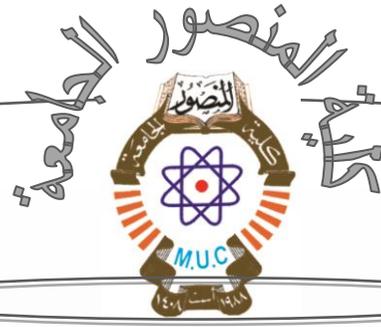


قسم الهندسة المدنية

المرحلة الثالثة



Al-Mansour University College

Civil Eng. Dept.

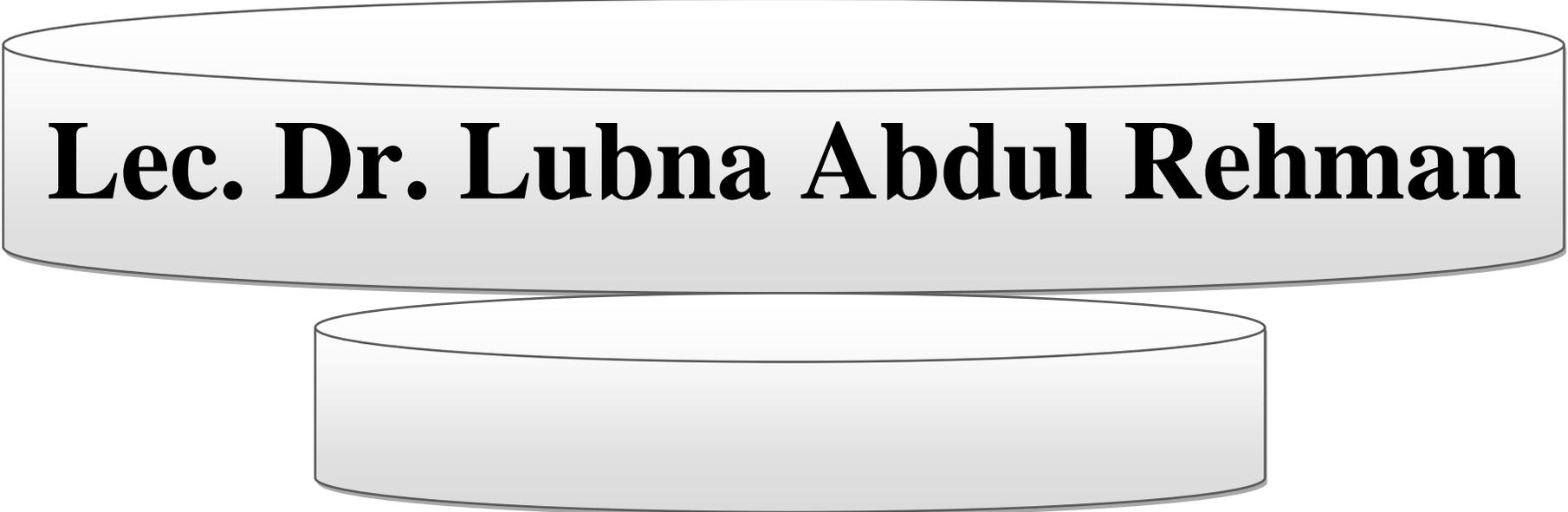
3rd. Stage

Soil Mechanics

2022 - 2023

ميكانيك التربة

Lec.2



Lec. Dr. Lubna Abdul Rehman

Ministry of Higher Education and Scientific Researches
Al-Mansour University College
Civil Engineering Department
Class 3



Basic Phase Relationships

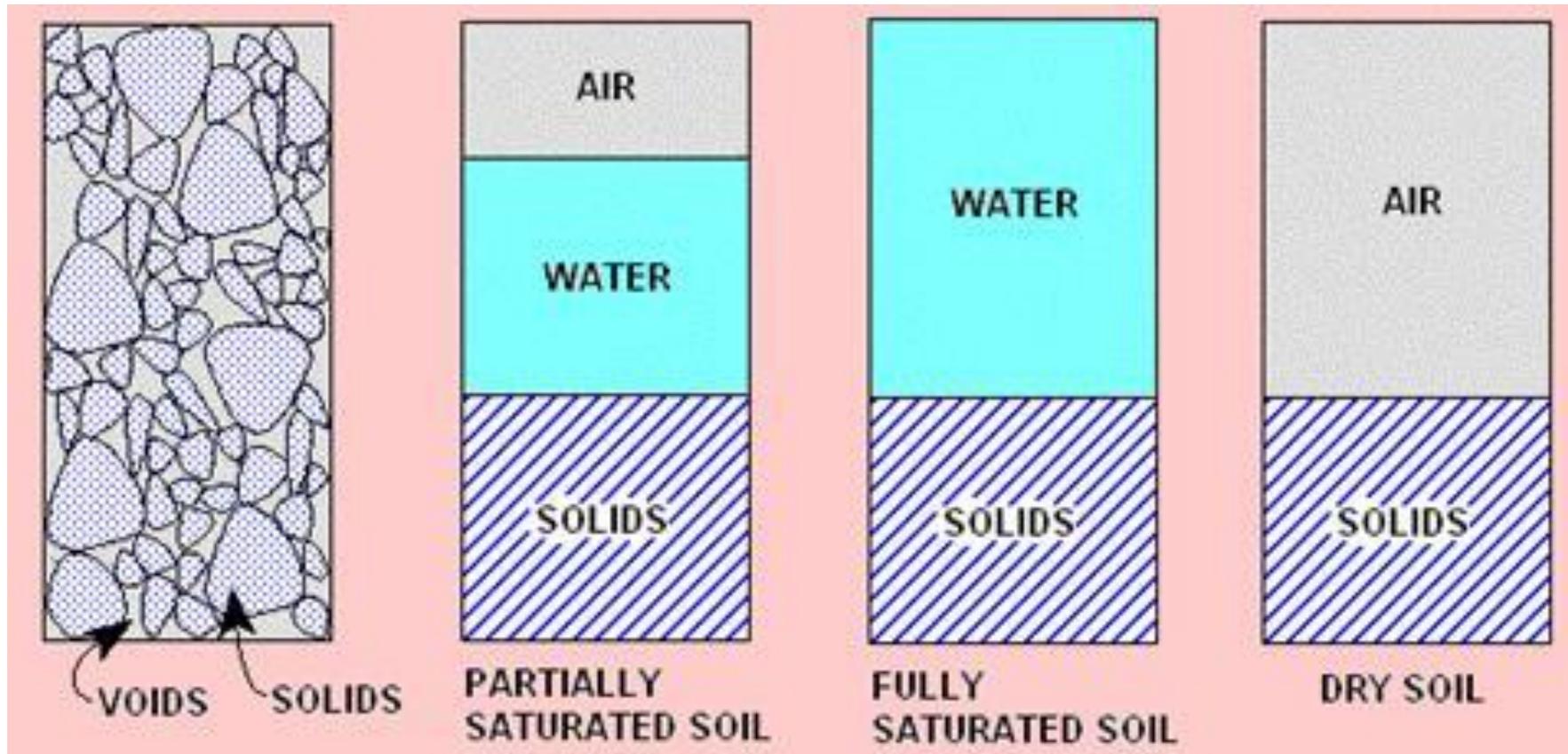
Dr. Lubna Abdulrahman

$$W_t = W_w + W_s$$

W_t : Total weight of soil W_w : Weight of water W_s : Weight of solid W_a : Weight of air ≈ 0

$$V_t = V_v + V_s = V_a + V_w + V_s$$

V_t Total Volume V_v : Volume of Void V_a : Volume of air V_w : Volume of water V_s : Volume of Solid



Basic Volume/Mass Relationships

Void ratio, $e = \frac{V_v}{V_s}$

Porosity, $n(\%) = \frac{V_v}{V_t} \times 100\%$

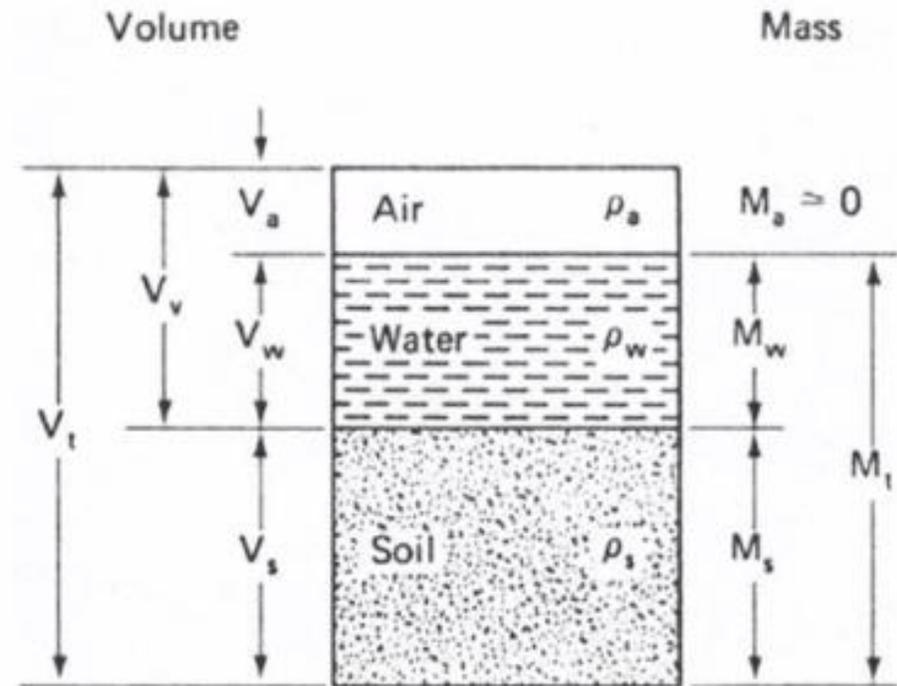
Degree of saturation, $S(\%) = \frac{V_w}{V_v} \times 100\%$

Water content, $\omega(\%) = \frac{M_w}{M_s} \times 100\%$

Specific gravity of solids, $G_s = \frac{M_s}{\rho_{\text{water}} V_s}$

Density, $\rho = \frac{M_t}{V}$

Dry density, $\rho_{\text{dry}} = \frac{M_s}{V}$



$$\rho_B = \rho_d(1+w)$$

$$n = \frac{e}{1+e}$$

$$Se = wG_s$$

$$e = \frac{n}{1-n}$$

Typical Values of Parameters:

$$G_s = 2.60-2.75$$

$$\gamma = 1.60-2.25 \text{ g/cc}$$

$$\gamma_s = 1.30-2.00 \text{ g/cc}$$

$$n = 0.25-0.45 \text{ (for sand)}$$

$$S = 0 \text{ (for dry soil)}-100\% \text{ (for fully saturated)}$$

Some Useful Correlation:

$$1- S.e = G_s \cdot \omega_c$$

$$2- n = \frac{e}{1+e}$$

$$3- e = \frac{n}{1-n}$$

$$4- A = n(1 - s)$$

$$5- A = \frac{e - \omega * G_s}{1+e}$$

$$6- \rho_t = \frac{G_s(1+\omega)}{1+e} \rho_w \quad \text{or} \quad \gamma_t = \frac{G_s(1+\omega)}{1+e} \gamma_w$$

$$7- \rho_t = \frac{G_s + s * e}{1+e} \rho_w \quad \text{or} \quad \gamma_t = \frac{G_s + s * e}{1+e} \gamma_w$$

$$8- \rho_s = \frac{G_s + e}{1+e} \rho_w \quad \text{or} \quad \gamma_s = \frac{G_s + e}{1+e} \gamma_w$$

$$9- \rho_{dry} = \frac{G_s}{1+e} \rho_w \quad \text{or} \quad \gamma_d = \frac{G_s}{1+e} \gamma_w$$

$$10- \rho_{eff.} = \dot{\rho} = \rho_{sat} - \rho_w$$

$$11- \gamma_{eff} = \dot{\gamma} = \frac{G_s - 1}{1+e} \gamma_w$$

Example 1:

In its condition a soil sample has a mass of 2290 g and a volume of $1.15 \cdot 10^{-3} \text{ m}^3$. After being completely dried in an oven the mass of the sample is 2035g. The value of G_s for the soil is 2.68. Determine the bulk density, unit weight, water content, void ratio, porosity, degree of saturation and air content.

Solution:

$$\rho_t = \frac{M}{V} = \frac{2.290}{1.15 \cdot 10^{-3}} = 1990 \text{ kg/m}^3 = 1.99 \frac{\text{Mg}}{\text{m}^3}$$

$$\text{Unit weight } \gamma = \frac{Mg}{V} = 1990 * 9.8 = 19500 \text{ N/m}^3 = 19.5 \text{ kN/m}^3$$

$$\text{Water content } \omega = \frac{M_w}{M_s} = \frac{2290 - 2035}{2035} = 0.125 \text{ or } 12.5\%$$

$$\gamma_t = \frac{G_s(1 + \omega_c)}{1 + e} \gamma_w$$

$$19.5 = \frac{2.68(1 + .125)}{1 + e} * 10$$

$$e = 0.538$$

$$\text{Porosity, } n = \frac{e}{1 + e} = \frac{0.538}{1.538} = 0.3490 \sim 0.35$$

$$S \cdot e = G_s \cdot \omega_c$$

$$\text{Degree of saturation } S = \frac{0.125 * 2.68}{0.538} = 62.267\%$$

$$\text{Air content, } A = n(1 - S) = 0.35(1 - .62) = 0.132$$

Example 2:

A moist soil has these values : $V=7.08 \times 10^{-3} \text{ m}^3$, $m = 13.95 \text{ kg}$, $w= 9.8\%$, $G_s=2.66$.

Determine:

ρ , ρ_t , e , n , $S(\%)$, volume occupied by water and the volume occupied by solid?

Solution:

$$\rho = \frac{m}{V} = \frac{13.95}{7.08 \times 10^{-3}} = 1970.3 \text{ kg/m}^3$$

$$\rho_d = \frac{\rho_{wet}}{1 + \omega} = \frac{1970.3}{1 + 0.098} = 1794.4 \frac{\text{kg}}{\text{m}^3}$$

$$\rho_d = \frac{G_s}{1 + e} \rho_w$$

$$1794.4 = \frac{2.66}{1 + e} * 1000 \longrightarrow e = 0.48$$

$$n = \frac{e}{1 + e} = \frac{0.48}{1.48} = 0.324$$

$$S \cdot e = G_s \cdot \omega \longrightarrow S \cdot 0.48 = 2.66 * 0.098 \longrightarrow S = 54.3\%$$

$$\rho_d = \frac{m_s}{v_t} \longrightarrow 1794.4 = \frac{m_s}{7.08 \times 10^{-3}} \longrightarrow m_s = 12.7 \text{ kg}$$

$$m_w = m - m_s \longrightarrow m_w = 13.95 - 12.7 = 1.25 \text{ kg}$$

$$\therefore v_w = \frac{m_w}{\rho_w} = \frac{1.25}{1000} = 0.00125 \text{ m}^3$$

$$v_s = v_t - v_w \longrightarrow v_s = 7.08 \times 10^{-3} - 0.00125 = 0.00583 \text{ m}^3$$

Example 3:

In the natural state, a moist soil has a volume of 0.0093 m^3 and weighs 177.6 N . The oven dry weight of the soil is 153.6 N . If $G_s=2.71$. Calculate the moisture content, moist unit weight, dry unit weight, void ratio, porosity and degree of saturation.

$$\text{Solution: } \omega_c = \frac{w_w}{w_s} = \frac{177.6-153.6}{153.6} = 15.6 \%$$

$$\gamma_t = \frac{W}{V} = \frac{177.6}{0.0093} = 19096 \frac{\text{N}}{\text{m}^3} = 19.1 \text{ kN/m}^3$$

$$\gamma_d = \frac{W_s}{V} = \frac{153.6}{0.0093} = 16516 \frac{\text{N}}{\text{m}^3} \sim 16.52 \text{ kN/m}^3$$

$$e = \frac{V_v}{V_s}, \quad V_s = \frac{W_s}{G_s \gamma_w} = \frac{0.1536}{2.71 \times 10} = 0.0058 \text{ m}^3$$

$$\therefore V_v = 0.0093 - 0.0058 = 0.0035 \text{ m}^3$$

$$e = \frac{0.0035}{0.0058} = 0.6 \longrightarrow n = \frac{e}{1+e} = \frac{0.6}{1+0.6} = 0.375$$

$$S.e = G_s \cdot \omega \longrightarrow S \cdot 0.6 = 2.71 \cdot 0.156 \longrightarrow S = 70.46\%$$

Example 4:

A soil specimen has a volume of 0.05 m³ and a mass of 87.5 kg. If the water content is 15% and specific gravity is 2.68.

Determine 1) void ratio 2) porosity 3) dry unit weight 4) saturated unit weight 5) degree of saturation.

Solution:

$$\rho_t = \frac{m_t}{v_t} = \frac{87.5}{0.05} = 1750 \text{ kg/m}^3$$

$$w_c = \frac{m_w}{m_s} = 0.15 = \frac{87.5 - m_s}{m_s} \longrightarrow m_s = 76 \text{ kg}$$

$$v_s = \frac{m_s}{G_s \rho_w} = \frac{76}{2.68 * 1000} = 0.028 \text{ m}^3$$

$$e = \frac{v_v}{v_s} = \frac{0.0216}{0.028} = 0.77, \quad n = \frac{e}{1+e} = \frac{0.77}{1+0.77} = 0.43$$

$$\gamma_{dry} = \frac{G_s}{1+e} \gamma_w = \frac{2.68}{1+0.77} 10 = 15.14 \text{ kN/m}^3$$

$$\gamma_{sat} = \frac{G_s + e}{1+e} \gamma_w = \frac{2.68 + 0.77}{1+0.77} 10 = 19.49 \text{ kN/m}^3$$

Example 5: Show that $\gamma_{sat} = \gamma_{d+\left(\frac{e}{1+e} * \gamma_w\right)}$

Solution: take the right hand side :

$$\gamma_{d+\left(\frac{e}{1+e} * \gamma_w\right)} = \frac{G_s}{1+e} * \gamma_w + \frac{e}{1+e} * \gamma_w = \frac{G_s + e}{1+e} * \gamma_w = \gamma_{sat}$$

Example 6:

Given mass of wet sample = 254 gm, void ratio = 0.6133, volume of air = 1.9 cm³, mass of solid = 210 gm. Determine degree of saturation, air content and dry unit weight.

Solution: $m_t = 254 \text{ gm}$, $m_s = 210 \text{ g}$ $\longrightarrow m_w = 254 - 210 = 44 \text{ gm}$

$$v_w = \frac{m_w}{\rho_w} = \frac{44}{1} = 44 \text{ cm}^3$$

$$v_v = v_w + v_a = 44 + 1.9 = 45.9$$

$$0.6133 = \frac{45.9}{v_s} \rightarrow \therefore v_s = 74 \text{ cm}^3$$

$$S = \frac{v_w}{v_v} = \frac{44}{45.9} = 95.8\% \rightarrow A = n(1 - s) = \frac{0.6133}{1 + 0.6133} (1 - 0.95) = 0.019$$

$$\rho_{dry} = \frac{m_s}{v_t}$$

$$v_t = v_w + v_{air} + v_s = 44 + 1.9 + 74.84 = 120 \text{ cm}^3$$

$$\therefore \rho_{dry} = \frac{210}{120} = 1.75 \frac{\text{gm}}{\text{cm}^3} \rightarrow \gamma_{dry} = 17.5 \text{ kN/m}^3$$

Example 7:

A soil specimen is 38 mm in diameter and 76 mm long and its natural condition weighs 168 gm when dried completely in an oven the specimen weighs 130.5 gm. The value of $G_s=2.73$. what is the degree of saturation of the specimen?

Solution: Dia = 38 mm = 3.8 cm

L= 76 mm = 7.6 cm

$$v_t = \left(\frac{3.8}{2}\right)^2 \pi * 7.6 = 86.192 \text{ cm}^3$$

$$m_w = 168 - 130.5 = 37.5 \text{ gm}$$

$$v_w = \frac{37.5}{1} = 37.5 \text{ cm}^3$$

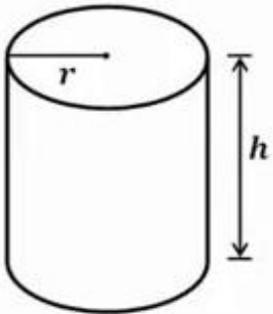
$$v_s = \frac{w_s}{G_s * \gamma_w} = \frac{130.5}{2.73 * 1} = 47.8 \text{ cm}^3$$

$$v_a = 86.192 - (37.5 + 47.80) = 0.889 \text{ cm}^3$$

$$v_v = v_w + v_a = 37.5 + 0.889 = 38.389 \text{ cm}^3$$

$$\therefore S = \frac{v_w}{v_v} = \frac{37.5}{38.389} = 97.6\%$$

Volume of a Cylinder



$$\text{Volume} = h \times \pi \times r^2$$

h = height of the cylinder

π (pi) = 3.14

r = radius of the base circle

Example 8:

Given: mass of wet sample = 254.1 gm, void ratio = 0.6133, volume of air = 1.9 cm³, mass of solids = 210 gm. **Determine:** Degree of saturation, Air content, dry unit weight.

Solution:

$$\text{Mass of water} = 254.1 - 210 = 44.1 \text{ gm}$$

$$\text{Volume of water} = \frac{w_w}{\gamma_w} = 44.1 \text{ cm}^3$$

$$e = \frac{v_v}{v_s} \rightarrow 0.6133 = \frac{v_v}{v_s} = \frac{v_w + v_a}{v_s} = \frac{44.1 + 1.9}{v_s}$$

$$0.6133 = \frac{46}{v_s} \rightarrow v_s = 75 \text{ cm}^3$$

$$S = \frac{v_w}{v_v} = \frac{44.1}{46} = 95.8\%$$

$$A = \frac{e}{1+e} (1 - s) = \frac{0.6133}{1+0.6133} (1 - 0.958) = 0.0157$$

$$v_t = v_v + v_s = 46 + 75 = 121 \text{ cm}^3$$

$$G_s = \frac{w_s}{v_s \gamma_w} = \frac{210}{75 * 1} = 2.8$$

$$\gamma_{dry} = \frac{G_s}{1+e} \gamma_w \rightarrow \gamma_{dry} = \frac{2.8}{1+0.6133} * 10 = 17.355 \text{ kN/m}^3$$

$$\text{Or } \rho_{dry} = \frac{m_s}{v_t} = \frac{210}{121} = 1.7355 \text{ gm/cm}^3$$

$$\therefore \gamma_{dry} = \rho_{dry} * g = 1.7355 * 10 = 17.355 \text{ kN/m}^3$$

Example 9:
A soil specimen
and water content

$$\gamma_{sat} = \frac{2.72 + 0.7}{1 + 0.7} * 10 = 20.11 \text{ kN/m}^3$$

$$\gamma_b = \gamma' = \gamma_{eff} = \gamma_{sat} - \gamma_w = 20.11 - 10 = 10.11 \text{ kN/m}^3$$

at, unit weight

$$\gamma_{at\ s=75\%} = \frac{2.72 + 0.75 * 0.7}{1 + 0.7} * 10 = 19.1 \text{ kN/m}^3$$

$$0.75 * 0.70 = 2.72 * \omega \quad \rightarrow \quad \omega = 19.3\%$$

Example 10 : Prove that $S \cdot e = G_S \cdot \omega_c$

Take the right hand side;

$$\begin{aligned} G_S \cdot \omega_c &= \frac{w_w}{w_s} * \frac{w_s}{v_s \gamma_w} \\ &= \frac{v_w \gamma_w}{v_s \gamma_w} * \frac{v_v}{v_v} = \frac{v_w}{v_v} * \frac{v_v}{v_s} = S * e \end{aligned}$$

Example 11: Show that $\gamma_{dry} = \frac{\gamma_t}{1+\omega}$

$$\frac{\gamma_t}{1+\omega} = \frac{w_t/v_t}{1+w_w/w_s} = \frac{w_t/v_t}{\frac{w_s+w_w}{w_s}} = \frac{\frac{w_t}{v_t}}{\frac{w_t}{w_s}} = \frac{w_s}{v_t}$$

Example 12 : Prove that $n = \frac{e}{1+e}$

$$\frac{e}{1+e} = \frac{v_v/v_s}{1+v_v/v_s} = \frac{\frac{v_v/v_s}{v_s+v_v}}{\frac{v_s}{v_s}} = \frac{v_v}{v_t} = e$$