Civil Engineering

1st. Year

كلية المنصور الجامعة / الصف الاول / الرياضيات / د عماد

تمارين خاصة بالملازمة (٢) .

Exercises To Solve :

بسط التعابير الاتية

[1] Simplify the expression :

① $\log_2\left(e^{(\ln 2)(\sin x)}\right)$ ② $\log_4\left(2^{e^x \sin x}\right)$

③ $\log_e\left(e^x\right)$ ④ $\log_9 3x$

⑤ $25^{\log_5(3x^2)}$ ⑥ $\dfrac{\log_9 x}{\log_3 x}$ ⑦ $\dfrac{\log_x a}{\log_{x^2} a}$

⑧ $\dfrac{\log_{\sqrt{10}} x}{\log_{\sqrt{2}} x}$

حل المعادلات الاتية

[2] Solve the equations :

① $8^{\log_8(3)} - e^{\ln 5} = x^2 - 7^{\log_7(3x)}$

② $3^{\log_3(x^2)} = 5e^{\ln x} - 3 \cdot 10^{\log_{10}(2)}$

③ $\ln e + 4^{-2\log_4(x)} = \dfrac{1}{x}\log_{10}(100)$

اوجد المشتقة

[3] Find $\dfrac{dy}{dx}$

① $y = (x+1)^x$ ② $y = x^{(x+1)}$ ③ $y = x^{\ln x}$

④ $y = (\ln x)^{\ln x}$ ⑤ $y = x\log_3\left(e^{(\sin x)\ln(3)}\right)$

⑥ $y = 3\log_8(\log_2 x)$ ⑦ $y = \log_7\left(\dfrac{\sin x \cos x}{e^x\, 2^x}\right)$

⑧ $y = \log_2\left(\dfrac{x^2 e^2}{2\sqrt{x+1}}\right)$ ⑨ $y = \log_5\left(\sqrt{\left(\dfrac{7x}{3x+2}\right)^{\ln 5}}\right)$.

Solutions

الحلول

[1] Simplify the expression :

1. $\log_2\left(e^{(\ln 2)(\sin x)}\right) = \dfrac{\ln\left(e^{\ln 2 \sin x}\right)}{\ln 2} = \dfrac{\ln 2 \sin x}{\ln 2} = \sin x.$

2. $\log_4\left(2^{e^x \sin x}\right) = \dfrac{\ln\left(2^{e^x \sin x}\right)}{\ln 4} = \dfrac{e^x \sin x \cancel{\ln 2}}{2\cancel{\ln 2}}$

$= \frac{1}{2} e^x \sin x$.

3. $\log_e(e^x) = \dfrac{\ln e^x}{\ln e} = \dfrac{x}{1} = x.$

4. $\log_9 3x = \dfrac{\ln 3x}{\ln 9} = \dfrac{\ln(3x)}{2\ln 3} = \dfrac{\ln 3 + \ln x}{2\ln 3}.$

5. $25^{\log_5(3x^2)} = e^{\log_5(3x^2)\cdot \ln(25)} = e^{\frac{\ln 3x^2}{\cancel{\ln 5}}\cdot 2\cancel{\ln 5}} = e^{2\ln 3x^2} = \cancel{e^{\ln}}(3x^2)^2$

$= 9x^4$

6. $\dfrac{\log_9 x}{\log_3 x} = \dfrac{\frac{\ln x}{\ln 9}}{\frac{\ln x}{\ln 3}} = \dfrac{\frac{\ln x}{2\ln 3}}{\frac{\ln x}{\ln 3}} = \dfrac{1}{2}.$

7. $\dfrac{\log_x a}{\log_{x^2} a} = \dfrac{\frac{\ln a}{\ln x}}{\frac{\ln a}{\ln x^2}} = \dfrac{\frac{\ln a}{\ln x}}{\frac{\ln a}{2\ln x}} = 2.$

8. $\dfrac{\log_{\sqrt{10}} x}{\log_{\sqrt{2}} x} = \dfrac{\frac{\ln x}{\ln\sqrt{10}}}{\frac{\ln x}{\ln\sqrt{2}}} = \dfrac{\ln\sqrt{2}}{\ln\sqrt{10}} = \dfrac{\ln 2^{1/2}}{\ln 10^{1/2}} = \dfrac{\frac{1}{2}\ln 2}{\frac{1}{2}\ln 10}$

$= \dfrac{\ln 2}{\ln 10}$

[2] Solve the equations:

1. $8^{\log_8(3)} - e^{\ln 5} = x^2 - 7^{\log_7(3x)}$

$e^{\log_8(3)\cdot \ln 8} - 5 = x^2 - e^{\log_7(3x)\cdot \ln 7}$

$e^{\frac{\ln 3}{\ln 8}\cdot \ln 8} - 5 = x^2 - e^{\frac{\ln(3x)}{\ln 7}\cdot \ln 7}$

$e^{\ln 3} - 5 = x^2 - e^{\ln 3x} \Rightarrow 3 - 5 = x^2 - 3x$

$\Rightarrow x^2 - 3x + 2 = 0 \Rightarrow (x-2)(x-1) = 0$

$\boxed{x=2}$ $\boxed{x=1}$

2. $3^{\log_3(x^2)} = 5e^{\ln x} - 3\cdot 10^{\log_{10}(2)}$

$e^{\log_3(x^2)\cdot \ln 3} = 5x - 3e^{\log_{10}(2)\cdot \ln 10}$

$e^{\frac{\ln x^2}{\ln 3}\cdot \ln 3} = 5x - 3e^{\frac{\ln 2}{\ln 10}\cdot \ln 10}$

$x^2 = 5x - 3(2) \Rightarrow x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$\boxed{x=3}$ $\boxed{x=2}$

3 $\ln e + 4^{-2\log_4(x)} = \frac{1}{x}\log_{10}(100)$

$$1 + e^{-2\log_4(x)\cdot\ln 4} = \frac{1}{x}\,\frac{\ln 100}{\ln 10}$$

$$1 + e^{-\frac{2\ln x}{\ln 4}\cdot\ln 4} = \frac{1}{x}\,\frac{2\ln 10}{\ln 10}$$

$$1 + e^{\ln x^{-2}} = \frac{1}{x}(2) \Rightarrow 1 + \frac{1}{x^2} = \frac{2}{x}$$

$$\Rightarrow \frac{x^2+1}{x^2} = \frac{2}{x} \Rightarrow 2x^2 = x^3 + x$$

$$\Rightarrow x^3 + x - 2x^2 = 0 \Rightarrow x^3 - 2x^2 + x = 0 \Rightarrow x(x^2 - 2x + 1) = 0$$

$$\Rightarrow \boxed{x = 0} \Rightarrow x^2 - 2x + 1 = 0$$

$$(x-1)(x-1) = 0$$

$$\boxed{x = +1} \quad \boxed{x = 1}$$

[4] Find $\frac{dy}{dx}$

5 $y = x\log_3\left(e^{(\sin x)\ln 3}\right) = x\,\frac{\ln\left(e^{\sin x\ln 3}\right)}{\ln 3} = x\,\frac{\sin x\,\cancel{\ln 3}}{\cancel{\ln 3}}$

$$\therefore y = x\sin x \Rightarrow \frac{dy}{dx} = x\cos x + \sin x\,(1).$$

6 $y = 3\log_8(\log_2 x) = 3\,\frac{\ln(\log_2 x)}{\ln 8} = 3\,\frac{\ln\left(\frac{\ln x}{\ln 2}\right)}{\ln 8}$

$$y = \cancel{3}\,\frac{[\ln(\ln x) - \ln(\ln 2)]}{\cancel{3}\ln 2} = \frac{1}{\ln 2}\left[\ln(\ln x) - \ln(\ln 2)\right]$$

$$\frac{dy}{dx} = \frac{1}{\ln 2}\left[\frac{\frac{1}{x}}{\ln x} - 0\right] = \frac{1}{\ln 2\,(x\ln x)}$$

7 $$y = \log_7\left(\frac{\sin x \cos x}{e^x \; 2^x}\right) = \frac{\ln\left(\frac{\sin x \cos x}{e^x \; 2^x}\right)}{\ln 7}$$

$$y = \frac{1}{\ln 7}\left[\ln(\sin x \cos x) - \ln(e^x \; 2^x)\right]$$

$$y = \frac{1}{\ln 7}\left[\ln(\sin x) + \ln(\cos x) - (\ln e^x + \ln 2^x)\right]$$

$$y = \frac{1}{\ln 7}\left[\ln(\sin x) + \ln(\cos x) - x - x\ln 2\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{\ln 7}\left[\frac{\cos x}{\sin x} + \frac{-\sin x}{\cos x} - 1 - \ln 2\right]$$

$$= \frac{1}{\ln 7}\left[\cot x - \tan x - 1 - \ln 2\right].$$

8 $$y = \log_2\left(\frac{x^2 e^2}{2\sqrt{x+1}}\right) = \frac{\ln\left(\frac{x^2 e^2}{2\sqrt{x+1}}\right)}{\ln 2}$$

$$y = \frac{1}{\ln 2}\left[\ln(x^2 e^2) - \ln(2\sqrt{x+1})\right]$$

$$y = \frac{1}{\ln 2}\left[2\ln x + 2\ln e - (\ln 2 + \tfrac{1}{2}\ln(x+1))\right]$$

$$y = \frac{1}{\ln 2}\left[2\ln x + 2 - \ln 2 - \tfrac{1}{2}\ln(x+1)\right]$$

$$\frac{dy}{dx} = \frac{1}{\ln 2}\left[\frac{2}{x} + 0 - 0 - \frac{1}{2}\,\frac{1}{x+1}\right]$$

$$\frac{dy}{dx} = \frac{2}{x\ln 2} - \frac{1}{2\ln 2\,(x+1)}$$

9 $$y = \log_5\left(\sqrt{\left(\frac{7x}{3x+2}\right)^{\ln 5}}\right)$$

$$y = \frac{\ln\left(\sqrt{\left(\frac{7x}{3x+2}\right)^{\ln 5}}\right)}{\ln 5} = \frac{1}{\ln 5}\left[\frac{1}{2}\ln\left(\frac{7x}{3x+2}\right)^{\ln 5}\right]$$

$$= \frac{1}{\cancel{\ln 5}}\left[\frac{\cancel{\ln 5}}{2}\ln\left(\frac{7x}{3x+2}\right)\right]$$

$$= \frac{1}{2}\left[\ln(7x) - \ln(3x+2)\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\cdot\left[\frac{7}{7x} - \frac{3}{3x+2}\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x} - \frac{3}{3x+2}\right].$$

د. عماد

H.W.1

/ المادة الرياضيات - ١ /

Civil Engineering
1st. Year

/ تمارين خاصة بالملزمة (١) / الصف الاول.

Exercises To solve :

[1] Find the domain and the range of the following functions :

(1) $y = f(x) = \sqrt{4 - x^2}$ (2) $y = \sqrt{x^2 - 16}$

(3) $y = \dfrac{1}{x-2}$ (4) $y = \dfrac{1}{x^2 - 9}$

(5) $y = \dfrac{x}{x^2+4}$ (6) $y = \dfrac{x-1}{x^2+2}$

(7) $y = x^2 + 4$ (8) $y = \dfrac{2x}{(x-2)(x+1)}$

(9) $y = \dfrac{x^2-1}{x^2+1}$ (10) $y = \dfrac{1}{\sqrt{9-x^2}}$

(11) $y = \sqrt{x^2+4}$ (12) $y = \dfrac{x}{x+3}$

(13) $y = \sqrt{x-4}$ (14) $y = \sqrt{\dfrac{x}{2-x}}$

[2] Find :

(1) $\lim\limits_{x \to 2} \dfrac{4 - x^2}{3 - \sqrt{x^2+5}}$ (2) $\lim\limits_{x \to \infty} \dfrac{3x-2}{9x+7}$

(3) $\lim\limits_{x \to 2} \dfrac{x-2}{\sqrt{x^2-4}}$ (4) $\lim\limits_{x \to 1} \dfrac{x-1}{x^2-1}$

(5) $\lim\limits_{x \to 2} \dfrac{x^2-4}{x^2-5x+8}$ (6) $\lim\limits_{x \to -1} \dfrac{x^2+3x+2}{x^2+4x+3}$

(7) $\lim\limits_{h \to 0} \dfrac{(x+h)^2 - x^2}{h}$ (8) $\lim\limits_{x \to 1} \dfrac{x-1}{\sqrt{x^2+3} - 2}$.

[3] Evaluate

(1) $\lim\limits_{x \to \infty} \dfrac{x+3}{x^2+5x+6}$ (2) $\lim\limits_{x \to \infty} \dfrac{x}{x^2+5}$

(3) $\lim\limits_{x \to \infty} \dfrac{2x+3}{4x-5}$ (4) $\lim\limits_{x \to \infty} \dfrac{2x^2+1}{6+x-3x^2}$

(5) $\lim\limits_{x \to \infty} \dfrac{x^2+5x+6}{x+1}$.

[4] Let $f(x) = x^2 - 3x$, find $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

[5] Let $f(x) = \sqrt{5x+1}$, find $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

when $x > -\frac{1}{5}$.

[6] If $f(x) = \dfrac{x-1}{x+1}$, prove that ① $f(\frac{1}{x}) = -f(x)$

② $f(-\frac{1}{x}) = \dfrac{-1}{f(x)}$

[7] If $f(x) = x^2 - x$, prove that ① $f(1+x) = f(-x)$

[8] If $f(x) = \dfrac{1}{x}$, prove $f(a) - f(b) = f\left(\dfrac{ab}{b-a}\right)$

[9] If $y = f(x) = \dfrac{(5x+3)}{(4x-5)}$ prove that $x = f(y)$.

H.W.2

كلية المنصور الجامعة

Civil Engineering

1st. Year

Exercises To Solve:

[1] Evaluate the following limits: (if there exist)

(1) $\lim_{x \to 0} \frac{\sin^2 2x}{x \sin^2 3x}$

(2) $\lim_{x \to 0} \frac{\sin ax}{\sin bx}$

(3) $\lim_{x \to 0} \frac{\sin x (1 - \cos x)}{x^2}$

(4) $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$

(5) $\lim_{x \to 0} \frac{1 - \cos x}{\sin x}$

(6) $\lim_{x \to 0} x \cot x$

(7) $\lim_{x \to 2} \sin\left(\frac{1}{x} - \frac{1}{2}\right)$

(8) $\lim_{x \to 0} (\sec x + \tan x)$

(9) $\lim_{x \to \pi} \cos^2 x$

(10) $\lim_{x \to 0} \frac{\sin(a+x) - \sin a}{x}$

(11) $\lim_{x \to 0} x \csc x$

(12) $\lim_{x \to \frac{\pi}{4}} \frac{\sin x}{\cos x}$

(13) $\lim_{x \to 0} \frac{\cos(a+x) - \cos a}{x}$

(14) $\lim_{x \to \pi} \sec(1 + \cos x)$

(15) $\lim_{x \to \infty} \sin\left(\frac{\pi x}{2 - 3x}\right)$

(16) $\lim_{x \to 0} \frac{x}{\cos(\frac{\pi}{2} - x)}$

(17) $\lim_{x \to \frac{\pi}{2}} \frac{\sin(x - \frac{\pi}{2})}{(\frac{\pi}{2} - x)}$

(18) $\lim_{x \to -\frac{\pi}{2}} \frac{\sin(\frac{\pi}{2} + x)}{(x + \frac{\pi}{2})}$

قوانين مهمة

(1) $\sin(x - \frac{\pi}{2}) = -\cos x$

(2) $\sin(x + \frac{\pi}{2}) = \cos x$

(3) $\cos(x - \frac{\pi}{2}) = \sin x$

(4) $\cos(x + \frac{\pi}{2}) = -\sin x$

(5) $\sin(\pi - x) = \sin x$

(6) $\cos(\pi - x) = -\cos x$

عمودي، أفقي، مائل

[2] Shows the asymptotes of

(1) $y = \frac{x}{x-1}$ (2) $y = \frac{8}{2-x^2}$

(3) $y = \frac{1}{2x-3}$ (4) $y = \frac{x^2-4}{x^2-1}$

(5) $y = \frac{x^2+1}{x^2-4x+3}$ (6) $y = \frac{x^2+8}{x^2-4}$

(7) $y = \frac{x-4}{x-5}$ (8) $y = x - \frac{1}{x}$

(9) $y = \frac{x}{2} + \frac{2}{x}$ (10) $y = x^2 + \frac{1}{x^2}$

(11) $y = \frac{x^2+1}{x^3-4x}$ (12) $y = \frac{x^3-4x}{x^3-x}$

H.W.3

كلية المنصور الجامعة

Civil Engineering

1st. Year

Exercises To solve :

[1] If $f(x) = \sin(x^2)$ and $y = f((2x-1)/(x+1))$. Find $\frac{dy}{dx}$.

[2] If $f(x) = \sqrt{3x^2 - 1}$ and $y = f(x^2)$. Find $\frac{dy}{dx}$.

[3] Given $y = 3\sin 2x$ and $x = u^2 + \pi$, find the value of $\frac{dy}{dx}$ when $u = 0$.

[4] Find $\frac{dy}{dx}$ if $x = \cos 3t$ and $y = \sin^2 3t$.

[5] If $y = x^2 + 1$ and $u = \sqrt{x^2+1}$, find $\frac{dy}{du}$.

[6] If $x = y^2 + y$ and $u = (x^2 + x)^{3/2}$, find $\frac{dy}{du}$.

[7] If $x = t - t^2$ and $y = t - t^3$, find the value of $\frac{dy}{dx}$ at $t = 1$.

[8] Find the value of

$$\lim_{\Delta x \to 0} \frac{[2-3(x+\Delta x)]^2 - [2-3x]^2}{\Delta x}, \quad \Delta x \neq 0.$$

[9] Find

(1) $\lim_{x \to \infty} \frac{x}{x-1}$, (2) $\lim_{s \to \infty} \left(\frac{s}{s+1}\right)\left(\frac{s^2}{5+s^2}\right)$

(3) $\lim_{x \to \infty} \left(\frac{1}{x}+1\right)\left(\frac{5x^2-1}{x^2}\right)$ (4) $\lim_{x \to \infty} \left(1+\cos\frac{1}{x}\right)$

(5) $\lim_{x \to \infty} \frac{\sin 2x}{x}$ (6) $\lim_{x \to \infty} \frac{x+\sin x}{x+\cos x}$

(7) $\lim_{x \to 0} \frac{|x|}{|x|+1}$ (8) $\lim_{x \to -5} \frac{x^2+3x-10}{x+5}$

[10] Find the domain and the range of the functions

(1) $y=\sqrt{\frac{1}{x}-1}$

(2) $y=\frac{x}{x-2}$, (3) $y=1+\frac{1}{x}$

(4) $y=\frac{1}{x-2}$, (5) $y=\frac{1}{x+1}$

(6) $y=\frac{x}{x+1}$ (7) $y=\sin^{-1}(5x-1)$

(8) $y=\cos^{-1}(3x+2)$ (9) $y=\sin^{-1}(2x+1)$

(10) $y=\sec^{-1}(3x+2)$ (11) $y=\csc^{-1}(3x+2)$

(12) $y=\sec^{-1}(3x+3)$ (13) $y=\csc^{-1}(2x+1)$

(14) $y=\sin x$ (15) $y=\cos x$

الصف الاول
الرياضيات I
د. عماد

H.W.4

Civil Engineering
1st. Year

Exercises To Solve: (سئلة خاصة عن المحاضرة (٢) (دوال زائدية + مشتقات زائدية))

[1] Find $\frac{dy}{dx}$ of the following functions:

(1) $y = \cosh^{-1}(x^2 + \sin^3 x)$. (4) $y = x^3 \cdot e^{-x^2} \cdot \cosh^3 x \cdot \sinh^{-1} x$

(2) $y = \sinh^{-1}(x^2 + \sin^2 x)$. (5) $y = 3^{\tanh^{-1} 2x} \cdot \ln(\sec 2x)$

(3) $y = 2^x \operatorname{sech} 2x$. (6) $y = (\tan^{-1} x)^{\frac{x \sinh x}{x^2+1}}$

[2] Find the asymptotes (الخاذيات) of the following functions

(1) $y = \tanh(\ln\sqrt{x})$. (2) $y = \coth(\ln\sqrt{x})$.

[3] If $y = (\sinh^{-1} x)^{e^x}$. Find $\frac{d e^{\sin^{-1} y}}{d \ln x}$.

[4] If $y = (\sinh x)^{\ln x}$. Find $\frac{d e^{\sinh^{-1} y}}{d e^x}$.

[5] Find $\frac{dy}{dx}$ of the following function:

$$y = \frac{(x^2+1)^{3/2} \cdot (x^2-1)^{-3/2} \cdot \tanh^{-1}(\sinh 2x)}{(x^2+4)^{2/3} \cdot \sinh^{-3}(2x)}$$

[6] prove that: $\cosh 2x = \cosh^2 x + \sinh^2 x$

$= 2\sinh^2 x + 1$

$= 2\cosh^2 x - 1$

[7] prove that: $\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$.

[8] if $y = a \cosh \frac{x}{a}$ then $\frac{d^2y}{dx^2} = \frac{1}{a}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$
(prove that).

[9] prove that: if $y = A \cosh bx + B \sinh bx$
where A, B, b Constants, then $\frac{d^2y}{dx^2} = b^2 y$.

[10] Find $\frac{dy}{dx}$ of the following functions:

(1) $y = \tanh^{-1}(\sin x)$.

(2) $x = a \operatorname{sech}^{-1}\left(\frac{y}{a}\right) - \sqrt{a^2 - y^2}$. , where a; constant

(3) $y = 2 \tanh^{-1}\left(\tan \frac{x}{2}\right)$.

(4) $y = \tanh(1 + x^2)$.

(5) $y = x \operatorname{sech} x^2$.

(6) $y = \operatorname{csch}^2(x^2+1)$.

(7) $y = \frac{1}{4} \sinh 2x - \frac{1}{2} x$.

(8) $y = \ln(\tanh 2x)$.

(9) $y = \ln(\tanh^{-1} 2x)$.

(10) $y = e^{\tanh 2x}$.

[11] prove that: if $x = \operatorname{sech}^{-1} y - \sqrt{1-y^2}$, then $\frac{dy}{dx} = \frac{-y}{\sqrt{1-y^2}}$

Notes: **الملاحظات :**

Notes: **الملاحظات :**

Notes: **الملاحظات :**

Notes: **الملاحظات :**