\sum d^2} = \frac{y}{d} \frac{Md}{\sum(x^2 + y^2)} = \frac{My}{\sum(x^2 + y^2)}$$

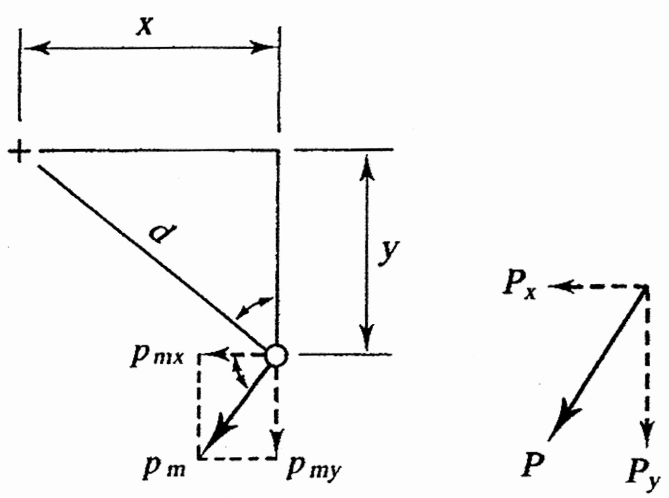
Similarly,

$$p_{my} = \frac{Mx}{\sum(x^2 + y^2)}$$

and the total fastener force is

$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2}$$

FIGURE 4



where

$$\sum p_x = p_{cx} + p_{mx}$$
$$\sum p_y = p_{cy} + p_{my}$$

Example 1 Determine the critical fastener force in the bracket connection shown in Figure 5.

Solution The centroid of the fastener group can be found by using a horizontal axis through the lower row and applying the principal of moments:

$$\bar{y} = \frac{2(5) + 2(8) + 2(11)}{8} = 6 \text{ in.}$$

The horizontal and vertical components of the load are

$$P_x = \frac{1}{\sqrt{5}}(50) = 22.36 \text{ kips} \leftarrow \quad \text{and} \quad P_y = \frac{2}{\sqrt{5}}(50) = 44.72 \text{ kips} \downarrow$$

Referring to Figure 6a, we can compute the moment of the load about the centroid:

$$M = 44.72(12 + 2.75) - 22.36(14 - 6) = 480.7 \text{ in.-kips} \qquad \text{(clockwise)}$$

FIGURE 5

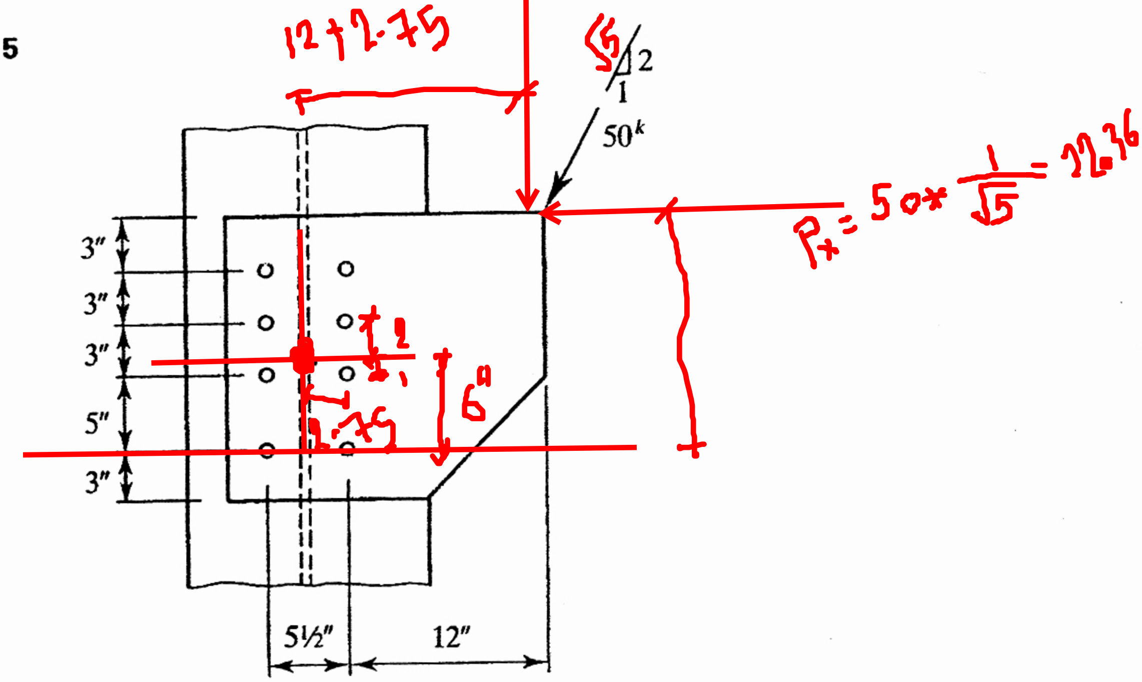


FIGURE 6

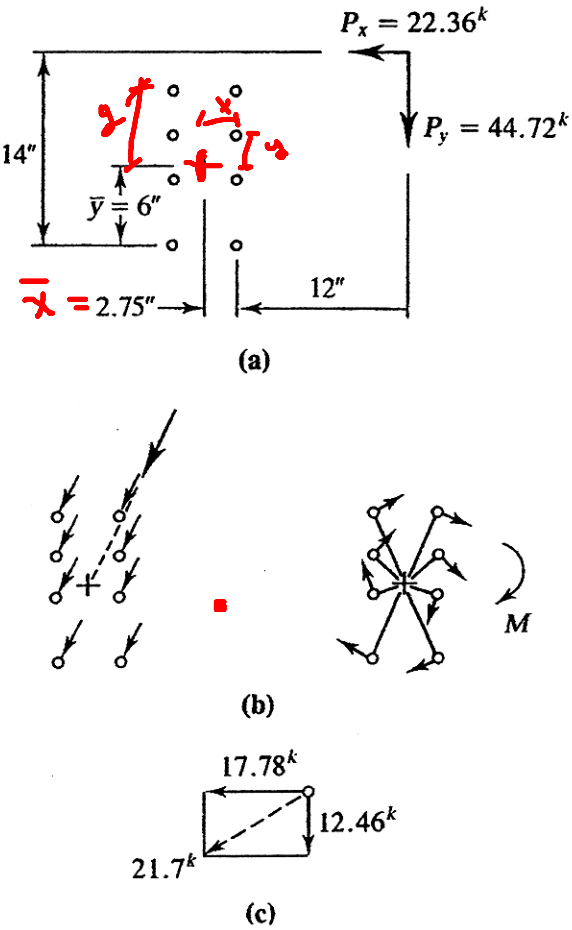


Figure 6b shows the directions of all component bolt forces and the relative magnitudes of the components caused by the couple. Using these directions and relative magnitudes as a guide and bearing in mind that forces add by the parallelogram law, we can conclude that the lower right-hand fastener will have the largest resultant force.

The horizontal and vertical components of force in each bolt resulting from the concentric load are

$$p_{cx} = \frac{22.36}{8} = 2.795 \text{ kips} \leftarrow \quad \text{and} \quad p_{cy} = \frac{44.72}{8} = 5.590 \text{ kips} \downarrow$$

For the couple,

$$\Sigma(x^2 + y^2) = 8(2.75)^2 + 2[(6)^2 + (1)^2 + (2)^2 + (5)^2] = 192.5 \text{ in.}^2$$

$$p_{mx} = \frac{My}{\Sigma(x^2 + y^2)} = \frac{480.7(6)}{192.5} = 14.98 \text{ kips} \leftarrow$$

$$p_{my} = \frac{Mx}{\Sigma(x^2 + y^2)} = \frac{480.7(2.75)}{192.5} = 6.867 \text{ kips} \downarrow$$

$$\sum p_x = 2.795 + 14.98 = 17.78 \text{ kips} \leftarrow$$
$$\sum p_y = 5.590 + 6.867 = 12.46 \text{ kips} \downarrow$$
$$p = \sqrt{(17.78)^2 + (12.46)^2} = 21.7 \text{ kips}$$ (see Figure 6c)

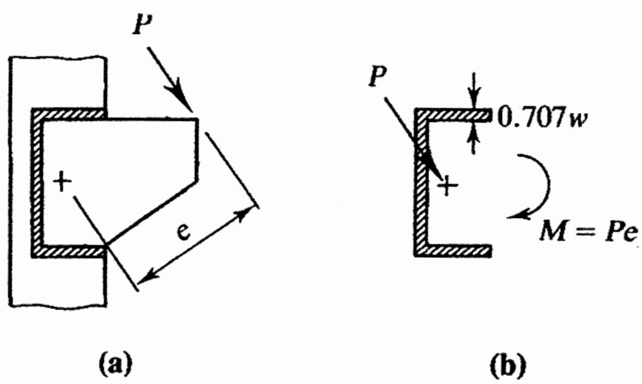
ECCENTRIC WELDED CONNECTIONS: SHEAR ONLY

Eccentric welded connections are analyzed in much the same way as bolted connections, except that unit lengths of weld replace individual fasteners in the computa-

Elastic Analysis

The load on the bracket shown in Figure 16a may be considered to act in the plane of the weld—that is, the plane of the throat. If this slight approximation is made, the load will be resisted by the area of weld shown in Figure 16b. Computations are simplified, however, if a unit throat dimension is used. The calculated load can then be multiplied by 0.707 times the weld size to obtain the actual load.

FIGURE 16



An eccentric load in the plane of the weld subjects the weld to both direct shear and torsional shear. Since all elements of the weld receive an equal portion of the direct shear, the direct shear stress is

$$f_1 = \frac{P}{L}$$

where L is the total length of the weld and numerically equals the shear area, because a unit throat size has been assumed. If rectangular components are used,

$$f_{1x} = \frac{P_x}{L} \quad \text{and} \quad f_{1y} = \frac{P_y}{L}$$

where P_x and P_y are the x- and y-components of the applied load. The shearing stress caused by the couple is found with the torsion formula

$$f_2 = \frac{Md}{J}$$

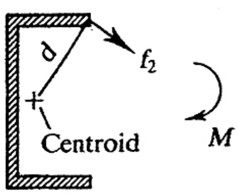
where

d = distance from the centroid of the shear area to the point where the stress is being computed
J = polar moment of inertia of that area

Figure 17 shows this stress at the upper right-hand corner of the given weld. In terms of rectangular components,

$$f_{2x} = \frac{My}{J} \quad \text{and} \quad f_{2y} = \frac{Mx}{J}$$

FIGURE 17



Also,

$$J = \int_A r^2 dA = \int_A (x^2 + y^2) dA = \int_A x^2 dA + \int_A y^2 dA = I_y + I_x$$

where I_x and I_y are the rectangular moments of inertia of the shear area. Once all rectangular components have been found, they can be added vectorially to obtain the resultant shearing stress at the point of interest, or

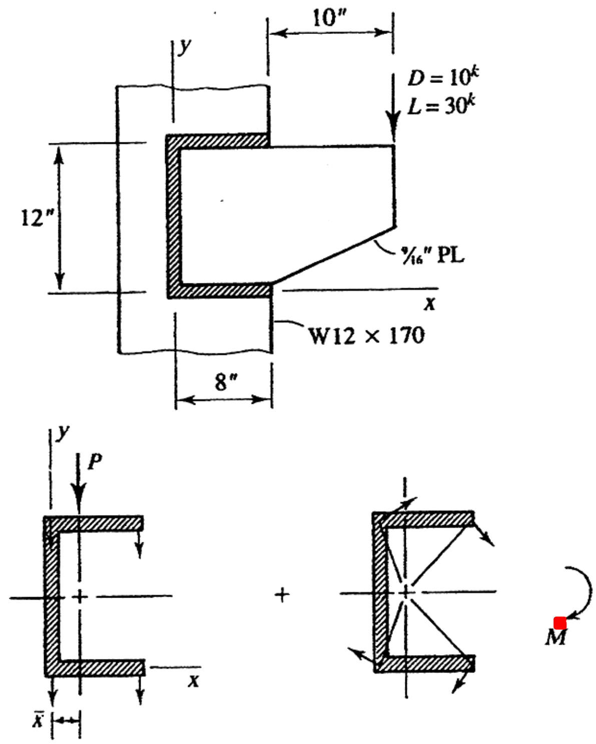
$$f_v = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2}$$

As with bolted connections, the critical location for this resultant stress can usually be determined from an inspection of the relative magnitudes and directions of the direct and torsional shearing stress components.

Example 5 Determine the size of weld required for the bracket connection in Figure 18. The service dead load is 10 kips, and the service live load is 30 kips. A36 steel is used for the bracket, and A992 steel is used for the column.

E70XX

FIGURE 18



LRFD Solution $P_u = 1.2D + 1.6L = 1.2(10) + 1.6(30) = 60$ kips

The eccentric load may be replaced by a concentric load and a couple, as shown in Figure 18. The direct shearing stress, in kips per inch, is the same for all segments of the weld and is equal to

$$f_{1y} = \frac{60}{8+12+8} = \frac{60}{28} = 2.143 \text{ kips/in.}$$

Before computing the torsional component of shearing stress, the location of the centroid of the weld shear area must be determined. From the principle of moments with summation of moments about the y-axis,

$$\bar{x}(28) = 8(4)(2) \qquad \text{or} \qquad \bar{x} = 2.286 \text{ in.}$$

The eccentricity e is $10 + 8 - 2.286 = 15.71$ in., and the torsional moment is

$$M = Pe = 60(15.71) = 942.6 \text{ in.-kips}$$

If the moment of inertia of each horizontal weld about its own centroidal axis is neglected, the moment of inertia of the total weld area about its horizontal centroidal axis is

$$I_x = \frac{1}{12}(1)(12)^3 + 2(8)(6)^2 = 720.0 \text{ in.}^4$$

Similarly,

$$I_y = 2\left[\frac{1}{12}(1)(8)^3 + 8(4 - 2.286)^2\right] + 12(2.286)^2 = 195.0 \text{ in.}^4$$

and

$$J = I_x + I_y = 720.0 + 195.0 = 915.0 \text{ in.}^4$$

Figure 18 shows the directions of both components of stress at each corner of the connection. By inspection, either the upper right-hand corner or the lower right-hand corner may be taken as the critical location. If the lower right-hand corner is selected,

$$f_{2x} = \frac{My}{J} = \frac{942.6(6)}{915.0} = 6.181 \text{ kips/in.}$$

$$f_{2y} = \frac{Mx}{J} = \frac{942.6(8-2.286)}{915.0} = 5.886 \text{ kips/in.}$$

$$f_v = \sqrt{(6.181)^2 + (2.143+5.886)^2} = 10.13 \text{ kips/in.}$$

Check the strength of the base metal. The bracket is the thinner of the connected parts and controls. the base metal shear *yield* strength per unit length is

$$\phi R_n = 0.6F_y t = 0.6(36)\left(\frac{9}{16}\right) = 12.2 \text{ kips/in.}$$

the base metal shear *rupture* strength per unit length is

$$\phi R_n = 0.45F_u t = 0.45(58)\left(\frac{9}{16}\right) = 14.7 \text{ kips/in.}$$

The base metal shear strength is therefore 12.2 kips/in. > 10.13kips/in. (OK)

the weld strength per inch is

$$\phi R_n = \phi(0.707 w F_W)$$

The matching electrode for A36 steel is E70. Because the load direction varies on each weld segment, the weld shear strength varies, but for simplicity, we will conservatively use $F_w = 0.6F_{EXX}$ for the entire weld. The required weld size is therefore

$$w = \frac{\phi R_n}{\phi(0.707)F_W} = \frac{10.13}{0.75(0.707)(0.6 \times 70)} = 0.455 \text{ in.}$$

Answer Use a ½-inch fillet weld, E70 electrode.

ASD Solution The total load is $P_a = D + L = 10 + 30 = 40$ kips.

The eccentric load may be replaced by a concentric load and a couple, as shown in Figure ..18. The direct shearing stress, in kips per inch, is the same for all segments of the weld and is equal to

$$f_{1y} = \frac{40}{8+12+8} = \frac{40}{28} = 1.429 \text{ kips/in.}$$

To locate the centroid of the weld shear area, use the principle of moments with summation of moments about the y-axis.

$$\bar{x}(28) = 8(4)(2) \quad \text{or} \quad \bar{x} = 2.286 \text{ in.}$$

The eccentricity e is $10 + 8 - 2.286 = 15.71$ in., and the torsional moment is

$$M = Pe = 40(15.71) = 628.4 \text{ in.-kips}$$

If the moment of inertia of each horizontal weld about its own centroidal axis is neglected, the moment of inertia of the total weld area about its horizontal centroidal axis is

$$I_x = \frac{1}{12}(1)(12)^3 + 2(8)(6)^2 = 720.0 \text{ in.}^4$$

Similarly,

$$I_y = 2\left[\frac{1}{12}(1)(8)^3 + 8(4-2.286)^2\right] + 12(2.286)^2 = 195.0 \text{ in.}^4$$

and

$$J = I_x + I_y = 720.0 + 195.0 = 915.0 \text{ in.}^4$$

Figure 18 shows the directions of both components of stress at each corner of the connection. By inspection, either the upper right-hand corner or the lower right-hand corner may be taken as the critical location. If the lower right-hand corner is selected,

$$f_{2x} = \frac{My}{J} = \frac{628.4(6)}{915.0} = 4.121 \text{ kips/in.}$$

$$f_{2y} = \frac{Mx}{J} = \frac{628.4(8-2.286)}{915.0} = 3.924 \text{ kips/in.}$$

$$f_v = \sqrt{(4.121)^2 + (1.429+3.924)^2} = 6.756 \text{ kips/in.}$$

Check the strength of the base metal. The bracket is the thinner of the connected parts and controls. the base metal shear *yield* strength per unit length is

$$\frac{R_n}{\Omega} = 0.4F_y t = 0.4(36)\left(\frac{9}{16}\right) = 8.10 \text{ kips/in.}$$

the base metal shear *rupture* strength per unit length is

$$\frac{R_n}{\Omega} = 0.3F_u t = 0.3(58)\left(\frac{9}{16}\right) = 9.79 \text{ kips/in.}$$

The base metal shear strength is therefore 8.10 kips/in. > 6.756 kips/in. (OK)

the weld strength per inch is

$$\frac{R_n}{\Omega} = \frac{0.707 w F_W}{\Omega}$$

The matching electrode for A36 steel is E70. Because the load direction varies on each weld segment, the weld shear strength varies, but for simplicity, we will conservatively use $F_W = 0.6F_{EXX}$ for the entire weld. The required weld size is, therefore,

$$w = \frac{\Omega(R_n/\Omega)}{0.707F_W} = \frac{\Omega(f_v)}{0.707F_W} = \frac{2.00(6.756)}{0.707(0.6 \times 70)} = 0.455 \text{ in.} \quad \therefore \text{ use } \frac{1}{2} \text{ in.}$$

Answer Use a ½-inch fillet weld, E70 electrode.

Eccentric Connection Analysis:

Two approaches exist for the solution of this problem: the traditional elastic analysis and the more accurate (but more complex) ultimate strength analysis. The first one will be illustrated in this chapter.

Elastic Analysis procedure:

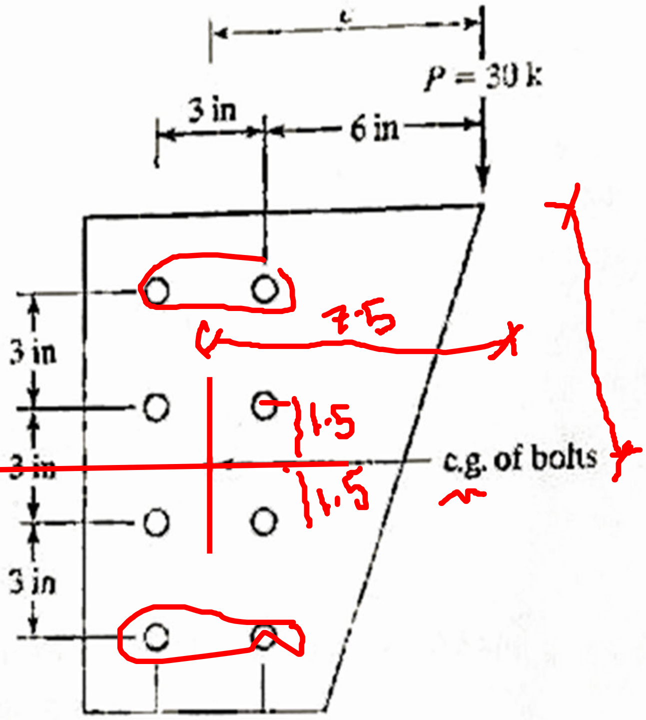
1. Find external load eccentricity e.
2. Find bolts centroids for two directions x and y.
3. Find the direct shear force $= \frac{applied\ force}{no.of\ bolts}$.
4. Find the couple moment = applied load x e .
5. Find the horizontal and vertical forces on the bolts .

$$P_{mx}(H) = \frac{M \times dy}{\Sigma(x2+y2)},\ P_{my}(v) = \frac{M \times dx}{\Sigma(x2+y2)}$$

6. Find the resultant force $R = \sqrt{\Sigma(fx)^2 + \Sigma(fy)^2}$
7. Bolts stress = Fnv = $\frac{R}{Bolt\ area}$

Example:

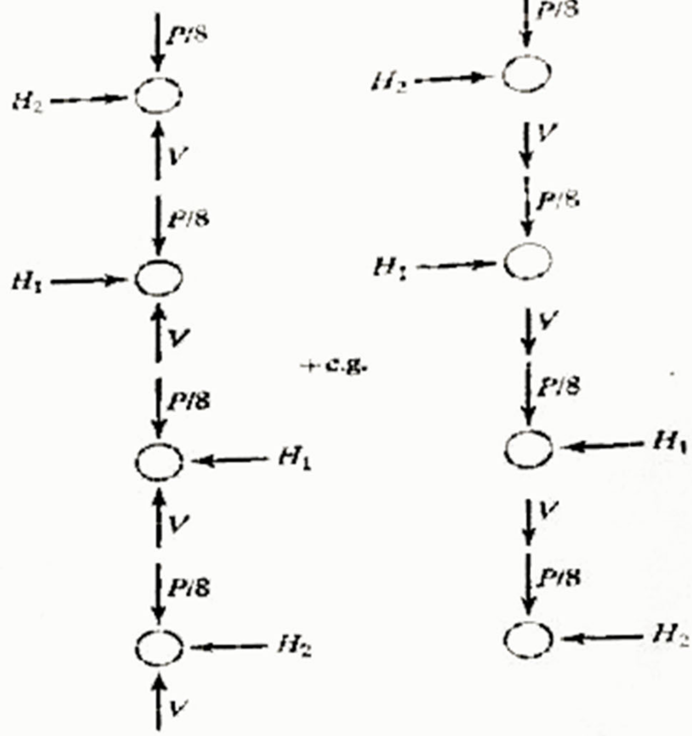
Determine the force in the most stressed bolt of the group shown in Fig. , using the elastic analysis method.



Solution:

A sketch of each bolt and the forces applied to it by the direct load and the clockwise moment are shown in Fig. From this sketch, the student can see that the upper right-hand bolt and the lower right-hand bolt are the most stressed and that their respective stresses are equal:

$$e = 6 + 1.5 = 7.5 \text{ in}$$



$$M = Pe = (30\ \text{k})(7.5\ \text{in}) = 225\ \text{in-k}$$

$$\Sigma d^2 = \Sigma x^2 + \Sigma y^2$$

$$\Sigma d^2 = (8)(1.5)^2 + (4)(1.5^2 + 4.5^2) = 108\ \text{in}^2$$

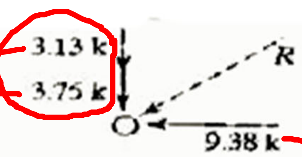
For lower right-hand bolt

$$H = \frac{My}{\Sigma d^2} = \frac{(225\ \text{in-k})(4.5\ \text{in})}{108\ \text{in}^2} = 9.38\ \text{k} \leftarrow$$

$$V = \frac{Mx}{\Sigma d^2} = \frac{(225\ \text{in-k})(1.5\ \text{in})}{108\ \text{in}^2} = 3.13\ \text{k} \downarrow$$

$$\frac{P}{8} = \frac{30\ \text{k}}{8} = 3.75\ \text{k} \downarrow$$

These components for the lower right-hand bolt are sketched as follows:

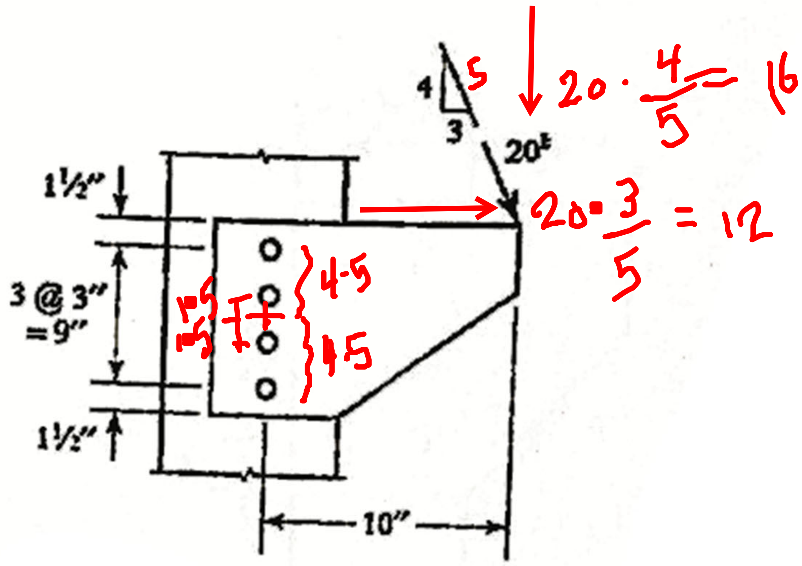


The resultant force applied to this bolt is

$$R = \sqrt{(3.13 + 3.75)^2 + (9.38)^2} = 11.63\ \text{k}$$

Example: 3

Use an elastic analysis to determine the maximum bolt shear force in the bracket connection of Figure



Solution:

Direct shear components:

$P_x = \frac{3}{5}(20) = 12$ kips, $P_y = \frac{4}{5}(20) = 16$ kips

$p_{cx} = \frac{12}{4} = 3$ kips → $p_{cy} = \frac{16}{4} = 4$ kips ↓

Eccentricity: $e_x = 10$ in., $e_y = 9 + 1.5 - 4.5 = 6$ in.

$M = 12(6) + 16(10) = 232$ in.-kips ↷

$\sum(x^2 + y^2) = 2[(4.5)^2 + (1.5)^2] = 45.0 \text{ in.}^2$

Top bolt is critical. $x = 0$, $y = 9/2 = 4.5$ in.

$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{232(4.5)}{45} = 23.2$ kips →

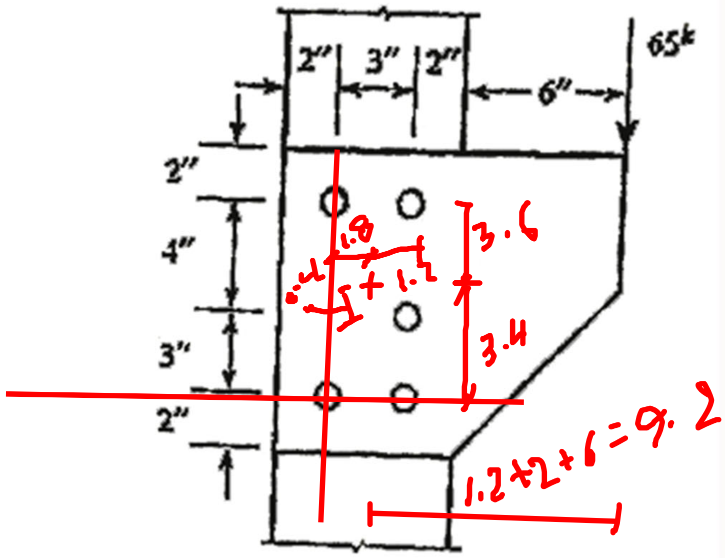
$\sum p_x = 3 + 23.2 = 26.2$ kips → $\sum p_y = 4$ kips ↓

$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(26.2)^2 + (4)^2} = 26.5$ kips

$p = 26.5$ kips

Example: 4

plate is used as a bracket and is attached to a column flange as shown in Figure Use an elastic analysis and compute the maximum bolt shear force.



Solution:

Direct shear component: $p_{cy} = \frac{65}{5} = 13.0$ kips ↓

Determine location of centroid with respect to lower left bolt:

$\bar{x} = \frac{3(3)}{5} = 1.8$ in., $\bar{y} = \frac{3 + 2(7)}{5} = 3.4$ in.

Eccentricity: $e_x = 3 + 2 + 6 - 1.8 = 9.2$ in.

$M = 65(9.2) = 598.0$ in.-kips ↻

$\sum(x^2 + y^2) = (1.8)^2(2) + (3 - 1.8)^2(3) + (3.4)^2(2) + (3.4 - 3)^2 + (7 - 3.4)^2(2)$

$= 60.0 \text{ in.}^2$

Top right bolt is critical. $x = 3 - 1.8 = 1.2$ in., $y = 3 + 4 - 3.4 = 3.6$ in.

$$p_{mx} = \frac{My}{\sum(x^2+y^2)} = \frac{598(3.6)}{60} = 35.88 \text{ kips} \rightarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2+y^2)} = \frac{598(1.2)}{60} = 11.96 \text{ kips} \downarrow$$

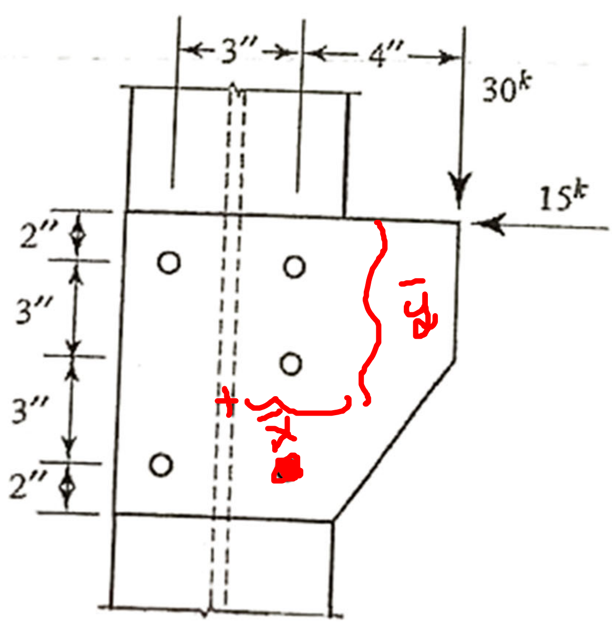
$$\sum p_x = 35.88 \text{ kips} \rightarrow \qquad \sum p_y = 13 + 11.96 = 24.96 \text{ kips} \downarrow$$

$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(35.88)^2 + (24.96)^2} = 43.71 \text{ kips}$$

$$\underline{p = 43.7 \text{ kips}}$$

Example: 5

A plate is used as a bracket and is attached to a column flange as shown in Figure. Use an elastic analysis and compute the maximum bolt shear force.



Solution:

Direct shear components:

$$p_{cx} = \frac{15}{5} = 3 \text{ kips} \leftarrow \qquad p_{cy} = \frac{30}{5} = 6 \text{ kips} \downarrow$$

Determine location of centroid with respect to lower right bolt:

$$\bar{x} = \frac{2(3)}{5} = 1.2 \text{ in.}, \qquad \bar{y} = 3 \text{ in.}$$

Eccentricity: $e_x = 1.2 + 4 = 5.2$ in., $e_y = 3 + 2 = 5$ in.

$$M = 30(5.2) - 15(5) = 81.0 \text{ in.-kips} \curvearrowleft$$

$$\sum(x^2 + y^2) = 3(1.2)^2 + 2(1.8)^2 + 4(3)^2 = 46.8 \text{ in.}^2$$

Lower right bolt is critical. $x = 1.2$ in., $y = 3$ in.

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{81(3)}{46.8} = 5.192 \text{ kips} \leftarrow$$

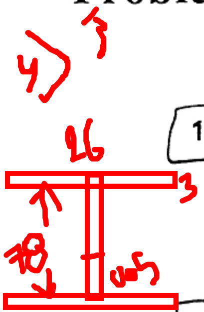
$$p_{my} = \frac{Mx}{\sum(x^2 + y^2)} = \frac{81(1.2)}{46.8} = 2.077 \text{ kips} \downarrow$$

$$\sum p_x = 3 + 5.192 = 8.192 \text{ kips} \leftarrow \qquad \sum p_y = 6 + 2.077 = 8.077 \text{ kips} \downarrow$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(8.192)^2 + (8.077)^2} = 11.5 \text{ kips}$$

$p = 11.5$ kips

Problems



Flexural Strength

10.4-1 Determine the nominal flexural strength of the following welded shape: The flanges are 3 inches × 26 inches, the web is 1/2 inch × 78 inches, and the member is simply supported, uniformly loaded, and has continuous lateral support. A572 Grade 50 steel is used.

10.4-2 Determine the nominal flexural strength of the following welded shape: The flanges are 1 inch × 10 inches, the web is 3/8 inch × 45 inches, and the member is simply supported, uniformly loaded, and has continuous lateral support. A572 Grade 50 steel is used.

10.4-3 Determine the nominal flexural strength of the following welded shape: The flanges are 7/8 inch × 12 inches, the web is 3/8 inch × 60 inches, and the member is simply supported, uniformly loaded, and has a span length of 40 feet. Lateral support is provided at the ends and at midspan. A572 Grade 50 steel is used.

10.4-4 Determine the nominal flexural strength of the following welded shape: The flanges are 3/4 inch × 18 inches, the web is 1/4 inch × 52 inches, and the member is simply supported, uniformly loaded, and has a span length of 50 feet. Lateral support is provided at the ends and at midspan. A572 Grade 50 steel is used.

10.4-5 An 80-foot-long plate girder is fabricated from a 1/2-inch × 78-inch web and two 3-inch × 22-inch flanges. Continuous lateral support is provided. The steel is A572 Grade 50. The loading consists of a uniform service dead load of 1.0 kip/ft (including the weight of the girder), a uniform service live load of 2.0 kips/ft, and a concentrated service live load of 500 kips at midspan. Stiffeners are placed at each end and at 4 feet, 16 feet, and 28 feet from each end. One stiffener is placed at midspan. Determine whether the flexural strength is adequate.

a. Use LRFD.

b. Use ASD.

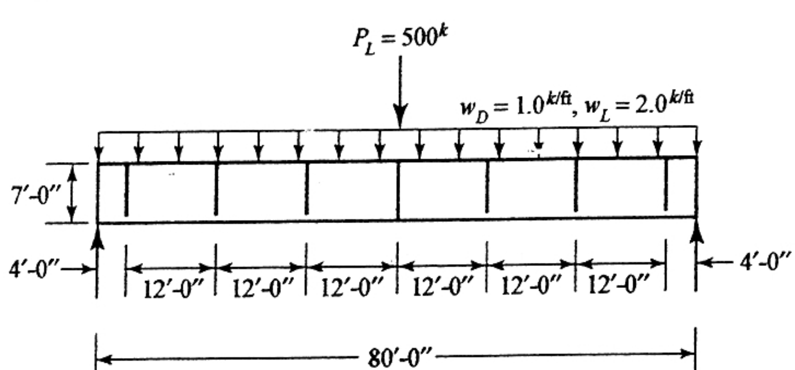


FIGURE P10.4-5

10.4-1

Check classification of shape.

$$\frac{h}{t_w} = \frac{78}{0.5} = 156, \qquad 5.70\sqrt{\frac{E}{F_y}} = 5.70\sqrt{\frac{29,000}{50}} = 137.3$$

Since $\frac{h}{t_w} > 5.70\sqrt{\frac{E}{F_y}}$, the web is slender and AISC Section F5 applies.

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h+t_f}{2}\right)^2 = \frac{1}{12}(0.5)(78)^3 + 2(3\times 26)\left(\frac{78+3}{2}\right)^2$$

$$= 2.757 \times 10^5 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2+t_f)} = \frac{275,700}{(78/2+3)} = 6564 \text{ in.}^3$$

Tension flange: $M_n = F_y S_x = 50(6564) = 3.282 \times 10^5$ in.-kips

Compression flange: LTB is not a factor in this problem. Check FLB:

$$\lambda = \frac{b_f}{2t_f} = \frac{26}{2(3)} = 4.333 < \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29,000}{50}} = 9.152$$

$\therefore F_{cr} = F_y = 50$ ksi

$$R_{pg} = 1 - \frac{a_w}{1200+300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \le 1.0$$

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{78(0.5)}{26(3)} = 0.5 < 10$$

$$R_{PG} = 1 - \frac{0.5}{1200+300(0.5)}\left(156 - 5.70\sqrt{\frac{29,000}{50}}\right) = 0.9931 < 1.0$$

$$M_n = R_{pg} F_{cr} S_x = 0.9931(50)6564 = 3.259 \times 10^5 \text{ in.-kips}$$

Compression flange strength controls. $M_n = 325900/12 = 2.716 \times 10^4$ ft-kips

$\phi_b M_n = 27,200$ ft-kips

10.4-2

Check classification of shape.

$$\lambda = \frac{h}{t_w} = \frac{45}{3/8} = 120, \qquad \lambda_r = 5.70\sqrt{\frac{E}{F_y}} = 5.70\sqrt{\frac{29{,}000}{50}} = 137.3$$

Since $\lambda < \lambda_r$, the web is not slender.

$$\lambda_p = 3.76\sqrt{\frac{E}{F_y}} = 3.76\sqrt{\frac{29000}{50}} = 90.55$$

Since , $\lambda_p < \lambda < \lambda_r$, the web is noncompact.

Flange: $$\lambda = \frac{b_f}{2t_f} = \frac{10}{2(1)} = 5 < \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{50}} = 9.15$$

∴ flange is compact. Since the flange is compact and the web is noncompact, AISC F4 applies (Table User Note F1.1), but AISC F5 may be conservatively used (F4 User Note).

Compression flange strength (because of symmetry, tension yielding will not control):

$$M_n = R_{pg} F_{cr} S_{xc}$$

Since the flange is compact, $F_{cr} = F_y = 50$ ksi, and LTB is not a factor in this problem.

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \le 1.0$$

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{45(3/8)}{10(1)} = 1.688 < 10$$

$$R_{PG} = 1 - \frac{1.688}{1200 + 300(1.688)}\left(120 - 5.7\sqrt{\frac{29000}{50}}\right) = 1.017 > 1.0 \therefore \text{use } 1.0$$

$$I_x = \frac{1}{12} t_w h^3 + 2A_f\left(\frac{h + t_f}{2}\right)^2 = \frac{1}{12}(3/8)(45)^3 + 2(10)\left(\frac{45 + 1}{2}\right)^2 = 13{,}430 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2 + t_f)} = \frac{13{,}430}{(45/2 + 1)} = 571.5 \text{ in.}^3$$

$$M_n = R_{pg}F_{cr}S_x = 1.0(50)(571.5) = 28,580 \text{ in.-kips} = 2380 \text{ ft-kips}$$

$$\underline{M_n = 2380 \text{ ft-kips}}$$

10.4-3

Check web width-thickness ratio:

$$\lambda = \frac{h}{t_w} = \frac{60}{3/8} = 160, \qquad \lambda_p = 3.76\sqrt{\frac{E}{F_y}} = 3.76\sqrt{\frac{29,000}{50}} = 90.55$$

$$\lambda_r = 5.70\sqrt{\frac{E}{F_y}} = 5.70\sqrt{\frac{29000}{50}} = 137.3$$

Since $\lambda > \lambda_r$, web is slender and AISC Section F5 applies. Compute the section modulus:

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h+t_f}{2}\right)^2 = \frac{1}{12}(3/8)(60)^3 + 2\left(\frac{7}{8}\times 12\right)\left(\frac{60+7/8}{2}\right)^2$$

$$= 2.621 \times 10^4 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2+t_f)} = \frac{26210}{(60/2+7/8)} = 848.9 \text{ in.}^3$$

From AISC Equation F5-10, the tension flange strength based on yielding is

$$M_n = F_y S_{xt} = 50(848.9) = 4.245 \times 10^4 \text{ in.-kips} = 3538 \text{ ft-kips}$$

The compression flange strength is given by AISC Equation F5-7:

$$M_n = R_{pg}F_{cr}S_{xc}$$

where the critical stress F_{cr} is based on either flange local buckling or yielding. For flange local buckling, the relevant slenderness parameters are

$$\lambda = \frac{b_f}{2t_f} = \frac{12}{2(7/8)} = 6.857, \qquad \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29000}{50}} = 9.152$$

Since $\lambda < \lambda_p$, there is no flange local buckling. The compression flange strength is therefore based on yielding, and $F_{cr} = F_y = 50$ ksi.

To compute the bending strength reduction factor R_{pg}, the value of a_w will be needed:

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{60(3/8)}{12(7/8)} = 2.143 < 10$$

From AISC Equation F5-6,

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \leq 1.0$$

$$= 1 - \frac{2.143}{1200 + 300(2.143)}\left(160 - 5.7\sqrt{\frac{29000}{50}}\right) = 0.9736$$

$$M_n = R_{pg}F_{cr}S_{xc} = 0.9736(50)(848.9) = 4.132 \times 10^4 \text{ in.-kips} = 3443 \text{ ft-kips}$$

Check lateral-torsional buckling.

$$\frac{h}{6} = \frac{60}{6} = 10 \text{ in.}, \qquad I = \frac{1}{12}(10)(3/8)^3 + \frac{1}{12}(7/8)(12)^3 = 126.0 \text{ in.}^4$$

$$A = 10(3/8) + 12(7/8) = 14.25 \text{ in.}^2, \qquad r_t = \sqrt{\frac{I}{A}} = \sqrt{\frac{126}{14.25}} = 2.974 \text{ in.}$$

$$L_b = 40/2 = 20 \text{ ft}$$

$$L_p = 1.1r_t\sqrt{\frac{E}{F_y}} = 1.1(2.974)\sqrt{\frac{29000}{50}} = 78.79 \text{ in.} = 6.566 \text{ ft}$$

$$L_r = \pi r_t\sqrt{\frac{E}{0.7F_y}} = \pi(2.974)\sqrt{\frac{29000}{0.7(50)}} = 268.9 \text{ in.} = 22.40 \text{ ft}$$

Since $L_p < L_b < L_r$, the girder is subject to inelastic lateral-torsional buckling. From AISC Equation F5-3,

$$F_{cr} = C_b\left[F_y - 0.3F_y\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq F_y$$

$$= 1.30\left[50 - (0.3 \times 50)\left(\frac{20 - 6.566}{22.40 - 6.566}\right)\right] = 48.46 \text{ ksi} \leq 50 \text{ ksi}$$

where $C_b = 1.30$ is from Figure 5.15 in the textbook. LTB has the lowest critical stress and controls.

$$M_n = R_{pg}F_{cr}S_{xc} = 0.9736(48.46)(848.9) = 4.005 \times 10^4 \text{ in.-kips} = 3338 \text{ ft-kips}$$

$$\underline{M_n = 3340 \text{ ft-kips}}$$

10.4-4

Check web width-thickness ratio:

$$\lambda = \frac{h}{t_w} = \frac{52}{1/4} = 208, \qquad \lambda_p = 3.76\sqrt{\frac{E}{F_y}} = 3.76\sqrt{\frac{29,000}{50}} = 90.55$$

$$\lambda_r = 5.70\sqrt{\frac{E}{F_y}} = 5.70\sqrt{\frac{29000}{50}} = 137.3$$

Since $\lambda > \lambda_r$, web is slender and AISC Section F5 applies. Compute the section modulus:

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h+t_f}{2}\right)^2 = \frac{1}{12}(1/4)(52)^3 + 2\left(\frac{3}{4}\times 18\right)\left(\frac{52+3/4}{2}\right)^2$$

$$= 2.171 \times 10^4 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2+t_f)} = \frac{21710}{(52/2+3/4)} = 811.6 \text{ in.}^3$$

From AISC Equation F5-10, the tension flange strength based on yielding is

$$M_n = F_y S_{xt} = 50(811.6) = 4.058 \times 10^4 \text{ in.-kips} = 3382 \text{ ft-kips}$$

The compression flange strength is given by AISC Equation F5-7:

$$M_n = R_{pg} F_{cr} S_{xc}$$

where the critical stress F_{cr} is based on either flange local buckling or yielding.

To compute the bending strength reduction factor R_{pg}, the value of a_w will be needed:

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{52(1/4)}{18(3/4)} = 0.9630 < 10$$

From AISC Equation F5-6,

$$R_{pg} = 1 - \frac{a_w}{1200+300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \leq 1.0$$

$$= 1 - \frac{0.9630}{1200+300(0.9630)}\left(208 - 5.7\sqrt{\frac{29000}{50}}\right) = 0.9543$$

For flange local buckling, the relevant slenderness parameters are

$$\lambda = \frac{b_f}{2t_f} = \frac{18}{2(3/4)} = 12.0, \qquad \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29000}{50}} = 9.152$$

case 7 ~~$\lambda_r = 0.95\sqrt{\frac{k_c E}{F_L}}$~~ X $\Rightarrow \lambda_r = 1\sqrt{\frac{E}{F_y}}$

$$k_c = \frac{4}{\sqrt{h/t_w}} = \frac{4}{\sqrt{52/0.25}} = 0.2774 < 0.35 \;\therefore\; \text{use } k_c = 0.35$$

$$F_L = 0.7F_y = 0.7(50) = 35.0 \text{ ksi}$$

$$\lambda_r = 0.95\sqrt{\frac{0.35(29000)}{35.0}} = 16.18$$

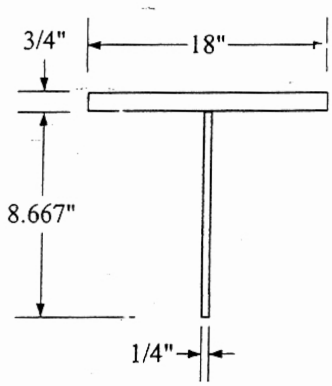
Since $\lambda_p < \lambda < \lambda_r$, the flange is noncompact, and FLB must be investigated.

$$F_{cr} = \left[F_y - 0.3F_y\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)\right] \quad \text{(AISC Eq. F5-8)}$$

$$= \left[50 - 0.3(50)\left(\frac{12.0 - 9.152}{16.18 - 9.152}\right)\right] = 43.92 \text{ ksi}$$

Check lateral-torsional buckling.

$$\frac{h}{6} = \frac{52}{6} = 8.667 \text{ in.}, \qquad I = \frac{1}{12}(3/4)(18)^3 + \frac{1}{12}(8.667)(1/4)^3 = 364.5 \text{ in.}^4$$



(not to scale)

$$A = 8.667(1/4) + 18(3/4) = 15.67 \text{ in.}^2, \qquad r_t = \sqrt{\frac{I}{A}} = \sqrt{\frac{364.5}{15.67}} = 4.823 \text{ in.}$$

$$L_b = 50/2 = 25.0 \text{ ft}$$

$$L_p = 1.1 r_t \sqrt{\frac{E}{F_y}} = 1.1(4.823)\sqrt{\frac{29000}{50}} = 127.8 \text{ in.} = 10.65 \text{ ft}$$

$$L_r = \pi r_t \sqrt{\frac{E}{0.7F_y}} = \pi(4.823)\sqrt{\frac{29000}{0.7(50)}} = 436.1 \text{ in.} = 36.34 \text{ ft}$$

Since $L_p < L_b < L_r$, the girder is subject to inelastic lateral-torsional buckling. From AISC Equation F5-3,

$$F_{cr} = C_b\left[F_y - 0.3F_y\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le F_y$$

$$= 1.30\left[50 - (0.3 \times 50)\left(\frac{25.0 - 10.65}{36.34 - 10.65}\right)\right] = 54.11 \text{ ksi} > 50 \text{ ksi} \therefore \text{ use } 50$$

ksi

where $C_b = 1.30$ is from Figure 5.15 in the textbook. FLB has the lowest critical stress and controls.

$$M_n = R_{pg}F_{cr}S_{xc} = 0.9543(43.92)(811.6)/12 = 2835 \text{ ft-kips}$$

$$\underline{M_n = 2840 \text{ ft-kips}}$$

10.4-5

Check classification of shape.

$$\frac{h}{t_w} = \frac{50}{0.25} \qquad 5.70\sqrt{\frac{E}{F_y}} = 1140.0\sqrt{\left(\frac{E}{F_y}\right)} = 5.70\sqrt{\frac{29,000}{50}} = 137.3$$

Since $\frac{h}{t_w} > 5.70\sqrt{\frac{E}{F_y}}$, the web is slender and the provisions of AISC F5 apply.

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h + t_f}{2}\right)^2 = \frac{1}{12}(0.5)(78)^3 + 2(3 \times 22)\left(\frac{78 + 3}{2}\right)^2$$

$$= 2.363 \times 10^5 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2 + t_f)} = \frac{236300}{(78/2 + 3)} = 5626 \text{ in.}^3$$

Tension flange: $M_n = F_y S_{xt} = 50(5626) = 2.813 \times 10^5$ in.-kips

Compression flange: LTB is not a factor in this problem. Check FLB:

$$\lambda = \frac{b_f}{2t_f} = \frac{22}{2(3)} = 3.667 < \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29,000}{50}} = 9.152$$

$\therefore F_{cr} = F_y = 50$ ksi

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \leq 1.0$$

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{78(0.5)}{22(3)} = 0.5909 < 10$$

$$R_{pg} = 1 - \frac{0.5909}{1200 + 300(0.5909)}\left(156 - 5.7\sqrt{\frac{29,000}{50}}\right) = 0.9920 < 1.0$$

$$M_n = R_{pg}F_{cr}S_{xc} = 0.9920(50)(5626) = 2.79 \times 10^5 \text{ in.-kips}$$

Compression flange strength controls. $M_n = 279000/12 = 2.325 \times 10^4$ ft-kips

(a) LRFD

$$\phi_b M_n = 0.90(23250) = 20,900 \text{ ft-kips}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(1.0) + 1.6(2) = 4.4 \text{ kips/ft}$$

$$P_u = 1.6P_L = 1.6(500) = 800.0 \text{ kips}$$

$$M_u = \frac{1}{8}w_u L^2 + \frac{P_u L}{4} = \frac{1}{8}(4.4)(80)^2 + \frac{800(80)}{4} = 19,500 \text{ ft-kips}$$

Since 19,500 ft-kips < 20,900 ft-kips, flexural strength is adequate

(b) ASD

$$\frac{M_n}{\Omega_b} = \frac{23250}{1.67} = 1.392 \times 10^4 \text{ ft-kips}$$

$$w_a = w_D + w_L = 1 + 2 = 3 \text{ kips/ft}$$

$$P_a = P_L = 500 \text{ kips}$$

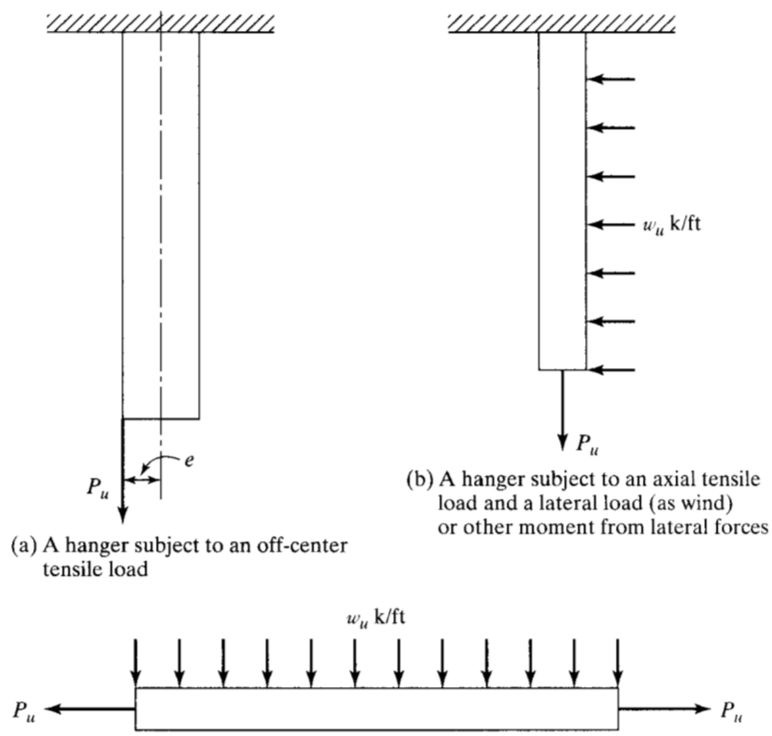
$$M_a = \frac{1}{8}w_a L^2 + \frac{P_a L}{4} = \frac{1}{8}(3)(80)^2 + \frac{500(80)}{4} = 1.24 \times 10^4 \text{ ft-kips}$$

Since 12,400 ft-kips < 13,900 ft-kips, flexural strength is adequate

Chapter 9: Bending and Axial Force

❖ <u>Members subjected to bending and axial tension:</u>

A few types of members subjected to both bending and axial tension are shown in the figure below. In section H1 of the AISC specification, the interaction equations that follows are given for symmetric shapes subjected to bending and axial tensile forces.



(a) A hanger subject to an off-center tensile load

(b) A hanger subject to an axial tensile load and a lateral load (as wind) or other moment from lateral forces

(c) A beam subject to a uniform gravity load and an axial tensile load

$$For\ \frac{P_r}{P_c} \geq 0.2, \qquad \frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \quad (AISC\ Equation\ H1-1a)$$

$$For\ \frac{P_r}{P_c} < 0.2, \qquad \frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \quad (AISC\ Equation\ H1-1b)$$

In which:

P_r = required axial tensile strength, P_u kips

P_c = design axial tensile strength ($\varphi_t P_n$) kips

M_r = required flexural strength, M_u kft

M_c = design flexural strength ($\varphi_b M_n$) kft

❖ Example 1:

A 50 ksi W12 × 40 tension member with no holes is subjected to the axial loads $P_D = 25$ k and $P_L = 30$ k, as well as the bending moments $M_{Dy} = 10$ ft-k and $M_{Ly} = 25$ ft-k. Is the member satisfactory if $L_b < L_p$?

Using a W12 × 40 ($A = 11.7\ \text{in}^2$)

LRFD	ASD
$P_r = P_u = (1.2)(25\text{ k}) + (1.6)(30\text{ k}) = 78\text{ k}$	$P_r = P_a = 25\text{ k} + 30\text{ k} = 55\text{ k}$
$M_{ry} = M_{uy} = (1.2)(10\text{ ft-k}) + (1.6)(25\text{ ft-k}) = 52\text{ ft-k}$	$M_{ry} = M_{ay} = 10\text{ ft-k} + 25\text{ ft-k} = 35\text{ ft-k}$
$P_c = \phi P_n = \phi_t F_y A_g = (0.9)(50\text{ ksi})(11.7\text{ in}^2) = 526.5\text{ k}$	$P_c = \dfrac{P_n}{\Omega_c} = \dfrac{F_y A_g}{\Omega_t} = \dfrac{(50\text{ ksi})(11.7\text{ in}^2)}{1.67} = 350.3\text{ k}$
$M_{cy} = \phi_b M_{py} = 63.0\text{ ft-k}$ (AISC Table 3-4)	$M_{cy} = \dfrac{M_{cy}}{\Omega_b} = 41.9\text{ ft-k}$ (AISC Table 3-4)
$\dfrac{P_r}{P_c} = \dfrac{78\text{ k}}{526.5\text{ k}} = 0.148 < 0.2$	$\dfrac{P_r}{P_c} = \dfrac{55\text{ k}}{350.3\text{ k}} = 0.157 < 0.2$
∴ **Must use AISC Eq. H1-1b**	∴ **Must use AISC Eq. H1-1b**
$\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$	$\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$
$\dfrac{78}{(2)(526.5)} + \left(0 + \dfrac{52}{63}\right) = 0.899 < 1.0$ **OK**	$\dfrac{55}{(2)(350.3)} + \left(0 + \dfrac{35}{41.9}\right) = 0.914 < 1.0$ **OK**

❖ Example 2:

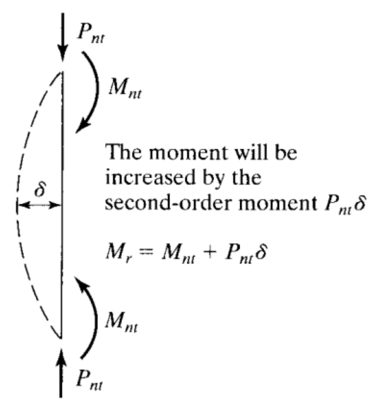
A W10 × 30 tensile member with no holes, consisting of 50 ksi steel and with $L_b = 12.0$ ft, is subjected to the axial service loads $P_D = 30$ k and $P_L = 50$ k and to the service moments $M_{Dx} = 20$ ft-k and $M_{Lx} = 40$ ft-k. If $C_b = 1.0$, is the member satisfactory?

Using a W10 × 30 ($A = 8.84\ \text{in}^2$, $L_p = 4.84$ ft and $L_r = 16.1$ ft, $\phi_b M_{px} = 137$ ft-k, BF for LRFD = 4.61 from AISC Table 3-2)

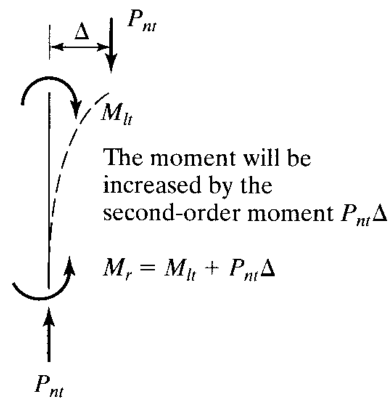
LRFD	ASD
$P_r = P_u = (1.2)(30\text{k}) + (1.6)(50\text{k}) = 116\text{ k}$	$P_r = P_a = 30\text{ k} + 50\text{ k} = 80\text{ k}$
$M_{rx} = M_{ux} = (1.2)(20\text{ ft-k}) + (1.6)(40\text{ ft-k})$	$M_{rx} = M_{ax} = 20\text{ ft-k} + 40\text{ ft-k}$
$= 88\text{ ft-k}$	$= 60\text{ ft-k}$
$P_c = \phi P_n = \phi_t F_y A_g = (0.9)(50\text{ ksi})(8.84\text{ in}^2)$	$P_c = \dfrac{P_n}{\Omega_c} = \dfrac{F_y A_g}{\Omega_c} = \dfrac{(50\text{ ksi})(8.84\text{ in}^2)}{1.67}$
$= 397.8\text{ k}$	$= 264.7\text{ k}$
$M_{cx} = \phi_b M_{nx} = C_b[\phi_b M_{px} - BF(L_b - L_p)]$	$M_{cx} = \dfrac{M_{nx}}{\Omega_b} = C_b\left[\dfrac{M_{px}}{\Omega_b} - BF(L_b - L_p)\right]$
$= 1.0[137 - 4.61(12.0 - 4.84)]$	$= 1.0[91.3 - (3.08)(12 - 4.84)]$
$= 104.0\text{ ft-k}$	$= 69.2\text{ ft-k}$
$\dfrac{P_r}{P_c} = \dfrac{116}{397.8} = 0.292 > 0.2$	$\dfrac{P_r}{P_c} = \dfrac{80}{264.7} = 0.302 > 0.2$
∴ **Must use AISC Eq. H1-1a**	∴ **Must use AISC Eq. H1-1a**
$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$	$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{rx}}{M_{cy}}\right) \le 1.0$
$\dfrac{116}{397.8} + \dfrac{8}{9}\left(\dfrac{88}{104.0} + 0\right)$	$\dfrac{80}{264.7} + \dfrac{8}{9}\left(\dfrac{60}{69.2} + 0\right)$
$= 1.044 > 1.0$ **N.G.**	$= 1.073 > 1.0$ **N.G.**

❖ First and Second Order Moment for Members Subjected to Axial Compression and Bending:

When a beam column is subjected to moment along its unsupported length, it will be displaced laterally in the plane of bending. The result will be an increase or secondary moment equal to the axial compression load times the lateral displacement or eccentricity. In the figure below, we can see that the member moment is increased by an amount (P_{nt} δ), where P_{nt} is the axial compression load determined by a first order analysis. This moment will cause additional lateral deflection, which will in turn cause a larger column moment, which will cause a larger lateral deflection, and so on until equilibrium is reached. M_r is the required moment strength of the member. M_{nt} is the first order moment, assuming no lateral translation of the frame.



If a frame is subjected to sidesway when the ends of the column can move laterally with respect to each other, additional secondary moments will result. In the figure below, the secondary moment produced due to sidesway is equal to $P_{nt}\,\Delta$. The moment Mr is assumed by the AISC specification to equal M_{lt} (which is the moment due to the lateral loads) plus the moment due to $P_{nt}\,\Delta$.



The required total flexural strength of a member must at least equal the sum of the first order and second order moments. Several methods are available for determining the required strength. The AISC specification chapter C states that we can either make a second order analysis to determine the maximum required strength or use a first order analysis or amplify the moments obtained with some amplification factors called B_1 and B_2.

- Approximate second order analysis:

You can find this method in appendix 8 of the AISC specification. Using this method we will make two first order analyses one an analysis where the frame is assumed to be braced so that it cannot sway. We will call this moment M_{nt} and will multiply them by a magnification factor B_1 to account for the Pδ effect. When we will analyze the frame again, allowing it to sway.

We will call these moments M_{lt} and will multiply them by a magnification factor B_2 to account for the $P\Delta$ effect.

The final moment in a member will equal,

$$M_r = B_1 M_{nt} + B_2 M_{lt} \quad (AISC\ equation\ C2-1a)$$

The final axial strength P_r must equal,

$$P_r = P_{nt} + B_2 P_{lt} \quad (AISC\ equation\ C2-1b)$$

- Magnification Factors:

The magnification factors are B_1 and B_2. With B_1, the analyst attempts to estimate the $P_{nt}\delta$ effect for a column, whether the frame is braced or unbraced against sidesway. With B_2, the analyst attempts to estimate the $P_{lt}\Delta$ effect in unbraced frames.

The horizontal deflection of a multistory building due to wind or seismic load is called drift (Δ). Drift is measured with drift index (Δ_H/L), where Δ_H is the first order lateral inter-story deflection and L is the story height. For the comfort of the occupants of the building, the index is usually limited at working loads to a value between 0.0015 and 0.0030, and at factored loads to about 0.0040.

The expression of B_1 was derived for a member braced against sidesway. It will be used only to magnify the M_{nt} moments (those moment computed assuming that there is no lateral translation of the frame).

$$B_1 = \frac{C_m}{1 - \alpha \frac{P_r}{P_{e1}}} \geq 1.0 \ (\, AISC\ Equation\ C2-2)$$

In this expression C_m is a term that is defined in the next section, α is a factor equal to 1 for the LRFD method; P_r is the required axial strength of the member, $and\ P_{e1}$ is the member Euler buckling strength calculated on the basis of zero sidesway.

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} \quad (\, AISC\ Equation\ C2-5)$$

One is permitted to use the first order estimate of P_r (that is, $P_r = P_{nt} + P_{lt}$) when calculating magnification factor B_1. Also, K_1 is the effective length factor in the plane of bending, determined based on the assumption of no lateral translation, and should be equal to 1.0 unless analysis justifies a smaller value.

In a similar fashion P_{e2} is the elastic critical buckling resistance for the story in question, determined by a sidesway buckling analysis. For this analysis, K_2L is the effective length in the plane of bending, based on the sidesway buckling analysis. For this case, the sidesway buckling resistance may be calculated with the following expression, in which Σ is used to include the entire column on that level or story.

$$\sum P_{e2} = \sum \frac{\pi^2 EI}{(K_2 L)^2} \quad (AISC\ equation\ C2-6a)$$

Furthermore, the AISC permits the use of the following alternative expression for calculating $\sum P_{e2}$

$$\sum P_{e2} = R_m \frac{\sum HL}{\Delta_H} \quad (AISC\ equation\ C2-6b)$$

Rm = 1 for braced frame system and 0.85 for moment frame system.

$\sum H$ = story shear produced by the lateral loads used to compute Δ_H, Kips

L = story length, in

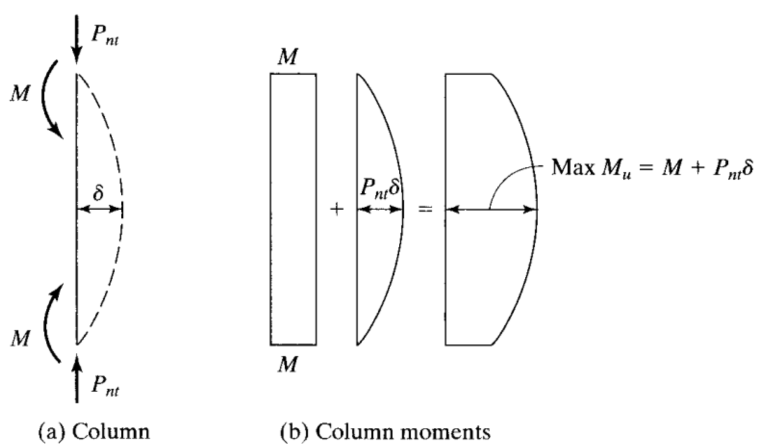
Δ_H = First order interstory drift due to the lateral loads

The value shown for $\sum P_{nt}$ and $\sum P_{e2}$ are for all of the columns on the floor in question. This is considered to be necessary because the B2 term is used to magnify column moments for sidesway. For sidesway to occur in a particular column, it is necessary for all the columns on the floor to sway simultaneously. The $\sum H$ value used in the first of the B2 expression represents the sum of the lateral loads acting above the floor being considered.

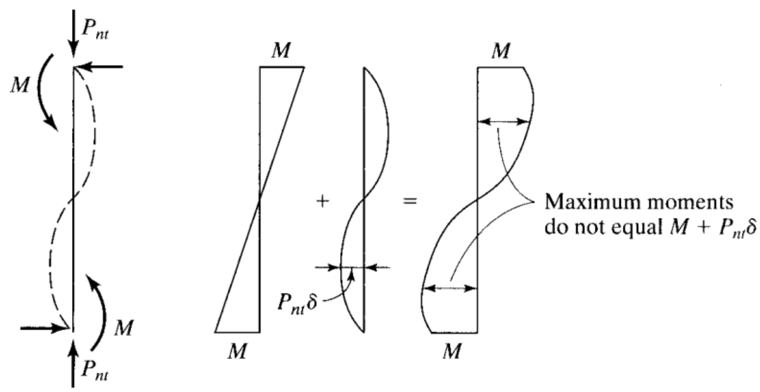
$$B_2 = \frac{1}{1 - \alpha \frac{\sum P_{nt}}{\sum P_{e2}}}$$

❖ Moment modification or C_m factors:

In the expression for B_1, a term C_m called the modification factor was included. The magnification factor B_1 was developed for the largest possible lateral displacement. On many occasions the displacement is not that large, and B_1 over magnifies the column moment. As a result, the moment may need to be reduced or modified with the C_m factor.



In the figure above, we have column bent in single curvature, with equal end moments such that the column bends laterally by an amount δ at mid depth. The maximum total moment occurs in the column clearly will equal M plus the increased moment $P_{nt}\delta$. As a result no modification is required and C_m = 1.0. An entirely different situation is considered in the figure below, where the end moments tend to bend the member in reveres curvature. The initial maximum moment occurs at one of the ends, and we should not increase by the value of $P_{nt}\delta$ because we will be overdoing the moment magnification. The purpose of the modification factor is to modify or reduce the magnified moment.



(a) Column (b) Column moments

Modification factor is based on the rotational restraint at the member ends and on the moment gradients in the member. The AISC specification C1 includes two categories of C_m.

In category 1, the members are prevented from joint translation or sidesway, and they are not subject to transverse loading between their ends. For such a member, the modification factor is based on an elastic first order analysis.

$$C_m = 0.6 - 0.4\frac{M_1}{M_2} \quad (AISC\ equation\ C2-4)$$

In this expression $\frac{M_1}{M_2}$ is the ratio of the smaller moment to the largest moment at the ends of the unbraced length in the plane of bending under consideration. The ratio is negative if the moments cause the member to bend in single curvature, and positive if they bend the member in reversed or double curvature.

Category 2 applies to members that are subjected to transverse loadings between the joints in the plane of loading. The AISC specification states that the value of Cm for this situation may be determined by rotational analysis or by setting it conservatively equal to 1.0. The value of Cm of category 2 may be determined for various end conditions and loads by the values given in Table C-C2.1.

$P_u = P_r$ = is the required column axial load

P_{e1} = is the elastic buckling load for a braced column.

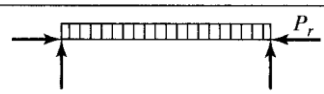
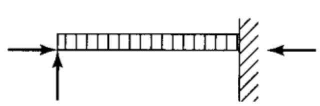
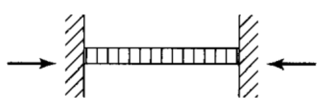
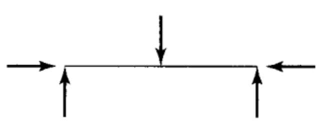
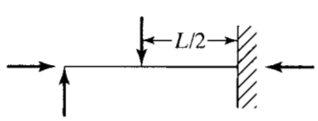
$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} \quad (\,AISC\ Equation\ C2-5)$$

- <u>Beam column in braced frame:</u>

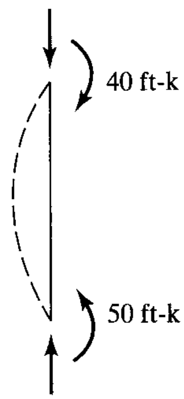
The same equations are used for member subjected to axial compression and bending as were used for member subjected to axial tension and bending. Pu is referring to compression force rather than tension force.

To analyze beam column or member subjected to both bending and axial compression, we need to make both first and second order moment analysis to obtain the bending moment. The first order moment is usually obtained by making an elastic analysis and consists for the moment M_{nt} (due to lateral loads – due to lateral translation)

Theoretically, if both the loads and frame are symmetrical M_{lt} will be zero. Similarly, if the frame is braced M_{lt} will be zero.

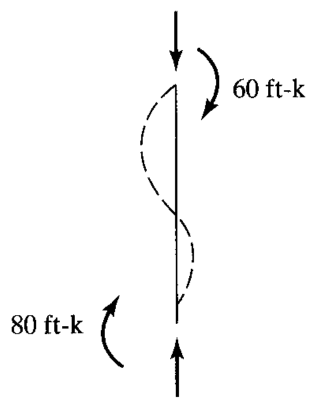
Case	ψ	C_m
	0	1.0
	−0.4	$1 - 0.4\dfrac{\alpha P_r}{P_{e1}}$
	−0.4	$1 - 0.4\dfrac{\alpha P_r}{P_{e1}}$
	−0.2	$1 - 0.2\dfrac{\alpha P_r}{P_{e1}}$
	−0.3	$1 - 0.3\dfrac{\alpha P_r}{P_{e1}}$
	−0.2	$1 - 0.2\dfrac{\alpha P_r}{P_{e1}}$

Source: Commentary on the Specification, Appendix 8–Table C–A–8.1, p16.1–525. June 22, 2010. "Copyright © American Institute of Steel Construction. Reprinted with permission. All rights reserved."



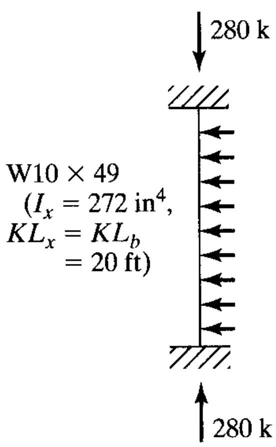
(a) No sidesway and no transverse loading.
Moments bend member in single curvature.

$$C_m = 0.6 - (0.4)\left(-\frac{40}{50}\right) = 0.92$$



(b) No sidesway and no transverse loading.
Moments bend member in reverse curvature.

$$C_m = 0.6 - 0.4\left(+\frac{60}{80}\right) = 0.30$$



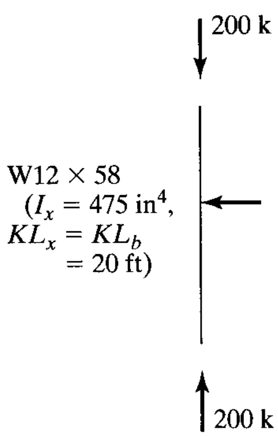
(c) Member has restrained ends and transverse loading and is bent about *x* axis.
C_m can be determined from Table 11.1 (AISC Table C-A-8.1) as follows:

$$\alpha P_r = 280 \text{ k}$$

$$P_{e1} = \frac{\pi^2 EI}{(KL)_x^2} = \frac{(\pi^2)(29 \times 10^3)(272)}{(12 \times 20)^2}$$

$$= 1351 \text{ k}$$

$$C_m = 1 - 0.4\left(+\frac{280}{1351}\right) = 0.92$$



(d) Member has unrestrained ends and transverse loading and is bent about *x* axis.
C_m can be determined from Table 11.1 (AISC Table C-A-8.1).

$$\alpha P_r = 200 \text{ k}$$

$$P_{e1} = \frac{(\pi^2)(29 \times 10^3)(475)}{(12 \times 20)^2} = 2360 \text{ k}$$

$$C_m = 1 - 0.2\left(+\frac{200}{2360}\right) = 0.98$$

❖ Example 3:

A 12-ft W12 × 96 (50 ksi steel) is used as a beam-column in a braced frame. It is bent in single curvature with equal and opposite end moments and is not subjected to intermediate transverse loads. Is the section satisfactory if $P_D = 175$ k, $P_L = 300$ k, and first-order $M_{Dx} = 60$ ft-k and $M_{Lx} = 60$ ft-k?

Solution. Using a W12 × 96 ($A = 28.2\ \text{in}^2$, $I_x = 833\ \text{in}^4$, $\phi_b M_{px} = 551$ ft-k, $L_p = 10.9$ ft, $L_r = 46.7$ ft, $BF = 5.78$ k for LRFD).

LRFD

$$P_{nt} = P_u = (1.2)(175) + (1.6)(300) = 690\ \text{k}$$

$$M_{ntx} = M_{ux} = (1.2)(60) + (1.6)(60) = 168\ \text{ft-k}$$

For a braced frame, let $K = 1.0$

$$\therefore (KL)_x = (KL)_y = (1.0)(12) = 12\ \text{ft}$$

$$P_c = \phi_c P_n = 1080\ \text{k (AISC Table 4-1)}$$

$$P_r = P_{nt} + \beta_2 P_{lt} = 690 + 0 = 690\ \text{k}$$

$$\frac{P_r}{P_c} = \frac{690}{1080} = 0.639 > 0.2$$

$\therefore$ Must use AISC Eq. H1-1a

$$C_{mx} = 0.6 - 0.4\frac{M_1}{M_2}$$

$$C_{mx} = 0.6 - 0.4\left(-\frac{168}{168}\right) = 1.0$$

LRFD

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_1L_x)^2} = \frac{(\pi^2)(29{,}000)(833)}{(1.0 \times 12 \times 12)^2}$$

$$= 11{,}498\ \text{k}$$

$$B_{1x} = \frac{C_{mx}}{1 - \frac{\alpha P_r}{P_{e1x}}} = \frac{1.0}{1 - \frac{(1.0)(690)}{11{,}498}} = 1.064$$

$$M_{rx} = B_{1x}M_{ntx} = (1.064)(168) = 178.8\ \text{ft-k}$$

Since $L_b = 12\ \text{ft} > L_p = 10.9\ \text{ft} < L_r = 46.6\ \text{ft}$

$\therefore$ **Zone 2**

$$\phi_b M_{px} = 1.0[551 - (5.78)(12 - 10.9)] = 544.6\ \text{ft-k}$$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right)$$

$$= \frac{690}{1080} + \frac{8}{9}\left(\frac{178.8}{544.6} + 0\right) = 0.931 < 1.0\ \text{OK}$$

$\therefore$ **Section is satisfactory.**

We can use table 6-1 and the following simplified equations to solve (example 3).

$$For\ pP_r \geq 0.2, \qquad pP_r + b_x M_{rx} + b_y M_{ry} \leq 1.0\ (Equation\ 6-1)$$

$$For\ pP_r < 0.2, \qquad \frac{1}{2} pP_r + \frac{9}{8}\left(b_x M_{rx} + b_y M_{ry}\right) \leq 1.0\ (Equation\ 6-2)$$

❖ <u>Example 4:</u>

Repeat Example 11-3, using the AISC simplified method of Part 6 of the Manual and the values for K, L, P_r and M_{rx} determined in that earlier example.

LRFD	ASD
$P_{e1x} = \dfrac{\pi^2 EI_x}{(K_1L_x)^2} = \dfrac{(\pi^2)(29{,}000)(833)}{(1.0 \times 12 \times 12)^2}$ $= 11{,}498$ k	$P_{e1x} = \dfrac{\pi^2 EI_x}{(K_1L_x)^2} = \dfrac{(\pi^2)(29{,}000)(833)}{(1.0 \times 12 \times 12)^2}$ $= 11{,}498$ k
$B_{1x} = \dfrac{C_{mx}}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \dfrac{1.0}{1 - \dfrac{(1.0)(690)}{11{,}498}} = 1.064$	$B_{1x} = \dfrac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \dfrac{1.0}{1 - \dfrac{(1.6)(475)}{11{,}498}} = 1.071$
$M_{rx} = B_{1x}M_{nt} = (1.064)(168) = 178.8$ ft-k	$M_{rx} = (1.071)(120) = 128.5$ ft-k
Since $L_b = 12$ ft $> L_p = 10.9$ ft $< L_r = 46.6$ ft	Since $L_b = 12$ ft $> L_p = 10.9$ ft $< L_r = 46.6$ ft
∴ **Zone 2**	∴ **Zone 2**
$\phi_b M_{px} = 1.0[551 - (5.78)(12 - 10.9)] = 544.6$ ft-k	$\dfrac{M_{px}}{\Omega_b} = 1.0[367 - 3.85\,(12 - 10.9)] = 362.7$ ft-k
$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right)$	$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) = \dfrac{475}{720} + \dfrac{8}{9}\left(\dfrac{128.5}{362.7} + 0\right)$
$= \dfrac{690}{1080} + \dfrac{8}{9}\left(\dfrac{178.8}{544.6} + 0\right) = 0.931 < 1.0$ OK	$= 0.975 < 1.0$ OK
∴ **Section is satisfactory.**	∴ **Section is satisfactory.**

❖ Example 5:

A 14-ft W14 × 120 (50 ksi steel) is used as a beam-column in a braced frame. It is bent in single curvature with equal and opposite moments. Its ends are rotationally restrained, and it is not subjected to intermediate transverse loads. Is the section satisfactory if $P_D = 70$ k, and $P_L = 100$ k and if it has the first-order moments $M_{Dx} = 60$ ft-k, $M_{Lx} = 80$ ft-k, $M_{Dy} = 40$ ft-k, and $M_{Ly} = 60$ ft-k?

Solution. Using a W14 × 120 ($A = 35.3$ in^2, $I_x = 1380$ in^4, $I_y = 495$ in^4, $Z_x = 212$ in^3, $Z_y = 102$ in^3, $L_p = 13.2$ ft, $L_r = 51.9$ ft, BF for LRFD $= 7.65$ k).

LRFD

$$P_{nt} = P_u = (1.2)(70) + (1.6)(100) = 244 \text{ k}$$

$$M_{ntx} = M_{ux} = (1.2)(60) + (1.6)(80) = 200 \text{ ft-k}$$

$$M_{nty} = M_{uy} = (1.2)(40) + (1.6)(60) = 144 \text{ ft-k}$$

For a braced frame $K = 1.0$

$$KL = (1.0)(14) = 14 \text{ ft}$$

$$P_c = \phi_c P_n = 1370 \text{ k (AISC Table 4-1)}$$

$$P_r = P_{nt} + \beta_2 P_{lt} = 244 + 0 = 244 \text{ k}$$

$$\frac{P_r}{P_c} = \frac{244}{1370} = 0.178 < 0.2$$

∴ Must use AISC Equation H1-1b

$$C_{mx} = 0.6 - 0.4\left(-\frac{200}{200}\right) = 1.0$$

$$P_{e1x} = \frac{(\pi^2)(29{,}000)(1380)}{(1.0 \times 12 \times 14)^2} = 13{,}995 \text{ k}$$

$$B_{1x} = \frac{1.0}{1 - \frac{(1.0)(244)}{13{,}995}} = 1.018$$

$$M_{rx} = (1.018)(200) = 203.6 \text{ ft-k}$$

$$C_{my} = 0.6 - 0.4\left(-\frac{144}{144}\right) = 1.0$$

$$P_{e1y} = \frac{(\pi^2)(29{,}000)(495)}{(1.0 \times 12 \times 14)^2} = 5020 \text{ k}$$

$$B_{1y} = \frac{1.0}{1 - \frac{(1.0)(244)}{5020}} = 1.051$$

$$M_{ry} = (1.051)(144) = 151.3 \text{ ft-k}$$

From AISC Table 6-1, for $KL = 14$ ft and $L_b = 14$ ft

$$p = 0.730 \times 10^{-3}, b_x = 1.13 \times 10^{-3},$$
$$b_y = 2.32 \times 10^{-3}$$

LRFD

$$\frac{1}{2}p\,P_r + \frac{9}{8}(b_x M_{rx} + b_y M_{ry}) \le 1.0$$

$$= \frac{1}{2}(0.730 \times 10^{-3})(244)$$

$$+ \frac{9}{8}(1.13 \times 10^{-3})(203.6)$$

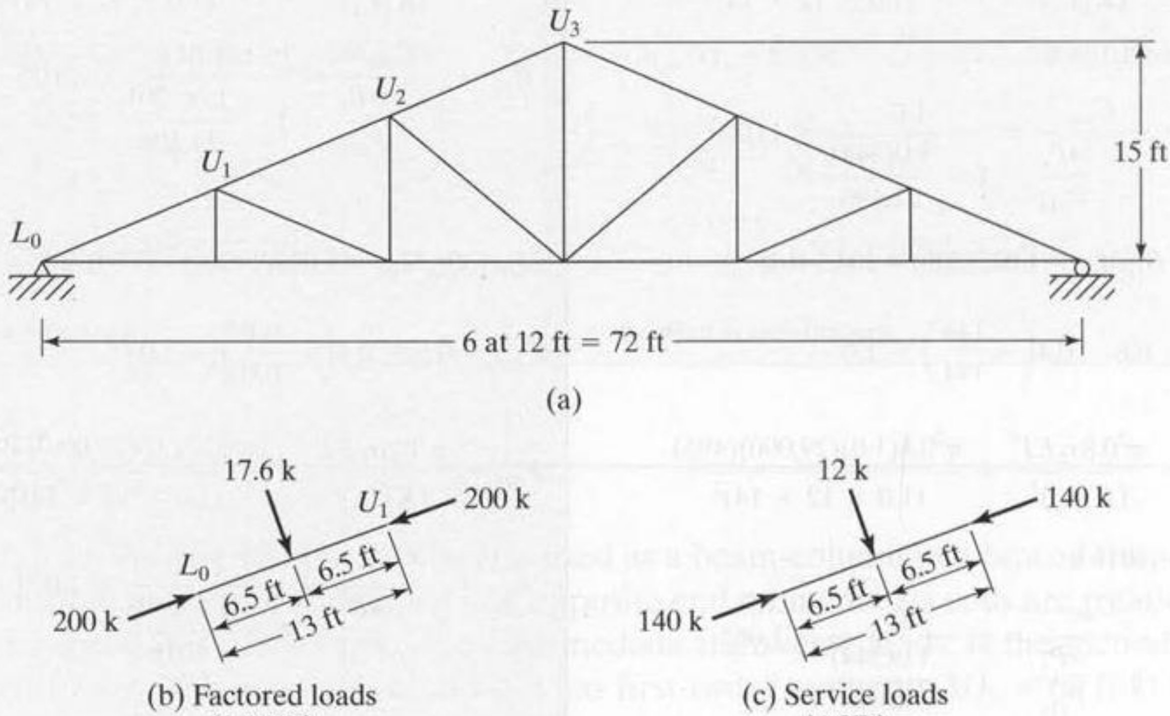
$$+ \frac{9}{8}(2.32 \times 10^{-3})(151.3)$$

$$= 0.743 \le 1.0 \quad \text{OK}$$

Section is satisfactory but perhaps overdesigned.

❖ Example 6:

For the truss shown in Fig. 11.7(a), a W8 × 35 is used as a continuous top chord member from joint L_0 to joint U_3. If the member consists of 50 ksi steel, does it have sufficient strength to resist the loads shown in parts (b) and (c) of the figure? The factored or LRFD loads are shown in part (b), while the service or ASD loads are shown in part (c). The 17.6 k and 12 k loads represent the reaction from a purlin. The compression flange of the W8 is braced only at the ends about the x-x axis, $L_x = 13$ ft, and at the ends and the concentrated load about the y-y axis, $L_y = 6.5$ ft and $L_b = 6.5$ ft.



(b) Factored loads (LRFD)

(c) Service loads (ASD)

Using a W8 × 35 ($A = 10.3$ in^2, $I_x = 127$ in^4, $r_x = 3.51$ in, $r_y = 2.03$ in, $L_P = 7.17$ ft, $\phi_b M_{Px} = 130$ ft-k, $\dfrac{M_{Px}}{\Omega_b} = 86.6$ ft-k, $r_x/r_y = 1.73$).

LRFD

$P_{nt} = P_u$ from figure $= 200 \text{ k} = P_r$

Conservatively assume $K_x = K_y = 1.0$. In truth, the K-factor is somewhere between $K = 1.0$ (pinned–pinned end condition) and $K = 0.8$ (pinned–fixed end condition) for segment L_oU_i

$$\left(\frac{KL}{r}\right)_x = \frac{(1.0)(12 \times 13)}{3.51} = 44.44 \leftarrow$$

$$\left(\frac{KL}{r}\right)_y = \frac{(1.0)(12 \times 6.5)}{2.03} = 38.42$$

From AISC Table 4-22, $F_y = 50$ ksi

$$\phi_c F_{cr} = 38.97 \text{ ksi}$$

$$\phi_c P_n = (38.97)(10.3) = 401.4 \text{ k} = P_c$$

$$\frac{P_r}{P_c} = \frac{200}{401.4} = 0.498 > 0.2$$

$\therefore$ Must use AISC Eq. H1-1a

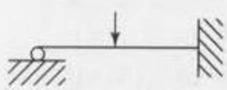
Computing P_{e1x} and C_{mx}

$$P_{e1x} = \frac{(\pi^2)(29{,}000)(127)}{(1.0 \times 12 \times 13)^2} = 1494 \text{ k}$$

From Table 11.1
For

$$C_{mx} = 1 - 0.2\left(\frac{1.0\,(200)}{1494}\right) = 0.973$$

For

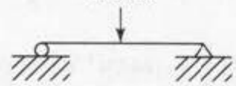


$$C_{mx} = 1 - 0.3\left(\frac{1.0\,(200)}{1494}\right) = 0.960$$

Avg $C_{mx} = 0.967$

Computing M_{ux}

For



$$M_{ux} = \frac{PL}{4} = \frac{(17.6)(13)}{4} = 57.2 \text{ ft-k}$$

For

$$M_{ux} = \frac{3\,PL}{16} = \frac{(3)(17.6)(13)}{16} = 42.9 \text{ ft-k}$$

Avg $M_{ux} = 50.05 \text{ ft-k} = M_{rx}$

$$B_{1x} = \frac{0.967}{1 - \dfrac{(1)(200)}{1494}} = 1.116$$

$M_r = (1.116)(50.05) = 55.86$ ft-k

Since $L_b = 6.5 \text{ ft} < L_p = 7.17$ ft

$\therefore$ Zone ①

$\phi_b M_{nx} = 130 \text{ ft-k} = M_{cx}$

Using Equation H1-1a

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0$$

$$\frac{200}{401.4} + \frac{8}{9}\left(\frac{55.86}{130} + 0\right) \le 1.0$$

$0.880 \le 1.0$ Section OK

From AISC Table 6-1

$(KL)_y = 6.5$ ft

$$(KL)_{yEQUIV} = \frac{(KL)_x}{r_x/r_y} = \frac{13}{1.73} = 7.51 \text{ ft} \leftarrow$$

$p = 2.50 \times 10^{-3}$, for $KL = 7.51$ ft

$b_x = 6.83 \times 10^{-3}$, for $L_b = 6.5$ ft

$p\,P_r + b_x\,M_{rx} + b_y\,M_{ry} \le 1.0$

$= (2.50 \times 10^{-3})\,(200) + (6.83 \times 10^{-3})\,(55.86) + 0$

$= 0.882 \le 1.0$ Section OK

Section is Satisfactory.

❖ Design of Beam Column Braced or Unbraced:

The design of beam column involves a trial and error procedure. A trail section is selected by a procedure and then checked with the appropriate interaction equation. If the section does not satisfy the equation, or if it is too much on the safe side (overdesigned), another section is selected and the interaction equation is applied again.

A common method used for selecting sections to resist both moment and axial loads is the equivalent axial load or effective axial load method. In this method the axial load P_u and the bending moments M_{ux}, M_{uy} are replaced with a fictitious concentric load P_{ueq}, equivalent to approximately to the actual axial load plus the moment effect.

Equations are used to convert the bending moment into an equivalent axial load P_u^-, which is added to the design axial load P_u. The total of P_u + P_u^- is equivalent or effective axial load P_{equ}, and it is used to enter the concentric column tables of part 4 of the AISC manual.

$$P_{equ} = P_u + M_{ux}m + M_{uy}mu$$

To apply this expression, a value of m is taken from the first approximation section of table 11-3, and u is assumed equal to 2. In applying the equation, the moments must be used in kft. The equation is solved for P_{equ}. After that a column is selected from the concentrically loaded column tables. Then the equation of P_{equ} is solved again with a revised value of m from the subsequent approximation part of the table, and the value of u is kept equal to 2.

TABLE 11.3 Preliminary Beam–Column Design F_y = 36 ksi, F_y = 50 ksi

Values of *m*														
F_y	36 ksi							50 ksi						
KL(ft)	10	12	14	16	18	20	22 and over	10	12	14	16	18	20	22 and over
1st Approximation														
All Shapes	2.0	1.9	1.8	1.7	1.6	1.5	1.3	1.9	1.8	1.7	1.6	1.4	1.3	1.2
Subsequent Approximation														
W4	3.1	2.3	1.7	1.4	1.1	1.0	0.8	2.4	1.8	1.4	1.1	1.0	0.9	0.8
W5	3.2	2.7	2.1	1.7	1.4	1.2	1.0	2.8	2.2	1.7	1.4	1.1	1.0	0.9
W6	2.8	2.5	2.1	1.8	1.5	1.3	1.1	2.5	2.2	1.8	1.5	1.3	1.2	1.1
W8	2.5	2.3	2.2	2.0	1.8	1.6	1.4	2.4	2.2	2.0	1.7	1.5	1.3	1.2
W10	2.1	2.0	1.9	1.8	1.7	1.6	1.4	2.0	1.9	1.8	1.7	1.5	1.4	1.3
W12	1.7	1.7	1.6	1.5	1.5	1.4	1.3	1.7	1.6	1.5	1.5	1.4	1.3	1.2
W14	1.5	1.5	1.4	1.4	1.3	1.3	1.2	1.5	1.4	1.4	1.3	1.3	1.2	1.2

Source: This table is from a paper in AISC *Engineering Journal* by Uang, Wattar, and Leet (1990).

❖ Example 7:

Select a trial W section for both LRFD and ASD for the following data: $F_y = 50$ ksi, $(KL)_x = (KL)_y = 12$ ft, $P_{nt} = 690$ k and $M_{ntx} = 168$ ft-k for LRFD, and $P_{nt} = 475$ k

LRFD	ASD
Assume B_1 and $B_2 = 1.0$	Assume B_1 and $B_2 = 1.0$
$\therefore\ P_r = P_u = P_{nt} + B_2(P_{lt})$	$\therefore\ P_r = P_a = P_{nt} + B_2(P_{lt})$
$P_u = 690 + 0 = 690$ k	$P_a = 475 + 0 = 475$ k
and, $M_{rx} = M_{ux} = B_1(M_{ntx}) + B_2(M_{ltx})$	and, $M_{rx} = M_{ax} = B_1(M_{ntx}) + B_2(M_{ltx})$
$M_{ux} = 1.0(168) + 0 = 168$ ft-k	$M_{ax} = 1.0(120) + 0 = 120$ ft-k
$P_{ueq} = P_u + M_{ux}\, m + M_{uy}\, mu$	$P_{aeq} = P_a + M_{ax} m + M_{ay} mu$
From "1st Approximation" part of Table 11.3	From "1st Approximation" part of Table 11.3
$m = 1.8$ for $KL = 12$ ft, $F_y = 50$ ksi	$m = 1.8$ for $KL = 12$ ft, $F_y = 50$ ksi
$u = 2.0$ (assumed)	$u = 2.0$ (assumed)
$P_{ueq} = 690 + 168(1.8) + 0 = 992.4$ k	$P_{aeq} = 475 + 120(1.8) + 0 = 691.0$ k
1st trial section: W12 × 96 ($\Phi_c P_n = 1080$ k) from AISC Table 4-1	1st trial section: W12 × 96 ($P_n/\Omega_c = 720$ k) from AISC Table 4-1
From "Subsequent Approximation" part of Table 11.3, W12's	From "Subsequent Approximation" part of Table 11.3, W12's
$m = 1.6$	$m = 1.6$
$P_{ueq} = 690 + 168(1.6) + 0 = 958.8$ k	$P_{aeq} = 475 + 120(1.6) + 0 = 667.0$ k
Try W12 × 87, ($\Phi_c P_n = 981$ k > 958.8 k)	**Try W12 × 96,** ($P_n/\Omega_c = 720$ k > 667.0 k)

Note: These are trial sizes. B_1 and B_2, which were assumed, must be calculated and these W12 sections checked with the appropriate interaction equations.

❖ Example 8:

Select a trial W section for both LRFD and ASD for an unbraced frame and the following data: $F_y = 50$ ksi, $(KL)_x = (KL)_y = 10$ ft.

For LRFD: $P_{nt} = 175$ k and $P_{lt} = 115$ k, $M_{ntx} = 102$ ft-k and $M_{ltx} = 68$ ft-k, $M_{nty} = 84$ ft-k and $M_{lty} = 56$ ft-k

For ASD: $P_{nt} = 117$ k and $P_{lt} = 78$ k, $M_{ntx} = 72$ ft-k and $M_{ltx} = 48$ ft-k, $M_{nty} = 60$ ft-k and $M_{lty} = 40$ ft-k

Solution

LRFD	ASD
Assume B_{1x}, B_{1y}, B_{2x} and $B_{2y} = 1.0$	Assume B_{1x}, B_{1y}, B_{2x} and $B_{2y} = 1.0$
$\therefore\ P_r = P_u = P_{nt} + B_2(P_{lt})$	$\therefore\ P_r = P_a = P_{nt} + B_2(P_{lt})$
$P_u = 175 + 1.0(115) = 290$ k	$P_a = 117 + 1.0(78) = 195$ k
and, $M_{rx} = M_{ux} = B_{1x}(M_{ntx}) + B_{2x}(M_{ltx})$	and, $M_{rx} = M_{ax} = B_{1x}(M_{ntx}) + B_{2x}(M_{ltx})$
$M_{ux} = 1.0(102) + 1.0(68) = 170$ ft-k	$M_{ax} = 1.0(72) + 1.0(48) = 120$ ft-k
and, $M_{ry} = M_{uy} = B_{1y}(M_{nty}) + B_{2x}(M_{lty})$	and, $M_{ry} = M_{ay} = B_{1y}(M_{nty}) + B_{2x}(M_{lty})$
$M_{uy} = 1.0(84) + 1.0(56) = 140$ ft-k	$M_{ay} = 1.0(60) + 1.0(40) = 100$ ft-k
$P_{ueq} = P_u + M_{ux}m + M_{uy}mu$	$P_{aeq} = P_a + M_{ax}\,m + M_{ay}mu$
From "1st Approximation" part of Table 11.3	From "1st Approximation" part of Table 11.3
$m = 1.9$ for $KL = 10$ ft, $F_y = 50$ ksi	$m = 1.9$ for $KL = 10$ ft, $F_y = 50$ ksi
$u = 2.0$ (assumed)	$u = 2.0$ (assumed)
$P_{ueq} = 290 + 170(1.9) + 140(1.9)(2.0) = 1145$ k	$P_{aeq} = 195 + 120(1.9) + 100(1.9)(2.0) = 803$ k
1st trial section from Table 4.1:	1st trial section from Table 4.1:
W14 → W14 × 99 ($\Phi_c P_n = 1210$ k)	W14 → W14 × 99 ($P_n/\Omega_c = 807$ k)
W12 → W12 × 106 ($\Phi_c P_n = 1260$ k)	W12 → W12 × 106 ($P_n/\Omega_c = 838$ k)
W10 → W10 × 112 ($\Phi_c P_n = 1280$ k)	W10 → W10 × 112 ($P_n/\Omega_c = 851$ k)
Suppose we decide to use a W14 section:	Suppose we decide to use a W14 section:
From "Subsequent Approximation" part of Table 11.3, W14's	From "Subsequent Approximation" part of Table 11.3, W14's
$m = 1.5$	$m = 1.5$
$P_{ueq} = 290 + 170(1.5) + 140(1.5)(2.0) = 965$ k	$P_{aeq} = 195 + 120(1.5) + 100(1.5)(2.0) = 675$ k
Try W14 × 90, ($\Phi_c P_n = 1100$ k > 965 k)	**Try W14 × 90,** ($P_n/\Omega_c = 735$ k > 675 k)

Note: These are trial sizes. B_{1x}, B_{1y}, B_{2x} and B_{2y}, which were assumed, must be calculated and these W14 sections checked with the appropriate interaction equations.

❖ Example 9:

Select the lightest W12 section for both LRFD and ASD for the following data: $F_y = 50$ ksi, $(KL)_x = (KL)_y = 12$ ft, $P_{nt} = 250$ k, $M_{ntx} = 180$ ft-k and $M_{nty} = 70$ ft-k for LRFD, and $P_{nt} = 175$ k, $M_{ntx} = 125$ ft-k and $M_{nty} = 45$ ft-k for ASD. $C_b = 1.0$, $C_{mx} = C_{my} = 0.85$.

LRFD

Assume $B_{1x} = B_{1y} = 1.0$, B_2 not required

$\therefore\ P_r = P_u = P_{nt} + B_2(P_{lt})$

$P_u = 250 + 0 = 250$ k

and, $M_{rx} = M_{ux} = B_1(M_{ntx}) + B_2(M_{ltx})$

$M_{ux} = 1.0(180) + 0 = 180$ ft-k

and, $M_{ry} = M_{uy} = B_1(M_{nty}) + B_2(M_{lty})$

$M_{uy} = 1.0(70) + 0 = 70$ ft-k

$P_{ueq} = P_u + M_{ux}m + M_{uy}mu$

From "Subsequent Approximation" part of Table 11.3, W12's

$m = 1.6$

$u = 2.0$ (assumed)

$P_{ueq} = 250 + 180(1.6) + 70(1.6)(2.0) = 762$ k

Try W12 × 72, $(\Phi_c P_n = 806 \text{ k} > 762 \text{ k})$ from Table 4.1

From Table 6.1 for $KL = 12$ ft and $L_b = 12$ ft

$p = 1.24 \times 10^{-3}, b_x = 2.23 \times 10^{-3}, b_y = 4.82 \times 10^{-3}$

$P_r/\Phi_c P_n = 250/806 = 0.310 > 0.2$
Use modified Equation H1-1a.

$1.24 \times 10^{-3}(250) + 2.23 \times 10^{-3}(180) + 4.82 \times 10^{-3}(70) = 1.049 > 1.0$ N.G.

Try W12 × 79, $(\Phi_c P_n = 887 \text{ k} > 762 \text{ k})$ from Table 4.1

From Table 6.1 for $KL = 12$ ft and $L_b = 12$ ft

$p = 1.13 \times 10^{-3}, b_x = 2.02 \times 10^{-3}, b_y = 4.37 \times 10^{-3}$

$1.13 \times 10^{-3}(250) + 2.02 \times 10^{-3}(180) + 4.37 \times 10^{-3}(70) = 0.952 < 1.0$ OK

LRFD

Check $B_{1x} = B_{1y} = 1.0$

$$P_{e1x} = \frac{\pi^2 EI^*}{(K_1L)^2} = \frac{\pi^2(29{,}000)(662)}{(1.0 \times 12 \times 12)^2} = 9138 \text{ k}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \frac{0.85}{1 - \dfrac{1.0(250)}{9138}} = 0.87 < 1;$$

$B_{1x} = 1.0$, OK

$$P_{e1y} = \frac{\pi^2 EI^*}{(K_1L)^2} = \frac{\pi^2(29{,}000)(216)}{(1.0 \times 12 \times 12)^2} = 2981 \text{ k}$$

$$B_{1y} = \frac{C_{my}}{1 - \dfrac{\alpha P_r}{P_{e1y}}} = \frac{0.85}{1 - \dfrac{1.0(250)}{2981}} = 0.93 < 1;$$

$B_{1y} = 1.0$, OK

With $B_{1x} = B_{1y} = 1.0$, section is sufficient based on previous check using modified Equation H1-1a.

Will perform additional check using Equation H1-1a:

For W12 × 79, $\Phi M_{px} = 446$ ft-k, $L_p = 10.8$ ft, $L_r = 39.9$ ft

$BF = 5.67$, $L_b = 12$ ft, Zone 2, $C_b = 1.0$, $\Phi M_{py} = 204$ ft-k

$\Phi M_{nx} = C_b [\Phi M_{px} - BF(L_b - L_p)]\ \Phi M_{px}$

$\Phi M_{nx} = 1.0 [446 - 5.67(12 - 10.8)] = 439.2$ ft-k

$\Phi M_{ny} = \Phi M_{py} = 204$ ft-k

Equation H1-1a:

$$\frac{250}{887} + \frac{8}{9}\left(\frac{180}{439.2} + \frac{70}{204}\right) = 0.951 < 1.0 \quad \text{OK}$$

Use W12 × 79, LRFD.

Chapter 8: Design of Beams for shear, deflection, etc

- Shear

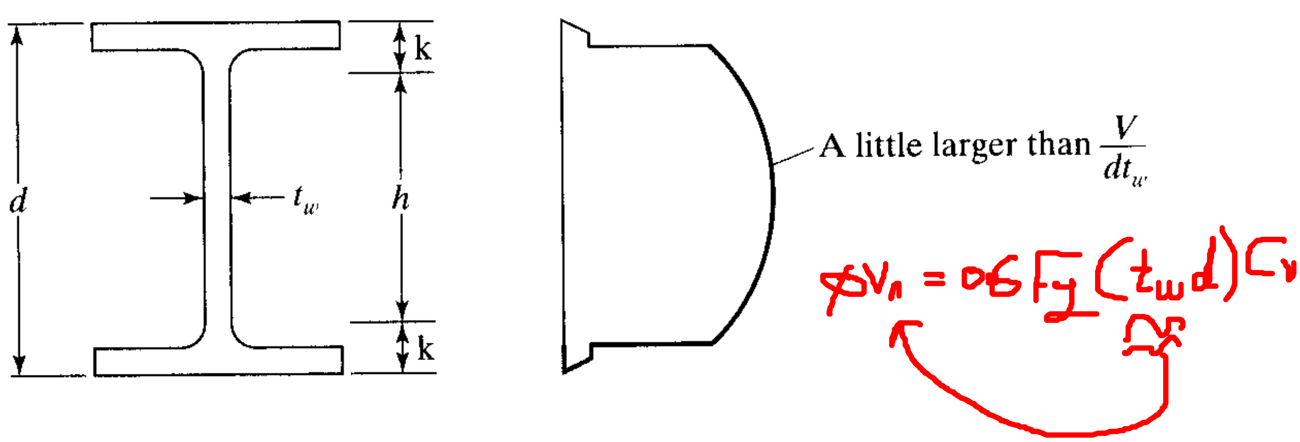
Generally, shear is not a problem is steel beams, because the webs of rolled shapes are capable of resisting large shear force. Perhaps it is well, however, to list here the most common situations where shear might be excessive:

1. Should large concentrated loads be placed near beam supports, they will cause large internal forces without corresponding increase in bending moments.
2. Where beams are notched or coped shear can be problem. For this case, shear forces must be calculated for the remaining beam depth. A similar discussion can be made where holes are cuts in beams webs for ductwork or other items.
3. Theatrically, very heavily loaded short beams can have excessive shears.
4. Shear may be a problem even for ordinary loading when very thin webs are used.

The shear stress formula $f_v = VQ/Ib$, where V is the external shear; Q is the statical moment of that portion of the section lying outside (either above or below) the line on which f_v is considered, taken about the neutral axis;

and b is the width of the section where the unit shear stress is desired. The figure below shows the variation in shear stress across the cross section of an I-shape member. It can be seen that the shear in I-shape section is primarily resisted by the web.

If the load is increased on an I-shape section until the bending yield stress is reached in the flange, the flange will be unable to resist shear stress and it will be carried in the web. If the moment is further increased, the bending yield stress will penetrate farther down into the web and the area of the web that can resist shear will be further decreased. Rather than assuming the nominal shear stress is resist by part of the web, the AISC specification assume that a reduced shear stress is resist by the entire web area. This web area, A_w, is equal to the overall depth of the member, d, times the web thickness, t_w.



The nominal shear strength of unstiffened or stiffened webs is specified as

$$V_n = 0.6F_yA_wC_v \qquad AISC\ Equation\ G2-1)$$

Using this equation for the webs of I-shapes members when $\frac{h}{t_w} \leq 2.24\sqrt{\frac{E}{F_y}}$, we find that $C_v = 1.0, and\ \varphi_v = 1.0$ (almost all current W, S, and HP shapes fall into this class. The exceptions are listed in Section G2 of the AISC specification.)

For the web of all doubly symmetric shapes, singly symmetric shapes, and channels, except round HSS, $\varphi_v = 0.9$ 1.0 is used to the design shear strength, φ_vV_n, the web shear coefficient, is determined from the following situations and substituted into AISC equation G2-1:

a. For $\frac{h}{t_w} \leq 1.10\sqrt{\frac{K_vE}{F_y}}$, $C_v = 1.0$ (AISC equation G2-3)

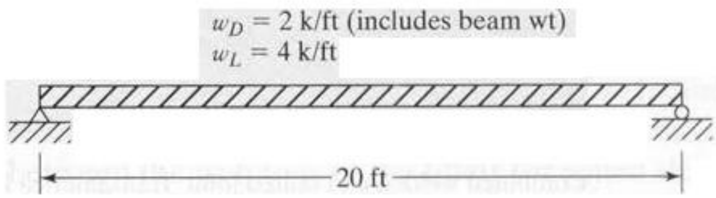
b. For $1.10\sqrt{\frac{K_vE}{F_y}} < \frac{h}{t_w} \leq 1.37\sqrt{\frac{K_vE}{F_y}}$, $C_v = \frac{1.10\sqrt{\frac{K_vE}{F_y}}}{\frac{h}{t_w}}$ (AISC equation G2-4)

c. For $\frac{h}{t_w} > 1.37\sqrt{\frac{K_vE}{F_y}}$, $C_v = \frac{1.51K_vE}{(\frac{h}{t_w})^2F_y}$ (AISC equation G2-5)

The web plate shear buckling coefficient, K_v, is specified in AISC specification G2.1b, parts (i) and (ii). For webs without transverse stiffeners with $\frac{h}{t_w} < 260$: K_v= 5. This is the case for must rolled I-shaped members designed by engineers.

❖ <u>Exaple1:</u>

A W21 × 55 with F_y = 50 ksi is used for the beam and loads of Fig. 10.4. Check its adequacy in shear,



Solution

Using a W21 × 55 (A = 16.2 in^2, d = 20.8 in, t_w = 0.375 in, and k_{des} = 1.02 in)

$$h = 20.8 - 2k_{des} = 20.8 - (2)(1.02) = 18.76 \text{ in}$$

$$\frac{h}{t_w} = \frac{18.76}{0.375} = 50.03 < 2.24\sqrt{\frac{29{,}000}{50}} = 53.95$$

$$\therefore C_v = 1.0,\ \phi_v = 1.0 \text{ and } \Omega_v = 1.50$$

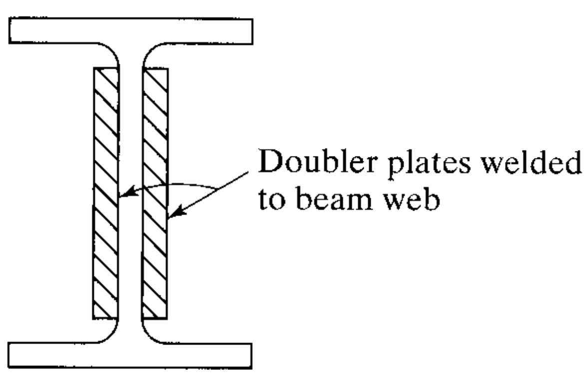
$$A_w = d\,t_w = (20.8 \text{ in})(0.375 \text{ in}) = 7.80 \text{ in}^2$$

$$\therefore V_n = 0.6\,F_y\,A_w\,C_v = 0.6\,(50 \text{ ksi})(7.80 \text{ in}^2)(1.0) = 234 \text{ k}$$

LRFD ϕ_v = 1.00
$w_u = (1.2)(2) + (1.6)(4) = 8.8$ k/ft
$V_u = \frac{8.8 \text{ k/ft } (20 \text{ ft})}{2} = 88$ k
$\phi_v V_n = (1.00)(234) = 234$ k
> 88 k **OK**

Notes:

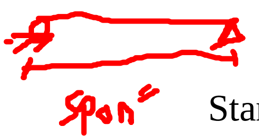
1. The values of $\varphi_v V_n$ with F_y = 50 ksi are given for W shapes in the manual table 3-2.
2. A very useful table 3-6 is provided in part 3 of the AISC manual for determining the maximum uniform load each W shape can support for various spans. The values given are for F_y = 50 ksi and are controlled by maximum moment or shares as specified by the LRFD.
3. Should V_u for a particular beam exceed the AISC specification shear strength of the member, the usual procedure will be to select a slightly heavier section. if it necessary to use a much heavier section than required for moment, doubler plates may be welded to the beam web, or stiffeners may be connected to the web in zones of high shear. Doubler plates must meet the width –thickness requirements for compact stiffened element, Section B4 of the AISC specification.



❖ Deflections:

The deflections of steel beams are usually limited to certain maximum values. Among the reasons for deflection limitation are the following:

1. Excessive deflection may damage other materials attached to or supported by the beams.
2. The appearance of structures is often damaged by excessive deflections.
3. Extreme deflections do not inspire confidence in the persons using a structure, although the structure may be completely safe from a strength standpoint.



Standard American practice for building has been to limit service live load deflections to approximately 1/360 of the span length. The 2010 AASHTO specifications limit deflection in steel beams and girders due to live load and impact to 1/800 of the span length. (for bridges in Urban areas that are shared by pedestrians, the AASHTO recommends a maximum value equal to 1/1000 of the span length).

The AISC specification does not specify exact maximum permissible deflections. There are so many different materials, types of structures, and loading that no one single set of deflection limitations is acceptable for all cases. Therefore, limitations must be set by the individual designer.

Note: the deflection limitation fall in the serviceability area. Therefore, deflections are determined for service loads.

❖ Example 2:

A W24 × 55 (I_x = 1350 in) has been selected for a 21-ft simple span to support a total service live load of 3 k/ft (including beam weight). Is the center line deflection of this section satisfactory for the service live load if the maximum permissible value is 1/360 of the span?

Solution. Use $E = 29 \times 10^6$ lb/in^2

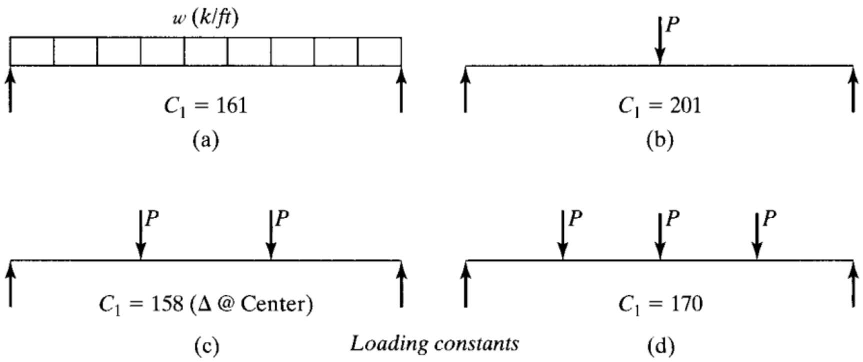
$$\Delta_{\text{℄}} = \frac{5wL^4}{384EI} = \frac{(5)(3000/12)(12 \times 21)^4}{(384)(29 \times 10^6)(1350)} = 0.335 \text{ in total load deflection}$$

$$< \left(\frac{1}{360}\right)(12 \times 21) = 0.70 \text{ in} \qquad \text{OK}$$

On the page 3-7 in the AISC manual, the following formula for determining maximum beam deflections for W, M, HP, S, C, and MC sections for several different loading conditions is presented:

$$\Delta = \frac{ML^2}{C_1 I_x}$$

In this expression, M is the maximum service load moment in (kft), C_1 is a constant whose value can determined from the figure above. L is the span length (ft), and I_x is the moment of inertia (in^4)



Loading constants

❖ Example 3:

Using the LRFD and ASD methods, select the lightest available section with F_y = 50 ksi to support a service dead load of 1.2 k/ft and a service live load of 3 k/ft for a 30-ft simple span. The section is to have full lateral bracing for its compression flange, and the maximum total service load deflection is not to exceed 1/1500 the span length.

Solution. After some scratch work, assume that beam wt = 167 lb/ft

LRFD

$w_u = 1.2(1.2 + 0.167) + (1.6)(3) = 6.44$ klf

$$M_u = \frac{(6.44 \text{ k/ft})(30 \text{ ft})^2}{8} = 724.5 \text{ ft-k}$$

From AISC Table 3-2, try W24 × 76 (I_x = 2100 in^4)

$$\text{Maximum permissible } \Delta = \left(\frac{1}{1500}\right)(12 \times 30) = 0.24 \text{ in}$$

$$\text{Actual } \Delta = \frac{ML^2}{C_1 I_x}$$

$$M = M_a = M_{service} = \frac{(4.37 \text{ k/ft})(30 \text{ ft})^2}{8}$$

$= 491.6$ ft-k

$$\Delta = \frac{(491.6)(30)^2}{(161)(2100)} = 1.31 \text{ in} > 0.24 \text{ in} \quad \textbf{N.G.}$$

Min I_x required to limit

Δ to 0.24 in

$$= \left(\frac{1.31}{0.24}\right)(2100) = 11{,}463 \text{ in}^4$$

From AISC Table 3-3

Use W40 × 167. (I_x = 11,600 in^4)

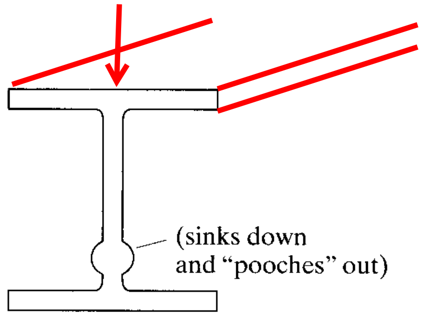
❖ Web under concentrated loads: 802

1. Local web yielding: The subject of local web yielding is applies to all concentrated loads, tensile or compressive. The nominal strength of the web of a beam at the web toe of the fillet when a concentrated load or reaction is applied is to be determined by one of the following two expressions, in which (k) is the distance outer edge of the flange to the web toe of the fillet, (l_b) is the length of bearing (in) of the force parallel to the plane of the web, (F_{yw}) is the specified minimum yield stress (ksi) of the web, and (t_w) is the thickness of the web. If the force is concentrated load or reaction that causes tension or compression and is applied at a distance greater than the member depth, d, from the end of the member, then

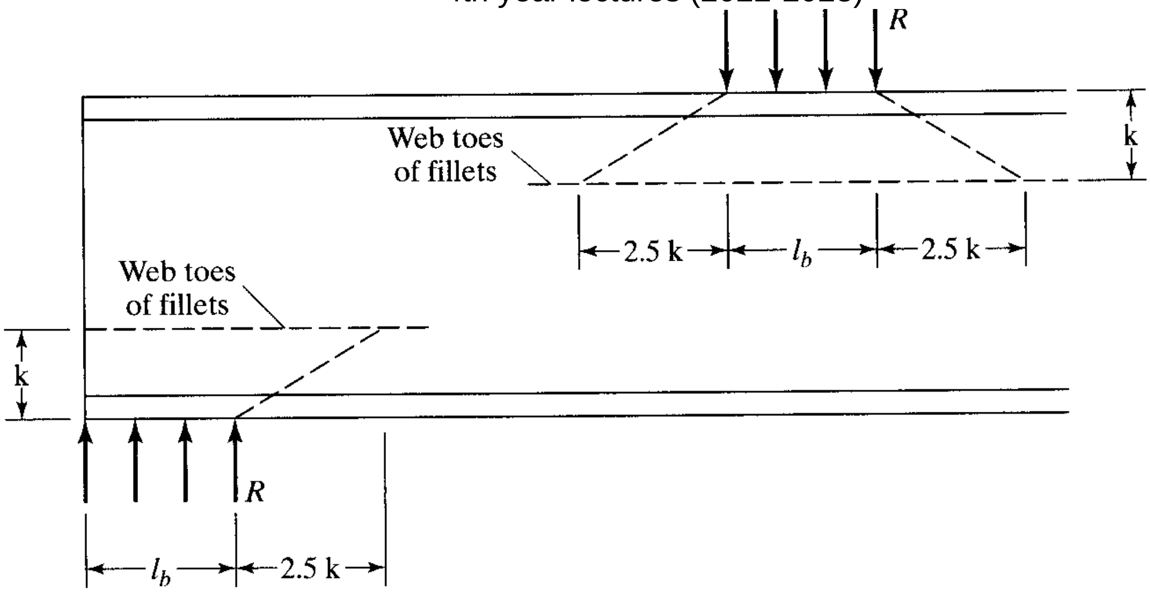
$$R_n = (5k + l_b)F_{yw}t_w \text{ , } \varphi = 1.0 \qquad (AISC\ Equation\ J10-2)$$

If the force is concentrated load or reaction applied at a distance d or less from the member end, then

$$R_n = (2.5k + l_b)F_{yw}t_w \text{ , } \varphi = 1.0 \qquad (AISC\ Equation\ J10-3)$$



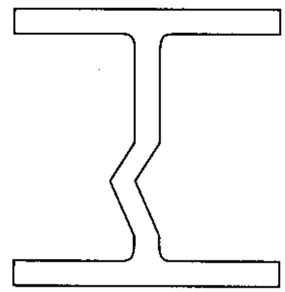
Local web yielding



The nominal strength R_n equals the length over which the force is assumed to be spread when it reaches the web toe of the fillet times the web thickness times the yield stress of the web. Should a stiffener extending for at least half the member depth or a doubler plate be provided on each side of the web at the concentrated load, it is not necessary to check for web yielding.

2. Web crippling: Should concentrated compressive loads be applied to a member with an unstiffened web (the load being applied in the plane of the web), the nominal web crippling strength is to be determined by the appropriate equation of the two that follow (in which d is the overall depth of the member). If one or two web stiffeners or one or two doubler plates are provided and extend for at least half of the web

depth, web crippling will not have to be checked. Research has shown when web crippling occurs; it is located in the part of the web adjacent to the loaded flange.



Web crippling

If the concentrated load is applied at a distance greater than or equal to d/2 from the end of the member, then

$$R_n = 0.8{t_w}^2\left[1+3\left(\frac{l_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}\ ,\varphi$$

$$= 0.75 \qquad (AISC\ Equation\ J10-4)$$

If the concentrated load is applied at a distance less than d/2 from the end of the member, then

$$for\ (\frac{l_b}{d}) \leq 0.2$$

$$R_n = 0.4{t_w}^2\left[1+3\left(\frac{l_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}\ ,\varphi$$

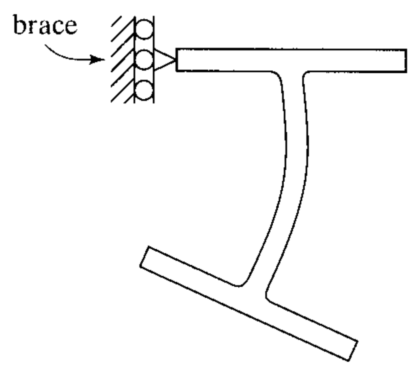
$$= 0.75 \qquad (AISC\ Equation\ J10-5a)$$

$$for\ (\frac{l_b}{d}) > 0.2$$

$$R_n = 0.4t_w^{\,2}\left[1+\left(4\frac{l_b}{d}-0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}\ ,\varphi$$

$$= 0.75 \quad (AISC\ Equation\ J10-5b)$$

3. Sidesway web buckling: Should compression be applied to laterally braced compression flange, the web will be but in compression and the tension flange may buckle as shown in the figure below.



Sidesway web buckling

It has been found that sidesway web buckling will not occur if the compression flange is restrained against rotation, with $(h/t_w)/(L_b/b_f) >$ 2.3, or if $(h/t_w)/(L_b/b_f) \le 1.7$ when the compression flange rotation is not restrained about its longitudinal axis. In these expressions, (h) is the web depth between the web toes of the fillet (d-2k) and (l_b) is the largest lateral unbraced length along either flange at the point of the load. Should member not be restrained against relative movement by stiffeners or lateral bracing and be subjected to concentrated compressive loads, their strength may be determined as follows:

When the loaded flange is braced against rotation and $(h/t_w)/(L_b/b_f) > 2.3$,

$$R_n = \frac{C_r t_w{}^3 t_f}{h^2}\left[1 + 0.4\left(\frac{\frac{h}{t_w}}{\frac{L_b}{b_f}}\right)^3\right], \varphi = 0.85 \quad (AISC\ Equation\ J10-6)$$

When the loaded flange is not restrained against rotation and $(h/t_w)/(L_b/b_f) \leq 1.7$,

$$R_n = \frac{C_r t_w{}^3 t_f}{h^2}\left[0.4\left(\frac{\frac{h}{t_w}}{\frac{L_b}{b_f}}\right)^3\right], \varphi = 0.85 \quad (AISC\ Equation\ J10-7)$$

It is not necessary to check equation J10-6 and J10-7 if the webs are subjected to distributed load. In these expressions,

C_r = 960000 ksi when $M_u < M_y$ at the location of the force.

C_r = 480000 ksi when $M_u \geq M_y$ at the location of the force.

4. Compression buckling of the web: This limit state relates to concentrated compressive loads applied to both flanges of a member. For this situation it is necessary to limit the slenderness ratio of the web to avoid the possibility of buckling. Should the concentrated load be larger than the value of $ØR_n$ given in next equation, it will be necessary to provide either one stiffener, a pair of stiffeners, or a doubler plates, extending for the full depth of the web and meeting the

requirements of AISC specification J10-8 (the equation to follow is applied to moment connection but not to bearing ones).

$$R_n = \frac{24t_w{}^3\sqrt{EF_{yw}}}{h}, \varphi = 0.9 \quad (AISC\ Equation\ J10-8)$$

If the concentrated force to be resisted is applied at a distance from the member end that is less than d/2, then the value of R_n is reduced by 50 percent.

- Stiffeners design:

If one of the web under concentrated loads checks is not satisfactory, then we need to design stiffeners at the location of the concentrated loads.

The stiffeners should be designed as axially compressed members in accordance with the requirements of section E6.2 and section J4.4. the member properties should be determined using an effective length of (kl = 0.75h) and a cross section composite of two stiffeners, and a strip of the web having a width of ($25t_w$) at interior stiffeners and ($12t_w$) at the ends of members.

$$the\ effective\ length = kl = 0.75h$$

$$b_s + \frac{t_w}{2} \geq \frac{b_f}{3}, \qquad t_s \geq \frac{t_f}{2}$$

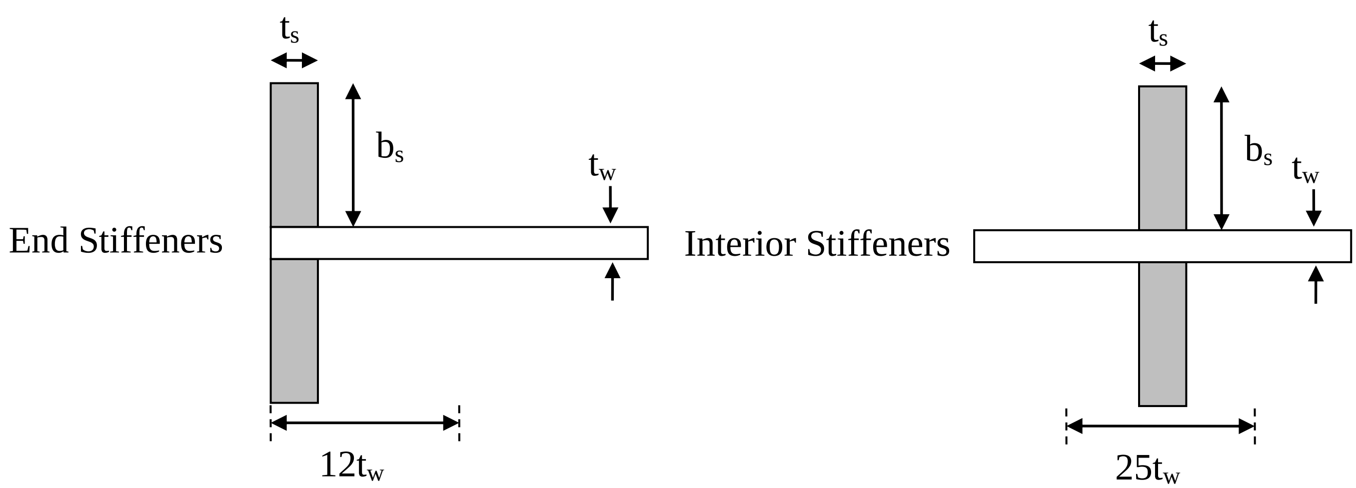
$$A_s = b_s * t_s$$

$$r_y = \sqrt{\frac{I_y}{A_s + 12{t_w}^2}} \qquad (or) r_y = \sqrt{\frac{I_y}{A_s + 25{t_w}^2}}$$

$$find \left(\frac{kl}{r_y}\right), then\ \varphi_c F_{cr}$$

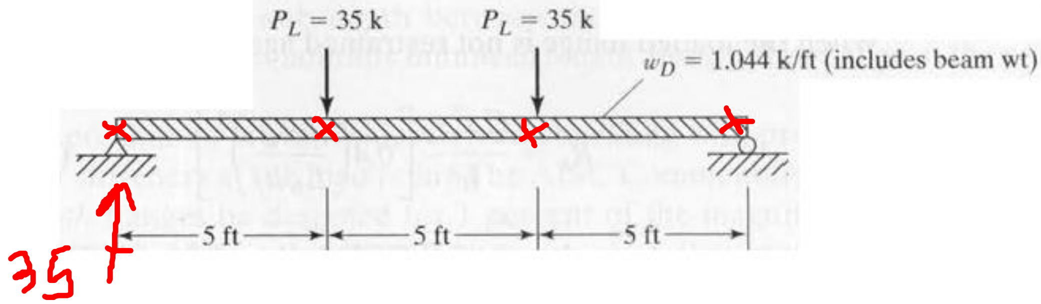
$$A_s + 12{t_w}^2 * \varphi_c F_{cr} = R \ \ (should\ be\ larger\ than\ the\ applied\ load)$$

$$A_s + 25{t_w}^2 * \varphi_c F_{cr} = R \ \ (should\ be\ larger\ than\ the\ applied\ load)$$



❖ <u>Example 4:</u>

A W21x44 has been selected for moment in the beam shown in the figure below. Lateral bracing is provided for both flanges at beam end and at concentrated loads. If the end bearing length is 3.5 in and the concentrated load bearing lengths are each 3.1 in, check the beam for web yielding, web crippling, and sidesway web buckling.



Solution

Using a W21 × 44 ($d = 20.7$ in, $b_f = 6.50$ in, $t_w = 0.350$ in, $t_f = 0.450$ in, $k = 0.950$ in)

LRFD
End reaction $R_u = (1.2)(1.044\text{ k/ft})\left(\frac{15\text{ ft}}{2}\right) + (1.6)(35\text{ k})$ $= 65.4$ k Concentrated load $P_u = (1.6)(35\text{ k}) = 56$ k

Local web yielding

(l_b = bearing length of reactions = 3.50 in, for concentrated loads $l_b = 3.00$ in)

At end reactions (AISC Equation J10-3)

$$R_n = (2.5k + l_b)F_{yw}t_w = (2.5 \times 0.950\text{ in} + 3.50\text{ in})(50\text{ ksi})(0.350\text{ in}) = 102.8\text{ k}$$

LRFD $\phi = 1.00$
$\phi R_n = (1.00)(102.8) = 102.8$ k > 65.4 k **OK**

At concentrated loads (AISC Equation J10-2)

$$R_n = (5\text{ k} + l_b)F_{yw}t_w = (5 \times 0.950\text{ in} + 3.00\text{ in})(50\text{ ksi})(0.350\text{ in}) = 135.6\text{ k}$$

LRFD $\phi = 1.00$
$\phi R_n = (1.00)(135.6) = 135.6\text{ k}$ $> 56\text{ k}$ **OK**

Web crippling

At end reactions (AISC Equation J10-5a) since $\frac{l_b}{d} \leq 0.20$

$$\frac{N}{d} = \frac{3.5}{20.7} = 0.169 < 0.20$$

$$R_n = 0.40t_w^2\left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}$$

$$= (0.40)(0.350\text{ in})^2\left[1 + 3\left(\frac{3.5\text{ in}}{20.7\text{ in}}\right)\left(\frac{0.350\text{ in}}{0.450\text{ in}}\right)^{1.5}\right]\sqrt{\frac{(29 \times 10^3\text{ ksi})(50\text{ ksi})(0.450\text{ in})}{0.350\text{ in}}}$$

$$= 90.3\text{ k}$$

LRFD $\phi = 0.75$
$\phi R_n = (0.75)(90.3) = 67.7\text{ k}$ $> 65.4\text{ k}$ **OK**

At concentrated loads (AISC Equation J10-4)

$$R_n = 0.80\,t_w^2\left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}$$

$$= (0.80)(0.350)^2\left[1 + 3\left(\frac{3.0}{20.7}\right)\left(\frac{0.350}{0.450}\right)^{1.5}\right]\sqrt{\frac{(29 \times 10^3)(50)(0.450)}{0.350}}$$

$$= 173.7\text{ k}$$

LRFD $\phi = 0.75$
$\phi R_n = (0.75)(173.7)$ $= 130.3 \text{ k} > 56 \text{ k}$ **OK**

Sidesway web buckling
The compression flange is restrained against rotation.

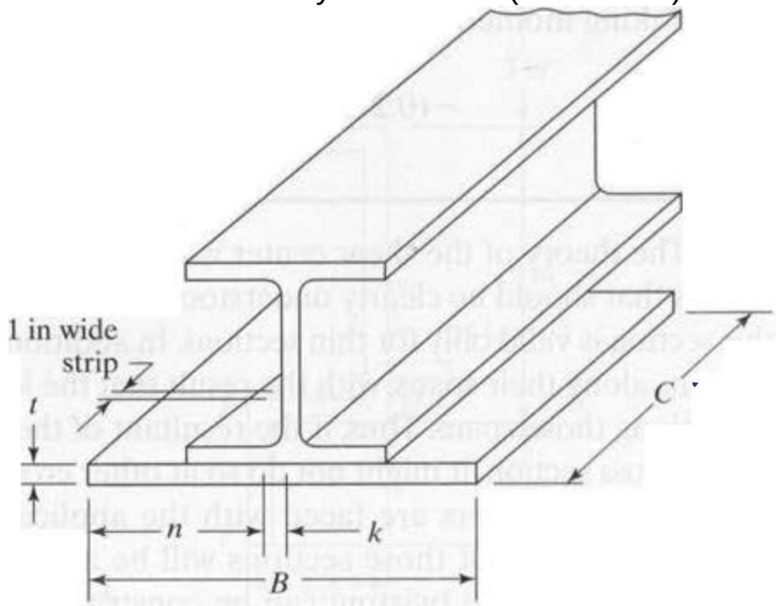
$$\frac{h}{t_w} \Big/ \frac{L_b}{b_f} = \frac{20.7 \text{ in} - 2 \times 0.950 \text{ in}}{0.350 \text{ in}} \Big/ \left(\frac{12 \text{ in/ft} \times 5 \text{ ft}}{6.50 \text{ in}} \right) = 5.82 > 2.3$$

∴ **Sidesway web buckling does not have to be checked.**

❖ <u>Design of beam bearing plate:</u>

When the ends of beams are supported by direct bearing on concrete or other masonry construction, it is necessary to distribute the beam reaction over the masonry by mean of beam bearing plate. The reaction is assumed to spread uniformly through the bearing plate to the masonry, and the masonry is assumed to push up against the plate with a uniform pressure equal to the reaction R_u over the area of the plate A_1. This pressure tends to curl up the plate and the bottom flange of the beam.

The determination of the true pressure distribution in a beam bearing plate is a very difficult task, and the uniform pressure distribution assumption is usually made.



The required thickness of a 1 in wide strip of plate can be determined as follows,

$$Z\ of\ a\ 1\ in\ wide\ piece\ of\ plate\ of\ t\ thickness = 1 * \frac{t}{2} * \frac{t}{4} * 2 = \frac{t^2}{4}$$

The moment Mu is computed at a distance k from the web centerline and is equated to $\varphi_b F_y Z$; the resultant equation is then solved for the required plate thickness.

LRFD $\phi_b = 0.90$
$M_u = \frac{R_u}{A_1} n \left(\frac{n}{2}\right) = \frac{R_u n^2}{2A_1}$ $\frac{R_u n^2}{2A_1} = \phi_b F_y \frac{t^2}{4}$ From which $t_{reqd} = \sqrt{\frac{2R_u n^2}{\phi_b A_1 F_y}}$

The design strength for bearing on concrete is to be taken equal to $\varphi_c P_p$ according to AISC Specification J8. This specification states that when a bearing plate extends for the full area of a concrete support, the bearing strength of the concrete can be determined as follows:

$$P_p = 0.85 f\acute{c} A_1 \quad LRFD\ equation\ J8-1$$

Should the bearing load be applied to an area less than the full area of the concrete support, $\varphi_c P_p$ is to be determined with the following equation, in which A_2 is the maximum area of the supporting surface that is geometrically similar the loaded area, with $\sqrt{A_2/A_1}$ having a maximum value of 2:

$$P_p = 0.85 f\acute{c} A_1 \sqrt{\frac{A_2}{A_1}} \leq 1.7 f\acute{c} A_1 \quad AISC\ Equation\ J8-2$$

In these expressions $f\acute{c}$ is the compression strength of the concrete in psi and A_1 is the area of the plate in^2

For the design of such plate, its required area A_1 can be determined by dividing the factored reaction R_u by $\varphi_c 0.85 f\acute{c}$

$$A_1 = \frac{R_u}{\varphi_c 0.85 f\acute{c}} \quad with\ \varphi_c = 0.65$$

After A_1 is determined, its length parallel to the beam and its width are selected. The length may not be less that the N required to prevent web yielding or crippling of the beam, nor may it may be less than about 3 ½ or 4 in for practical construction reasons. It may not be greater than the thickness of the wall or other support.

❖ Example 5:

A W18 × 71 beam ($d = 18.5$ in, $t_w = 0.495$ in, $b_f = 7.64$ in, $t_f = 0.810$ in, $k = 1.21$ in) has one of its ends supported by a reinforced-concrete wall with $f'_c = 3$ ksi. Design a bearing plate for the beam with A36 steel, for the service loads $R_D = 30$ k and $R_L = 50$ k. The maximum length of end bearing ⊥ to the wall is the full wall thickness = 8.0 in.

Solution

Compute plate area A_1.

LRFD $\phi_c = 0.65$	ASD $\Omega_c = 2.31$
$R_u = (1.2)(30) + (1.6)(50) = 116$ k	$R_a = 30 + 50 = 80$ k
$A_1 = \frac{R_u}{\phi_c 0.85 f'_c} = \frac{116}{(0.65)(0.85)(3)}$	$A_1 = \frac{\Omega_c R_a}{0.85 f'_c} = \frac{(2.31)(80)}{(0.85)(3)}$
$= 70.0$ in^2	$= 72.5$ in^2
Try PL 8 × 10 (80 in^2).	Try PL 8 × 10 (80 in^2).

Check web local yielding.

$$R_n = (2.5k + l_b)F_{yw}t_w \quad \text{(AISC Equation J10-3)}$$

$$= (2.5 \times 1.21 + 8)(36)(0.495) = 196.5 \text{ k}$$

LRFD $\phi = 1.00$	ASD $\Omega = 1.50$
$R_u = \phi R_n = (1.00)(196.5)$ $= 196.5\text{ k} > 116\text{ k}$ **OK**	$R_a = \dfrac{R_n}{\Omega} = \dfrac{196.5}{1.50}$ $= 131\text{ k} > 80\text{ k}$ **OK**

Check web crippling.

$$\frac{l_b}{d} = \frac{8}{18.5} = 0.432 > 0.2 \quad \therefore \text{ Must use AISC Equation (J10-5b)}$$

$$R_n = 0.40t_w^2\left[1 + \left(\frac{4l_b}{d} - 0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}$$

$$= (0.40)(0.495)^2\left[1 + \left(\frac{4 \times 8}{18.5} - 0.2\right)\left(\frac{0.495}{0.810}\right)^{1.5}\right]\sqrt{\frac{(29 \times 10^3)(36)(0.810)}{0.495}}$$

$$= 221.7\text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$R_u = \phi R_n = (0.75)(221.7)$ $= 166\text{ k} > 116\text{ k}$ **OK**	$R_a = \dfrac{R_n}{\Omega} = \dfrac{221.7}{2.00} = 111\text{ k}$ $>80\text{ k}$ **OK**

Determine plate thickness.

$$n = \frac{10}{2} - 1.21 = 3.79\text{ in}$$

LRFD $\phi_b = 0.90$	ASD $\Omega_b = 1.67$
$t = \sqrt{\dfrac{2R_u n^2}{\phi_b A_1 F_y}} = \sqrt{\dfrac{(2)(116)(3.79)^2}{(0.9)(80)(36)}}$ $= 1.13\text{ in}$ **Use PL $1\frac{1}{4} \times 8 \times 10$ (A36).**	$t = \sqrt{\dfrac{2R_a n^2 \Omega_b}{A_1 F_y}} = \sqrt{\dfrac{(2)(80)(3.79)^2(1.67)}{(80)(36)}}$ $= 1.15\text{ in}$ **Use PL $1\frac{1}{4} \times 8 \times 10$ (A36).**

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If we were to check to see if the flange thickness alone is sufficient, we would have $\left(\text{with } n = \frac{b_f}{2} - k\right) = \frac{7.64}{2} - 1.21 = 2.61$ in.

LRFD $\phi_b = 0.90$	ASD $\Omega_b = 1.67$
$t = \sqrt{\frac{(2)(116)(2.61)^2}{(0.9)(8 \times 7.64)(36)}}$	$t = \sqrt{\frac{(2)(80)(2.61)^2(1.67)}{(8)(7.64)(36)}}$
$= 0.893$ in $> t_f = 0.810$ in for W18 × 71 **N.G.**	$= 0.910 > t_f = 0.810$ in for W18 × 71 **N.G.**

∴ Flange t_f is not sufficient alone for either LRFD or ASD designs.