Al-Mansour University College

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Civil Eng. Dept.

المرحلة الرابعة

4th. Stage

Reinforced Concrete Design II

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Lec.1

تصميم خرسانة مسلحة

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Reinforced Concrete Design II

References:

- Design of Reinforced Concrete. Ninth Edition (2014) , Jack C. McCormac, Russell H. Brown. John Wiley & Sons, Inc.

- Design of Concrete Structures, 14th Edition (2010) Arthur Nilson. David Darwin & Charles W. Dolan , McGraw-Hill Co. Inc.

- Prestressed Concrete, A fundamental approach, Edward G. Nawy. Updated Fifth Edition (2010)

Design Code:

- "*Building Code Requirements for Structural Concrete (ACI 318M-11) and Commentary*", American Concrete Institute, Farmington Hills, Michigan, 2011.

Flat Plate Construction—Pharr Road Condominiums, Atlanta, Georgia.

2. Analysis of Two-Way Slabs

Two-way slabs bend under load into dish-shaped surfaces, so there is bending in both principal directions. As a result, they must be reinforced in both directions by layers of bars that are perpendicular to each other. A theoretical elastic analysis for such slabs is a very complex problem because of their highly indeterminate nature. Numerical techniques such as finite difference and finite elements are required, but such methods require sophisticated software to be practical in design. The methods described in this chapter can be done by hand or with simple spreadsheets, and are sufficiently accurate for most design problems.

The design of two-way slabs is generally based on empirical moment coefficients, which, although they might not accurately predict stress variations, result in slabs with satisfactory overall safety factors. In other words, if too much reinforcing is placed in one part of a slab and too little somewhere else, the resulting slab behavior will probably still be satisfactory. *The total amount of reinforcement in a slab seems more important than its exact placement.*

DESIGN OF TWO WAY SLABS

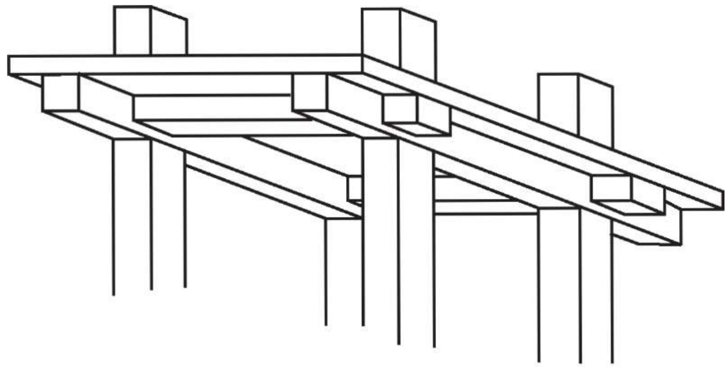
1. Introduction

In general, slabs are classified as being one way or two way. Slabs that primarily deflect in one direction are referred to as *one-way slabs*. Simple-span. When slabs are supported by columns arranged generally in rows so that the slabs can deflect in two directions, they are usually referred to as *two-way slabs*.

Two-way slabs can be strengthened by the addition of beams between the columns, by thickening the slabs around the columns (*drop panels*), and by flaring the columns under the slabs (*column capitals)*. These situations are shown in Figures below.

Flat plates: are solid concrete slabs of uniform depths that transfer loads directly to the supporting columns without the aid of beams or capitals or drop panels. Flat plates can be constructed quickly because of their simple formwork and reinforcing bar arrangements. They need the smallest overall story heights to provide specified headroom requirements, and they give the most flexibility in the arrangement of columns and partitions. They also provide little obstruction to light and have high fire resistance because there are few sharp corners where spalling of the concrete might occur. Flat plates are probably the most commonly used slab system today for multistory reinforced concrete hotels, motels, apartment houses & hospitals. However, flat plates present a possible problem in transferring the shear at the perimeter of the columns. In other words, there is a danger that the columns may punch through the slabs. For heavy industrial loads or long spans, however, some other type of floor system may be used.

Flat slabs: include two-way reinforced concrete slabs with capitals, drop panels, or both. These slabs are very satisfactory for heavy loads and long spans. Although the formwork is more expensive than for flat plates, flat slabs will require less concrete and reinforcing than would be required for flat plates with the same loads and spans. They are particularly economical for warehouses, parking and industrial buildings, and similar structures, where exposed drop panels or capitals are acceptable.



Two way slab with beams

Flat Plate Structure, London

restrained edges (where the slab is constructed integrally with a very stiff reinforced concrete wall so that the little rotation occurs at the slab-to-wall connection).

13.6.3.3 — In an end span, total factored static moment, M_o, shall be distributed as follows:

	(1)	(2)	(3)	(4)	(5)
			Slab without beams between interior supports		
	Exterior edge unrestrained	Slab with beams between all supports	Without edge beam	With edge beam	Exterior edge fully restrained
Interior negative factored moment	0.75	0.70	0.70	0.70	0.65
Positive factored moment	0.63	0.57	0.52	0.50	0.35
Exterior negative factored moment	0	0.16	0.26	0.30	0.65

The next problem is to estimate what proportion of these moments is taken by the column strips and what proportion is taken by the middle strips. Which will be clarified in the following tables of the ACI-code:

13.6.4 — Factored moments in column strips

13.6.4.1 — Column strips shall be proportioned to resist the following portions in percent of interior negative factored moments:

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

Also, designers may design slabs on the basis of numerical solutions, yield-line analysis, or other theoretical methods, provided that it can be clearly demonstrated that they have met all the necessary safety and service ability criteria required by the ACI Code. However, using approximate analyses for design have proved to be very satisfactory under service loads. Furthermore, they have been proved to have appreciable overload capacity.

3. Design of Two-Way Slabs by the ACI Code

The ACI Code (13.5.1.1) specifies two methods for designing two-way slabs. These are:

- Direct Design Method:

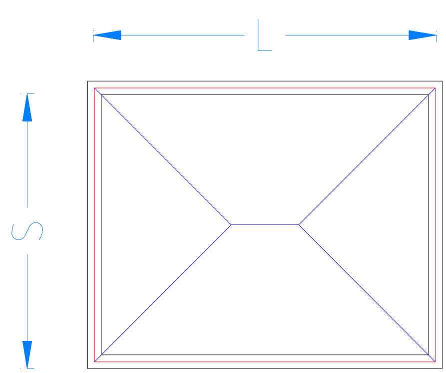
The code (13.6) provides a procedure with which a set of moment coefficients can be determined. The method, in effect, involves a single-cycle moment distribution analysis of the structure based on (a) the estimated flexural stiffnesses of the slabs, beams (if any), and columns and (b) the torsional stiffnesses of the slabs and beams (if any) transverse to the direction in which flexural moments are being determined. Some types of moment coefficients have been used satisfactorily for many years for slab design. They do not, however, give very satisfactory results for slabs with unsymmetrical dimensions and loading patterns.

- Equivalent Frame Method

The same stiffness values used for the direct design method are used for the equivalent frame method. This method is very satisfactory for symmetrical frames as well as for those with unusual dimensions or loadings.

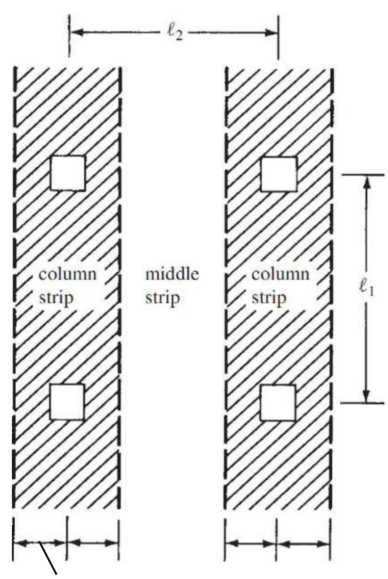
The loads on supporting beam s for two way panels may be assumed as the load within tributary areas of the panel bounded by the intersections of (45)^o lines from the corner with the median of the panel parallel to the long side. The bending moments may be determined approximately by using an equivalent uniform load per unit length of the beam as follows:

- For short beam: $\frac{w.S}{3}$
- For long beam: $\frac{w.S}{3}(\frac{3-m^2}{2})$ where $m=\frac{S}{L}$



positive and negative moments are estimated in each strip. The *column strip* is a slab with a width on each side of the column centerline equal to one-fourth the smaller of the panel dimensions l_1 or l_2. It includes beams if they are present. The middle strip is the part of the slab between the two column strips.

The part of the moments assigned to the column and middle strips may be assumed to be uniformly spread over the strips. The percentage of the moment assigned to a column strip depends on the effective stiffness of that strip and on its *aspect ratio,* (where l_1 is the length of span, center to center, of supports in the direction in which moments are being determined and l_2 is the span length, center to center, of supports in the direction transverse to l_1). Note that the Figure below shows column and middle strips in only one direction. A similar analysis must be performed in the perpendicular direction. The resulting analysis will result in moments in both directions.



0.25l_1 or 0.25l_2 whichever is smaller

- **Depth Limitations:**

1) Slabs without Interior Beams

For a slab without interior beams spanning between its supports and with a ratio of its long span to short span not greater than 2.0, the minimum thickness can be taken from [Table 9.5(c) in the ACI code]. The values selected from the table, however, must not be less than the following values (ACI 9.5.3.2):

- Slabs without drop panels 125 mm.
- Thickness of those slabs with drop panels outside the panels 100 mm.

TABLE 9.5(c)—MINIMUM THICKNESS OF SLABS WITHOUT INTERIOR BEAMS*

	Without drop panels‡			With drop panels‡		
	Exterior panels		Interior panels	Exterior panels		Interior panels
f_y, MPa†	Without edge beams	With edge beams§		Without edge beams	With edge beams§	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

*For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.
†For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.
‡Drop panels as defined in 13.2.5.
§Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

In this Table, some of the values are given for slabs with drop panels. To be classified as a drop panel, according to Sections 13.3.7 and 13.2.5 of the code, a panel must:

(a) Extend horizontally in each direction from the centerline of the support no less than one-sixth the distance, center to center, of supports in that direction and

(b) Project vertically below the slab a distance no less than one-fourth the thickness of the slab away from the drop panel.

In this table, l_n is the length of the clear span in the long direction of two-way construction, measured face to face of the supports in slabs without beams and face to face of beams or other supports in other cases.

Very often slabs are built without interior beams between the columns but with edge beams running around the perimeter of the building. These beams are very helpful in stiffening the slabs and reducing the deflections in the exterior slab panels. The stiffness of slabs with edge beams is expressed as a function of α_f, which follows.

The letter α_f is used to represent the ratio of the flexural stiffness ($E_{cb}I_b$) of a beam section to the flexural stiffness of the slab ($E_{cs}I_s$) whose width equals the distance between the centerlines of the panels on each side of the beam. If no beams are used, as in the case for the flat plate, α_f will equal 0. For slabs with beams between columns along exterior edges, α_f for the edge beams may not be < 0.8.

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} \qquad \text{(ACI Equation 13-3)}$$

where

E_{cb} = the modulus of elasticity of the beam concrete

E_{cs} = the modulus of elasticity of the column concrete

I_b = the gross moment of inertia about the centroidal axis of a section made up of the beam and the slab on each side of the beam extending a distance equal to the projection of the beam above or below the slab (whichever is greater) but not exceeding four times the slab thickness (ACI 13.2.4)

I_s = the moment of inertia of the gross section of the slab taken about the centroidal axis and equal to $h^3/12$ times the slab width, where the width is the same as for α

13.2.4 — For monolithic or fully composite construction, a beam includes that portion of slab on each side of the beam extending a distance equal to the projection of the beam above or below the slab, whichever is greater, but not greater than four times the slab thickness.

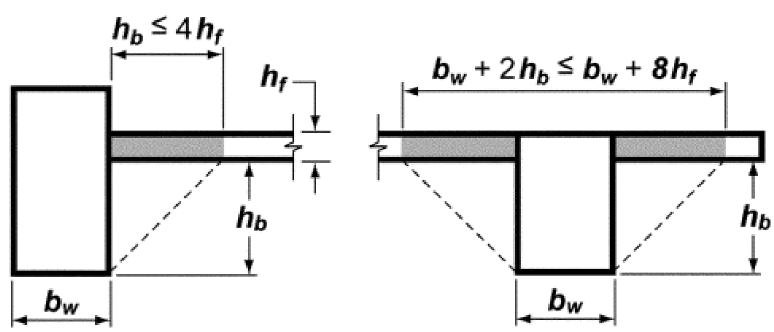
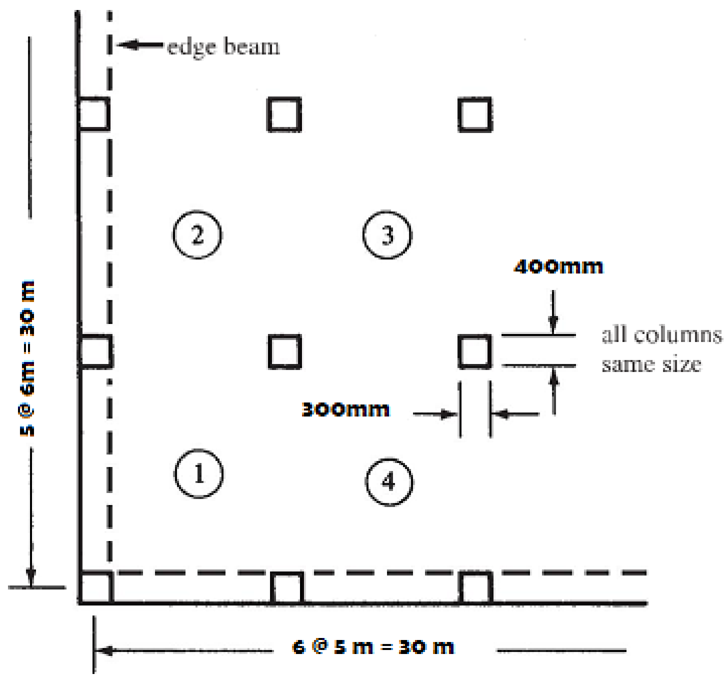


Fig. R13.2.4—Examples of the portion of slab to be included with the beam under 13.2.4.

<u>Example 3:</u>

Using the ACI Code, determine the minimum permissible total thicknesses required for the slabs in panels (3) and (2) for the floor system shown in Figure. Edge beams are used around the building perimeter, and they are 300mm wide and extend vertically for 200mm below the slab as shown. No drop panels are used, and the concrete in the slab is the same as that used in the edge beams. f_y = 420MPa.

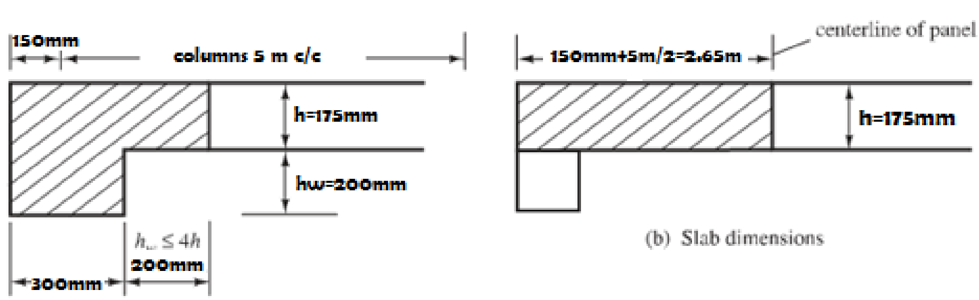


Solution:

Assume h=175mm and compute α_f with reference to figure below, centroid of beam cross section is at:

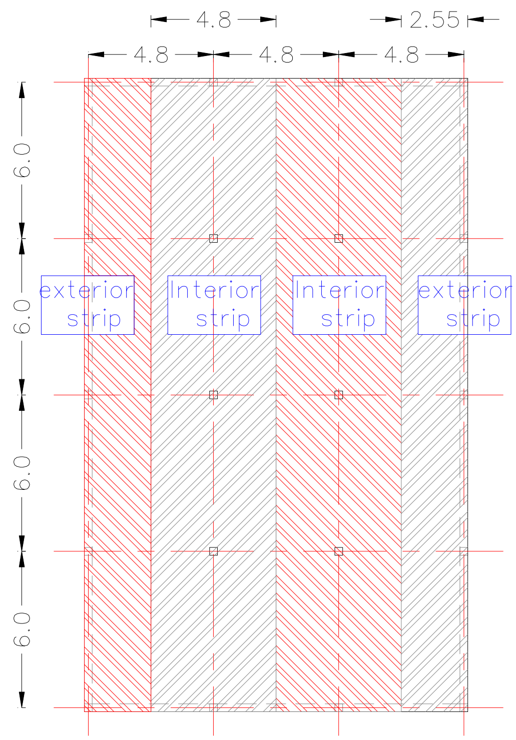
$$\bar{y} = \frac{\sum A.\bar{y}}{\sum A} = \frac{(300)(200)(100) + (175)(500)(200 + \frac{175}{2})}{(300)(200) + (175)(500)} \cong 211mm \text{ from bottom}$$

=164mm from top



(a) Edge beam dimensions

(b) Slab dimensions



For interior longitudinal strip (N-S) direction:

$l_1 = 6000mm$, $l_n = 5700mm$

$l_2 = 4800mm$

$$M_o = \frac{q_u l_2 l_n^2}{8} \frac{(14)(4.8)(5.7)^2}{8} \cong 272.9kN.m$$

a) Internal spans

Dividing the moment into negative and positive portions:

Negative design moment: $(0.65)(272.9) = 177.4kN.m$

Positive design moment: $(0.35)(272.9) = 95.5kN.m$

For $\frac{l_2}{l_1} = \frac{4.8}{6} = 0.8$, $\alpha_{f1}\frac{l_2}{l_1} = 0$

- Negative design moment =177.4$kN.m$

Column strip moment = slab moment = (0.75)(177.4)=133.1$kN.m$

Middle strip moment = 177.4-133.1=44.1$kN.m$

- Positive design moment =95.5$kN.m$

Column strip moment = slab moment = (0.6)(95.5)=57.3$kN.m$

Middle strip moment = 95.5-57.3=38.2$kN.m$

b) End spans

Interior Negative design moment: $(0.7)(272.9) = 191kN.m$

Positive design moment: $(0.5)(272.9) = 136.5kN.m$

Exterior Negative design moment: $(0.3)(272.9) = 81.9kN.m$

For $\frac{l_2}{l_1} = \frac{4.8}{6} = 0.8$, $\alpha_{f1}\frac{l_2}{l_1} = 0$

- Interior Negative design moment =191$kN.m$

Column strip moment = slab moment = (0.75)(191)=143.3$kN.m$

Middle strip moment = 191-143.3=47.7$kN.m$

- Positive design moment =136.5$kN.m$

Column strip moment = slab moment = (0.6)(136.5)=81.9$kN.m$

Middle strip moment = 136.5-81.9=54.6$kN.m$

$$I_s = \frac{(2650)(175)^3}{12} \cong (1.18)(10)^9\, mm^4$$

$$\therefore \alpha_f = \frac{EI_b}{EI_s} = \frac{(E)(1.67)(10)^9}{(E)(1.18)(10)^9} = 1.42 > 0.8$$

$\therefore$ This is an edge beam as defined by the footnote of table 9.5c of ACI code.

Min. h=l_n/33= $\frac{6m - 0.4m}{33} = 0.17m = 170mm$

2) Slabs with Interior Beams

To determine the minimum thickness of slabs with beams spanning between their supports on all sides, Section 9.5.3.3 of the code must be followed. Involved in the expressions presented there are span lengths, panel shapes, flexural stiffness of beams, steel yield stresses, and so on. In these equations, the following terms are used:

l_n = the clear span in the long direction, measured face to face, of beams.

β = the ratio of the long to the short clear span

α_{fm} = the average value of the ratios of beam-to-slab stiffness on all sides of a panel.

The minimum thickness of slabs or other two-way construction may be obtained by substituting into the equations to follow, which are given in Section 9.5.3.3 of the code. In the equations, the quantity β is used to take into account the effect of the shape of the panel on its deflection, while the effect of beams (if any) is represented by α_{fm}.

a) For $\alpha_{fm} < 0.2$, the minimum thicknesses are obtained as they were for slabs without interior beams spanning between their supports.

b) For $0.2 < \alpha_{fm} < 2.0$, the thickness shall not be less than 125mm. or:

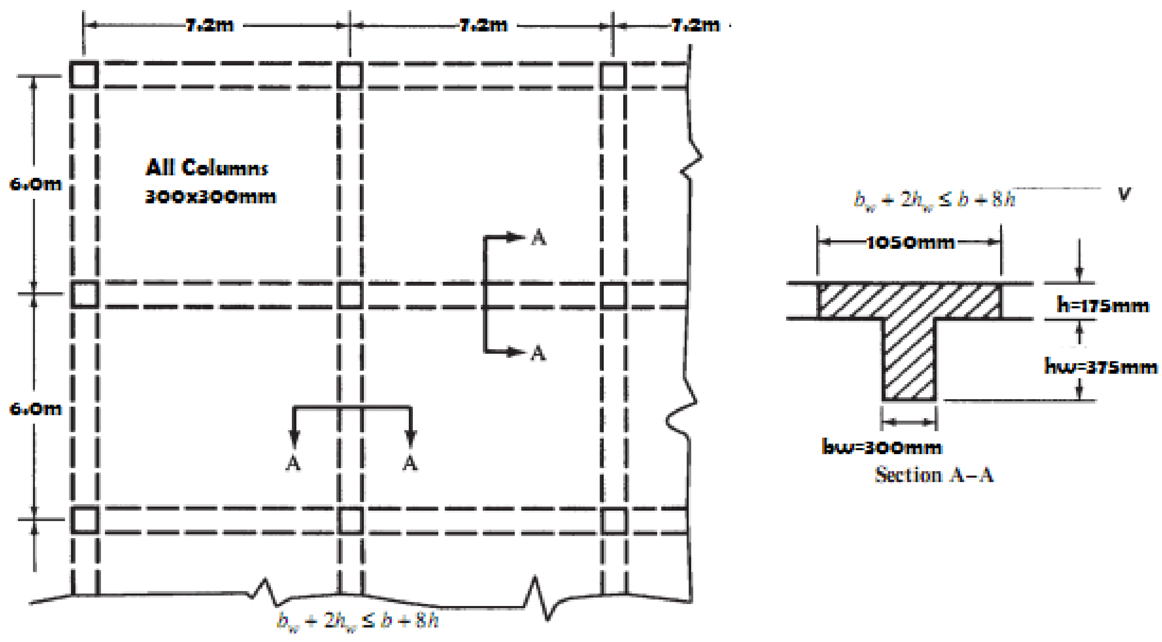
$$h = \frac{l_n(0.8 + \frac{f_y}{1400})}{36 + 5\beta(\alpha_{fm} - 0.2)} \qquad \text{(ACI equation 9-12)}$$

c) For $\alpha_{fm} > 2.0$, the thickness shall not be less than 90mm. or:

$$h = \frac{l_n(0.8 + \frac{f_y}{1400})}{36 + 9\beta} \qquad \text{(ACI equation 9-13)}$$

Example 4:

The two-way slab shown in Figure below has been assumed to have a thickness of 175mm. Section A-A in the figure shows the beam cross section. Check the ACI equations to determine if the slab thickness is satisfactory for an interior panel. f'_c = 21MPa , f_y = 420MPa.



For 7.2m long beam,

$$\bar{y} = \frac{\sum A.\bar{y}}{\sum A} = \frac{(300)(375)(187.5) + (175)(1050)(375 + \frac{175}{2})}{(300)(375) + (175)(1050)} \cong 358mm \text{ from bottom}$$

$$I_b = \frac{(300)(375)^3}{12} + (300)(375)(358 - \frac{375}{2})^2 + \frac{(1050)(175)^3}{12} + (1050)(175)(550 - \frac{175}{2} - 358)^2$$

$$I_b \cong (7.06)(10)^9 mm^4$$

$$I_s = \frac{(6000)(175)^3}{12} \cong (2.68)(10)^9 mm^4$$

$$\therefore \alpha_f = \frac{EI_b}{EI_s} = \frac{(E)(7.06)(10)^9}{(E)(2.68)(10)^9} = 2.63$$

For 6m long beam,

$$I_b \cong (7.06)(10)^9 mm^4$$

$$I_s = \frac{(7200)(175)^3}{12} \cong (3.22)(10)^9 mm^4$$

$$\therefore \alpha_f = \frac{EI_b}{EI_s} = \frac{(E)(7.06)(10)^9}{(E)(3.22)(10)^9} = 2.2$$

$$\alpha_{fm} = \frac{2 \times (2.63 + 2.2)}{4} = 2.42$$

$\because \alpha_{fm} = 2.42 > 2.0 \Rightarrow \therefore$ Use equation 9-13 to determine slab thickness,

$$h = \frac{(7200-300)(0.8+\frac{420}{1400})}{36+9(\frac{7200-300}{6000-300})} \cong 162mm$$

$$h \geq 90mm$$

Then h should be 170mm.

<u>Note that the for an edge or corner panels, each edge of the panel would have had a different</u> α_f .

- **Distribution of Moments in Slabs:**

The total moment, M_o, that is resisted by a slab equals the sum of the maximum positive and negative moments in the span. It is the same as the total moment that occurs in a simply supported beam. For a uniform load per unit area, q_u, it is as follows:

$$M_o = \frac{(q_u l_2)(l_1)^2}{8}$$

In this expression, l_1 is the span length, center to center, of supports in the direction in which moments are being taken and l_2 is the length of the span transverse to l_1, measured center to center of the supports.

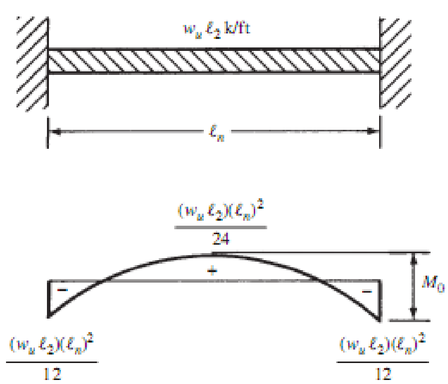
The moment that actually occurs in such a slab has been shown by experience and tests to be somewhat less than the value determined by the above M_o expression. For this reason, l_1 is replaced with l_n, the clear span measured face to face of the supports in the direction in which moments are taken. The code

(13.6.2.5) states that l_n may not be taken to be less than 65% of the span l_1 measured center to center of supports. If l_1 is replaced with l_n, the expression for M_o, which is called the *static moment,* becomes

$$M_o = \frac{(q_u l_2)(l_n)^2}{8} \qquad \text{(ACI Equation 13-4)}$$

When the static moment is being calculated in the long direction, it is convenient to write it as M_{ol}, and in the short direction as M_{os}.

It is next necessary to know what proportions of these total moments are positive and what proportions are negative. If a slab was completely fixed at the end of each panel, the division would be as it is in a fixed-end beam, two-thirds negative and one-third positive, as shown in Figure below.



This division is reasonably accurate for interior panels where the slab is continuous for several spans in each direction with equal span lengths and loads. In effect, the rotation of the interior columns is assumed to be small, and moment values of $0.65M_o$ for negative moment and $0.35M_o$ for positive moment are specified by the code (13.6.3.2).

For exterior spans, the code (13.6.3.3) provides a set of percentages for dividing the total factored static moment into its positive and negative. These divisions, which are shown in the Table below, include values for unrestrained edges (where the slab is simply supported on a masonry or concrete wall) and for

Using h=200mm, Check if α_f for the edge beams is > 0.8.

For the beam section shown with flange extension = 200 mm

$$y_c = \frac{(300)(500)(250)+(200)(200)(300)}{(300)(500)+(200)(200)} = 260.5mm$$

$$I_{beam} = \frac{(300)(500)^3}{12} + (500)(300)(260.5-250)^2 + \frac{(200)(200)^3}{12} + (200)(200)(300-260.5)^2$$

$$I_{beam} = (3.34)(10)^9 mm^4$$

$$I_{slab} = \frac{(2550)(200)^3}{12} = (1.7)(10)^9 mm^4$$

For the other direction: $I_{slab} = \frac{(3150)(200)^3}{12} = (2.1)(10)^9 mm^4$

$$\alpha_{f1} = \frac{(3.34)(10)^9}{(1.7)(10)^9} = 1.96 > 0.8, \ \alpha_{f2} = \frac{(3.34)(10)^9}{(2.1)(10)^9} = 1.59 > 0.8$$

$$\therefore t_{min} = \frac{l_n}{33} = \frac{5700}{33} = 172mm < 200\text{mm} \quad \text{O.K.}$$

Flat plate slab thickness should be > 125mm O.K.

D.L = (0.2)(24)+1.4+1.4=7.6 kN/m^2

W_u = (1.2)(7.6)+(1.6)(3)=13.92 **say** 14 kN/m^2

Assume 12mm bars for slab reinforcement,

$$d_{N-S} = 200-20-\frac{12}{2} = 174mm$$

$$d_{E-W} = 200-20-12-\frac{12}{2} = 162mm$$

$$d_{avg.} = \frac{174+162}{2} = 168mm$$

3) For a complete design of slab, **Shear strength of slab should be checked first**.
4) Moment calculations.

   The slab shall be divided to design strips or panels as shown, each strip, in turn, contains column and middle strips.

- Exterior Negative design moment =81.9$kN.m$

$$C = \sum (1 - 0.63\frac{x}{y})(\frac{x^3 y}{3})$$

$$C = \left[1 - (0.63)(\frac{300}{500})(\frac{(300)^3(500)}{3})\right]$$

$$+\left[1 - (0.63)(\frac{200}{200})(\frac{(200)^3(200)}{3})\right] = (3)(10)^9 mm^4$$

$$I_s = \frac{(4800)(200)^3}{12} = (3.2)(10)^9 mm^4$$

$$\beta_t = \frac{C}{2l_s} = \frac{(3)(10)^9}{(2)(3.2)(10)^9} = 0.47$$

Column strip moment = slab moment = (0.95)(81.9)=77.8$kN.m$

Middle strip moment = 81.9-77.8=4.1$kN.m$

For exterior longitudinal strip (N-S) direction:

$l_1 = 6000mm$, $l_n = 5700mm$

$l_2 = 2550mm$

$$M_o = \frac{q_u l_2 l_n^2}{8} \frac{(14)(2.55)(5.7)^2}{8} \cong 145kN.m$$

a) **Internal spans**

Dividing the moment into negative and positive portions:

Negative design moment: $(0.65)(145) = 94.3kN.m$

Positive design moment: $(0.35)(145) = 50.8kN.m$

For $\frac{l_2}{l_1} = \frac{4.8}{6} = 0.8$, $\alpha_{f1}\frac{l_2}{l_1} = (1.96)(0.8) = 1.57 > 1.0$

- Negative design moment =94.3$kN.m$

Column strip moment = (0.81)(94.3)=76.4$kN.m$

(85% to the beam=(0.85)(76.4)=64.9$kN.m$, rest to the slab=76.4-64.9=11.5$kN.m$)

Middle strip moment = 94.3-76.4=17.9$kN.m$

- Positive design moment =50.8$kN.m$

Column strip moment = (0.81)(50.8)=41.1$kN.m$

(85% to the beam=(0.85)(41.1)=35$kN.m$, rest to the slab=41.1-35=6.1$kN.m$)

Middle strip moment = 50.8-41.1=9.7$kN.m$

b) <u>End spans</u>

Interior Negative design moment: $(0.7)(145) = 101.5kN.m$

Positive design moment: $(0.5)(145) = 72.5kN.m$

Exterior Negative design moment: $(0.3)(145) = 43.5kN.m$

For $\frac{l_2}{l_1} = \frac{4.8}{6} = 0.8$, $\alpha_{f1}\frac{l_2}{l_1} = (1.96)(0.8) = 1.57$

- Interior negative design moment =$101.5kN.m$

Column strip moment = (0.81)(101.5)=82.2$kN.m$

(85% to the beam=(0.85)(82.2)=69.9$kN.m$, rest to the slab=82.2-69.9=12.3$kN.m$)

Middle strip moment = 101.5-82.2=19.3$kN.m$

- Positive design moment =$72.5kN.m$

Column strip moment = (0.81)(72.5)=58.7$kN.m$

(85% to the beam=(0.85)(58.7)=49.9$kN.m$, rest to the slab=8.9$kN.m$)

Middle strip moment = 72.5-58.7=13.8$kN.m$

- Exterior negative design moment =$43.5kN.m$

$$\beta_t = \frac{C}{2l_s} = \frac{(3)(10)^9}{(2)(3.2)(10)^9} = 0.47$$

Column strip moment = (0.95)(43.5)=41.3$kN.m$

(85% to the beam=(0.85)(41.3)=35.1$kN.m$, rest to the slab=6.2$kN.m$)

Middle strip moment = 43.5-41.3=2.2$kN.m$

<u>Direct Loading on beam</u>

a) <u>Internal spans</u>

$$w_u = (0.5-0.2)(0.3)(24)(1.2) + (15.5)(1.2) = 21.2kN/m$$

$$M_o = \frac{w_u l_n^2}{8} = \frac{(21.2)(5.7)^2}{8} = 86.1kN.m$$

Total beam negative moment = 64.9+(0.65)(86.1)=120.9kN.m

Total beam positive moment = 35+(0.35)(86.1)=65.1kN.m

b) <u>End spans</u>

$$w_u = 21.2kN/m$$

$$M_o = \frac{w_u l_n^2}{8} = \frac{(21.2)(5.7)^2}{8} = 86.1kN.m$$

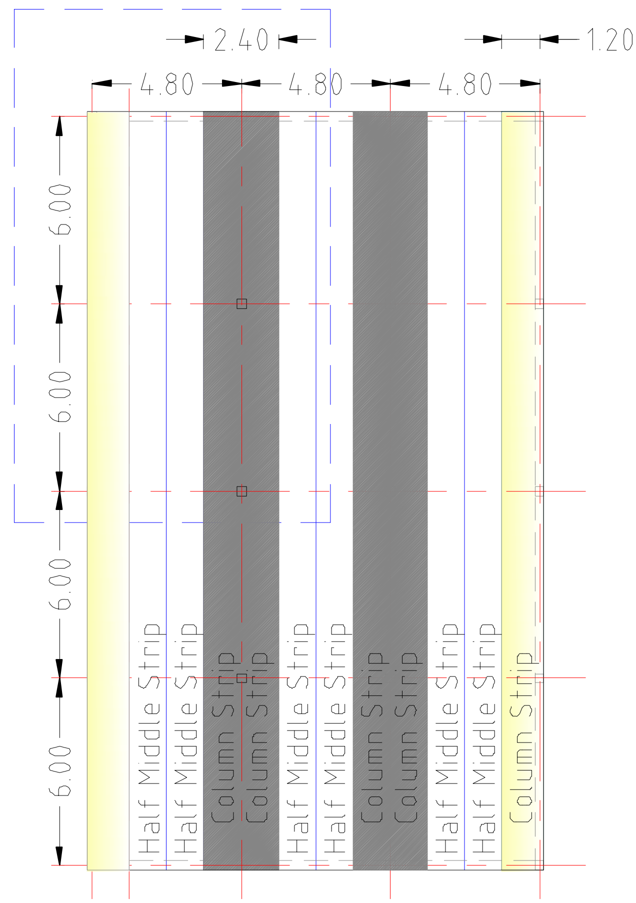
Total int. negative beam moment = 69.9+(0.7)(86.1)=130.2kN.m

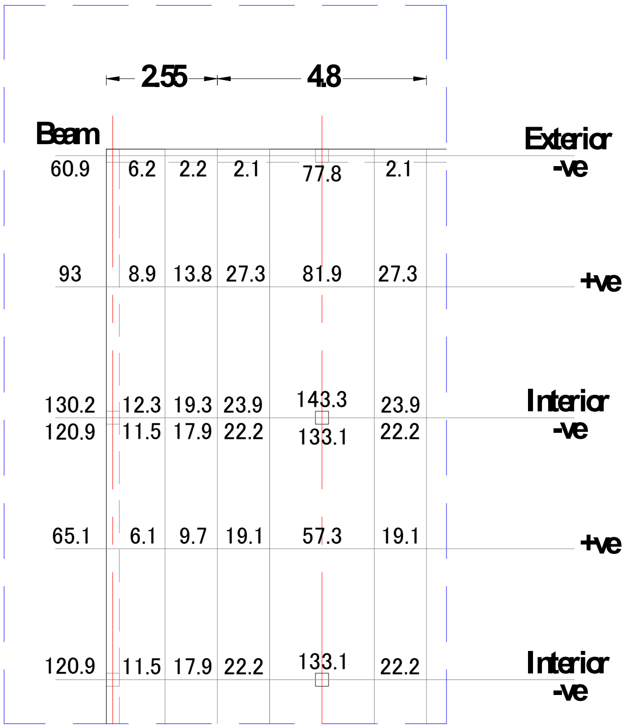
Total positive beam moment = 49.9+(0.5)(86.1)=93kN.m

Total ext. negative beam moment =35.1+(0.3)(86.1)=60.9kN.m

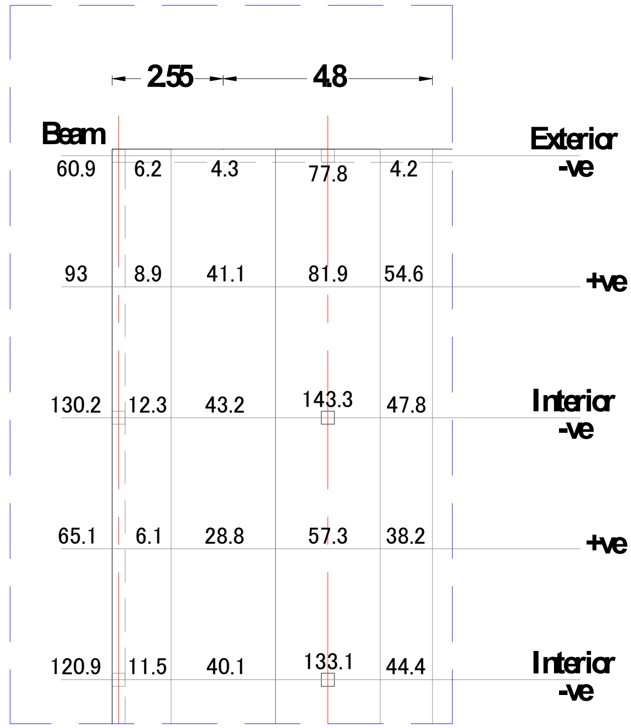
Notes:

- Each middle strip shall be designed to resist the sum of moments assigned to its two half strips.
- At a common spans end, design for the larger moment.
- In addition to moment assigned to a beam, beams shall be proportioned to resist moments caused by loads applied directly on beam (Such as brick walls) as shown in the previous example.





Summary of Moments (kN.m)



Design Moments (kN.m)

Reinforcement:

dir.	Location	Column strip						
		M_u kN.m	Strip width m	R_u N/mm^2	ρ	A_s mm^2/m	No. of bars	Spacing mm
N-S	Disc. Edge	77.8	2.4	1.2	0.0029	506	11	220
	Mid-span	81.9	2.4	1.25	0.0031	533	11	210
	Cont. Edge	143.3	2.4	2.2	0.0055	954	20	115
	Mid-span	57.3	2.4	0.9	0.0021	370	8	305
	Cont. Edge	133.1	2.4	2	0.0051	883	19	125

Sample of calculations:

$$R_u = \frac{M_u}{\phi b d^2} \quad \text{(b=2400, d=174mm, Ø=0.9)}$$

$$\mu = \frac{f_y}{0.85 f_c^{'}} \Rightarrow \rho = \frac{1}{\mu}\left[1-\sqrt{1-\frac{2R_u\mu}{f_y}}\right]$$

$$A_s = \rho b d > (A_s)_{\min.} = 0.0018bt = 360mm^2/m$$

$$No.\ of\ bars = \frac{strip.width \times A_s}{1000 A_{bar}},\ \text{Spacing} = \frac{strip.width}{No.of.bars}$$

dir.	Location	Middle strip (H.W)						
		M_u kN.m	Strip width m	R_u N/mm^2	ρ	A_s mm^2/m	No. of bars	Spacing mm
N-S	Disc. Edge	4.2	2.4					
	Mid-span	54.6	2.4					
	Cont. Edge	47.8	2.4					
	Mid-span	38.2	2.4					
	Cont. Edge	44.4	2.4					

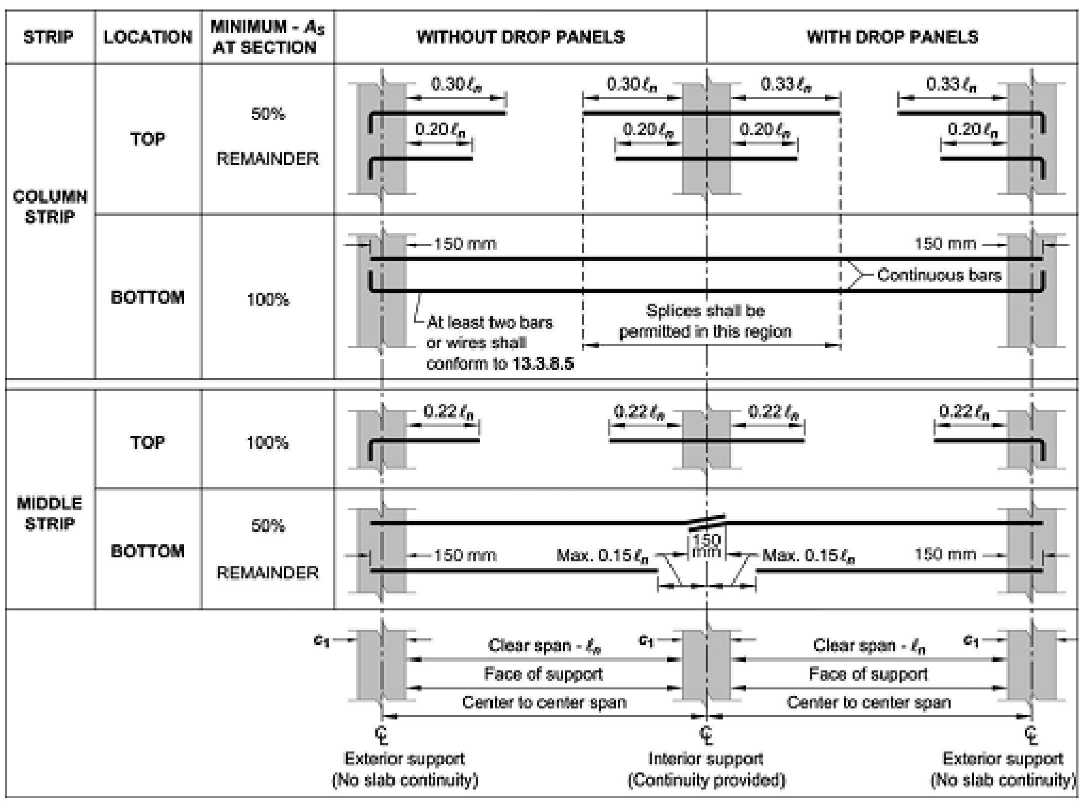
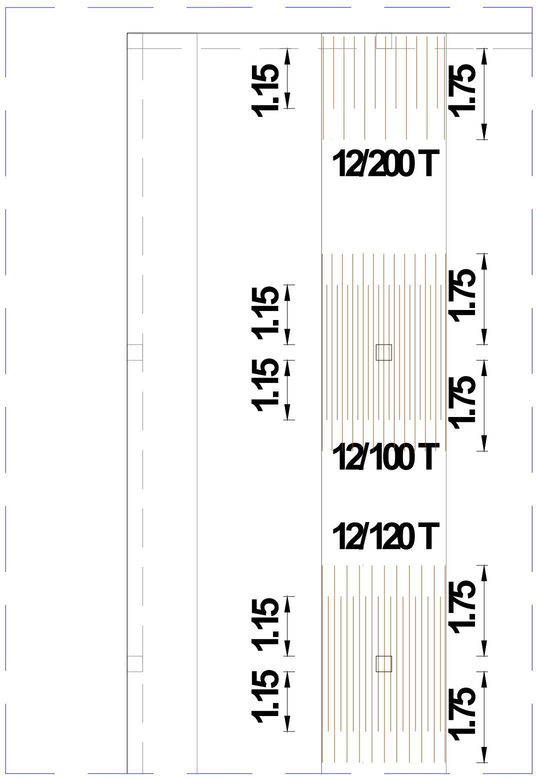
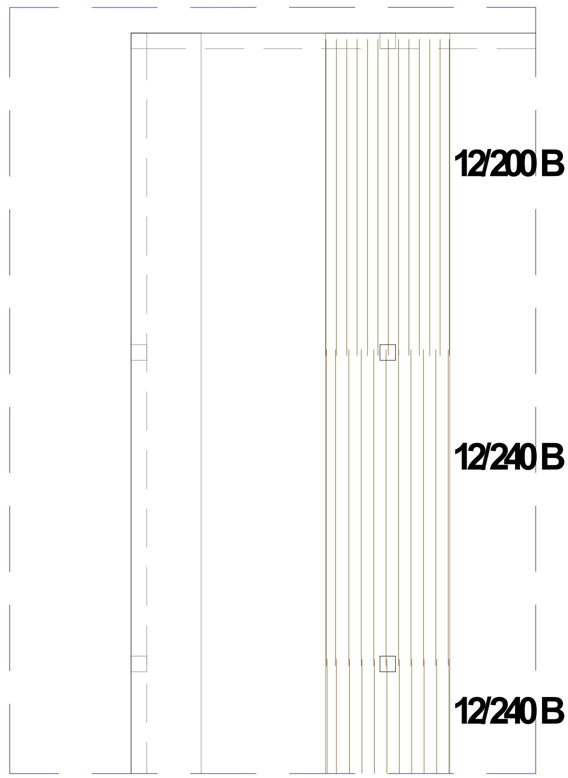


Fig. 13.3.8—Minimum extensions for reinforcement in slabs without beams. (See 12.11.1 for reinforcement extension into supports).



Column Strip Top Reinf. Distribution



Column Strip Bottom Reinf. Distribution

$$\%^{-}_{\text{int col}} = 75 + 30\left(\frac{\alpha_{f1}\ell_2}{\ell_1}\right)\left(1 - \frac{\ell_2}{\ell_1}\right)$$

$$\%^{+} = 60 + 30\left(\frac{\alpha_{f1}\ell_2}{\ell_1}\right)\left(1.5 - \frac{\ell_2}{\ell_1}\right)$$

$$\%^{-}_{\text{ext col}} = 100 - 10\beta_t + 12\left(\frac{\alpha_{f1}\ell_2}{\ell_1}\right)\left(1 - \frac{\ell_2}{\ell_1}\right)$$

In the preceding three equations, if $\beta_t > 2.5$, use 2.5, and if $\alpha_{f1}\ell_2/\ell_1 > 1$, use 1.

Table 8.10.5.7.1—Portion of column strip M_u in beams

$\alpha_{f1}\ell_2/\ell_1$	Distribution coefficient
0	0
≥1.0	0.85

Note: Linear interpolation shall be made between values shown.

Factored shear in slab systems with beams (Code 13.6.8):

- Beams with $\alpha_{f1}l_2/l_1$ equal to or greater than 1.0 shall be proportioned to resist shear caused by factored loads on tributary areas which are bounded by 45-degree lines drawn from the corners of the panels and the centerlines of the adjacent panels parallel to the long sides.
- In proportioning beams with $\alpha_{f1}l_2/l_1$ less than 1.0 to resist shear, linear interpolation, assuming beams carry no load at $\alpha_{f1} = 0$, shall be permitted.
- In addition to shears calculated above, beams shall be proportioned to resist shears caused by factored loads applied directly on beams.

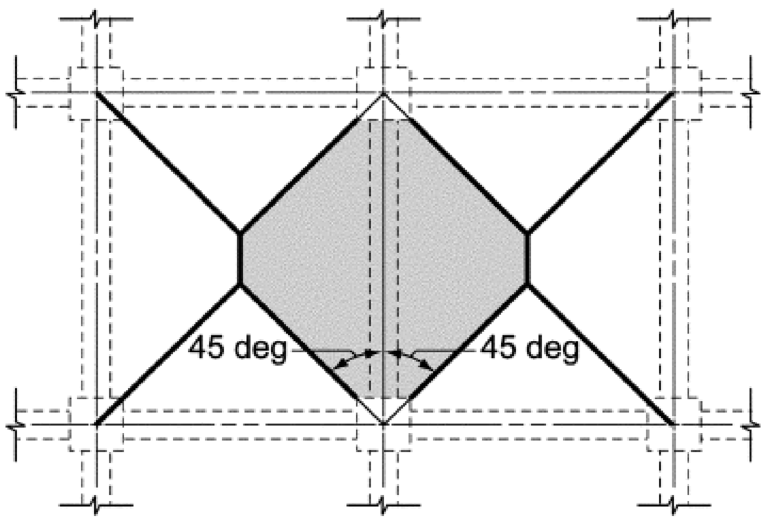


Fig. R13.6.8—Tributary area for shear on an interior beam.

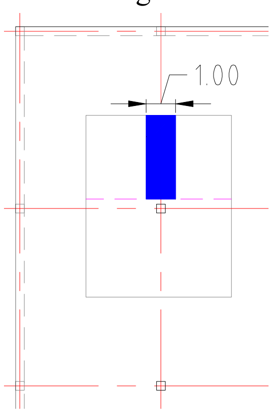
Shear in flat slab systems and flat plates:

Two kinds of shear may be critical in the design of flat slabs, flat plates and footings.

- One way shear (Beam shear): the familiar type of shear. Seldom controls in two way slab systems and may be critical in long narrow slabs or footings.

$$V_u / 1m - width = (w_u)(\frac{l_n}{2} - d) \le \phi 0.17\sqrt{f_c'}b_w d$$

$$(b_w = 1000mm)$$



- Two way shear (Punching shear): The critical section is taken at perpendicular to the slab at a distance d/2 from the periphery of the supports (columns, column capitals….etc). The shear force V_u to be resisted is the total factored load on the area bounded by panel centerlines around the columns less the load applied within the area defined by the critical shear perimeter. This force should be smaller than the reduced concrete capacity $ØV_c$. V_c is the smallest of :

(a) $$V_c = 0.17\left(1+\frac{2}{\beta}\right)\lambda\sqrt{f_c'}\,b_o d \quad (11\text{-}31)$$

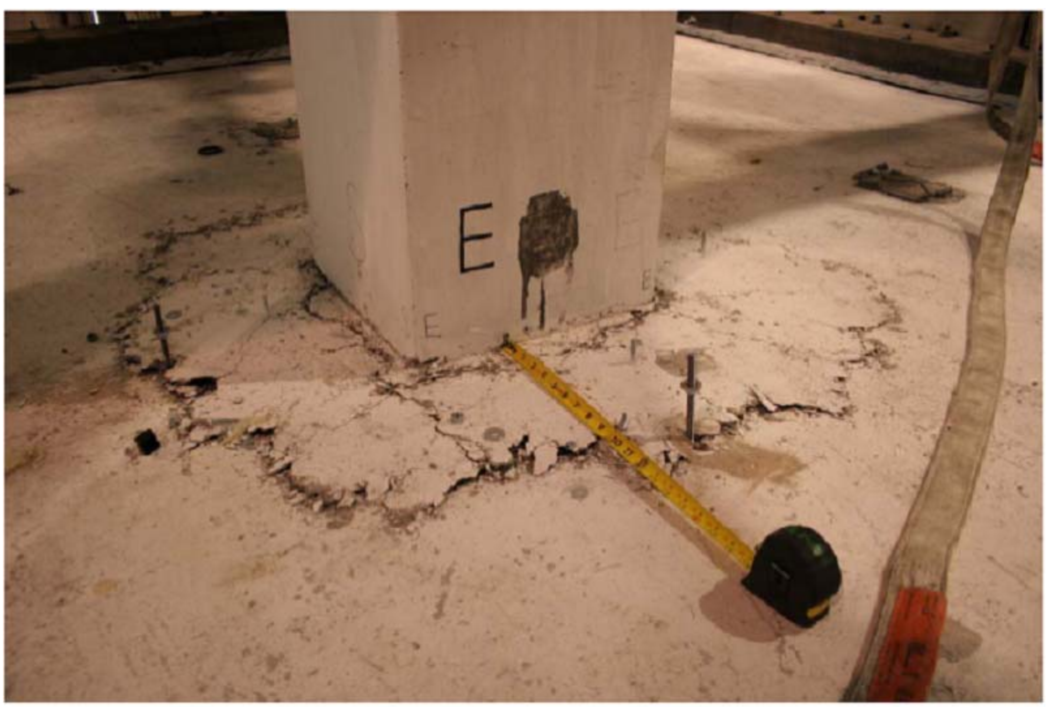
where β is the ratio of long side to short side of the column, concentrated load or reaction area;

(b) $$V_c = 0.083\left(\frac{\alpha_s d}{b_o}+2\right)\lambda\sqrt{f_c'}\,b_o d \quad (11\text{-}32)$$

where α_s is 40 for interior columns, 30 for edge columns, 20 for corner columns; and

(c) $$V_c = 0.33\lambda\sqrt{f_c'}\,b_o d \quad (11\text{-}33)$$

And λ is taken as 1.0 for normal concrete.



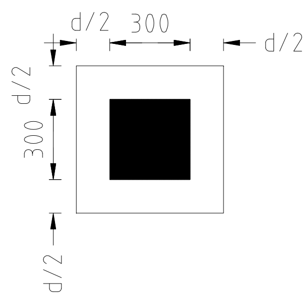
For Example 6 (interior column):

For beam shear,

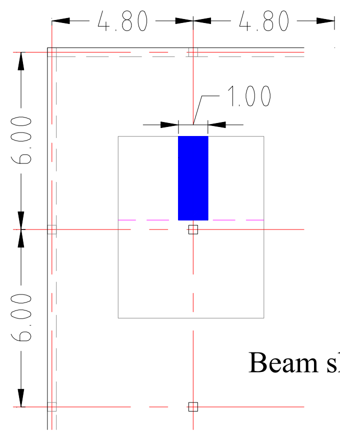
v_u at d from the face of column:

$$V_u = (\frac{14}{1000})(\frac{6000}{2} - \frac{300}{2} - 174) = 37.5kN/1m - width \qquad \text{(Long direction (N-S))}$$

$$\phi V_c = \phi 0.17\sqrt{f_c'} b_w d$$
$$= (0.75)(0.17\sqrt{28})(1000)(174)/1000 = 117.4 > 37.5kN/1m - width$$



Punching shear



Beam shear

Punching shear:

$$b_o = (4)(300 + \frac{d}{2} + \frac{d}{2}) = (4)(300 + 168) = 1872mm$$

$$V_u = (14)[(6)(4.8) - (0.468)(0.468)] = 400.1kN$$

$$\phi V_c = \phi 0.33\sqrt{f_c'} b_o d$$
$$= (0.75)(0.33\sqrt{28})(1872)(168)/1000 = 411.9kN > 400.1kN$$

O.K.

Example 7: Check the punching shear for the same slab of the example 6 for a factored load of (15.5)kN/m^2

$$b_o = (4)(300 + \frac{d}{2} + \frac{d}{2}) = (4)(300 + 168) = 1872mm$$

$$V_u = (15.5)[(6)(4.8) - (0.468)(0.468)] = 443kN$$

Then the drop panel should extend to (6/6=1m) & (4.8/6=0.8m) from the column center. The drop panel should project at least (200/4=50mm) below the slab.

Check the punching shear at distance (d/2) from the face of column:

$$d_{avg.} = 168 + 50 = 218mm$$

$$b_o = (4)(300 + \frac{d}{2} + \frac{d}{2}) = (4)(300 + 218) = 2072mm$$

$$V_u = (15.5)[(6)(4.8) - (0.518)(0.518)] = 442.2kN$$

$$\phi V_c = \phi 0.33\sqrt{f_c'}b_o d$$

$$= (0.75)(0.33\sqrt{28})(2072)(218)/1000 = 591.6kN > 442.2kN$$ **O.K.**

Check the punching shear at distance (d/2) from the face of drop panel:

Plan area of drop panel is: (2x1.6=3.2m^2)

$$d_{avg.} = 168mm$$

$$b_o = (2)(2000 + d + 1600 + d) = (2)(3936) = 7872mm$$

$$V_u = (15.5)[(6)(4.8) - (2.168)(1.768)] = 387kN$$

$$\phi V_c = \phi 0.33\sqrt{f_c'}b_o d$$

$$= (0.75)(0.33\sqrt{28})(7872)(168)/1000 = 1732kN > 387kN$$ **O.K.**

Openings in Slab Systems

According to the code (13.4), openings can be used in slab systems if adequate strength is provided and if all serviceability conditions of the ACI, including deflections, are met.

1. If openings are located in the area common to intersecting middle strips, it will be necessary to provide the same total amount of reinforcing in the slab that would have been there without the opening.
2. For openings in intersecting column strips, the width of the openings may not be more than one-eighth the width of the column strip in either span. An amount of reinforcing equal to that interrupted by the opening must be placed on the sides of the opening.
3. Openings in an area common to one column strip and one middle strip may not interrupt more than one-fourth of the reinforcing in

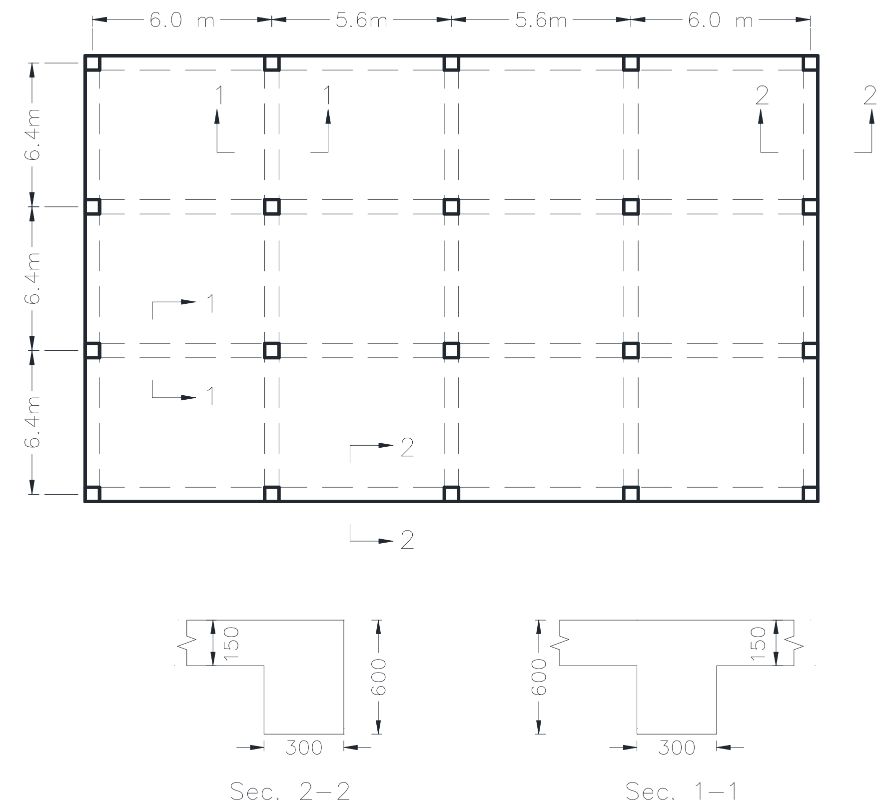
either strip. An amount of reinforcing equal to that interrupted shall be placed around the sides of the opening.

4. The shear requirements of Section 11.11.6 of the code must be met.

11.11.6.1 — For slabs without shearheads, that part of the perimeter of the critical section that is enclosed by straight lines projecting from the centroid of the column, concentrated load, or reaction area and tangent to the boundaries of the openings shall be considered ineffective.

Example :

For the slab shown in Figure, Compute the design moments for the N-S direction. The slab is to support a live load of 3.4 kN/m^2 and a super imposed dead load of 3.8 kN/m^2. The columns are 300mm x 300mm & the slab thickness is 150mm.



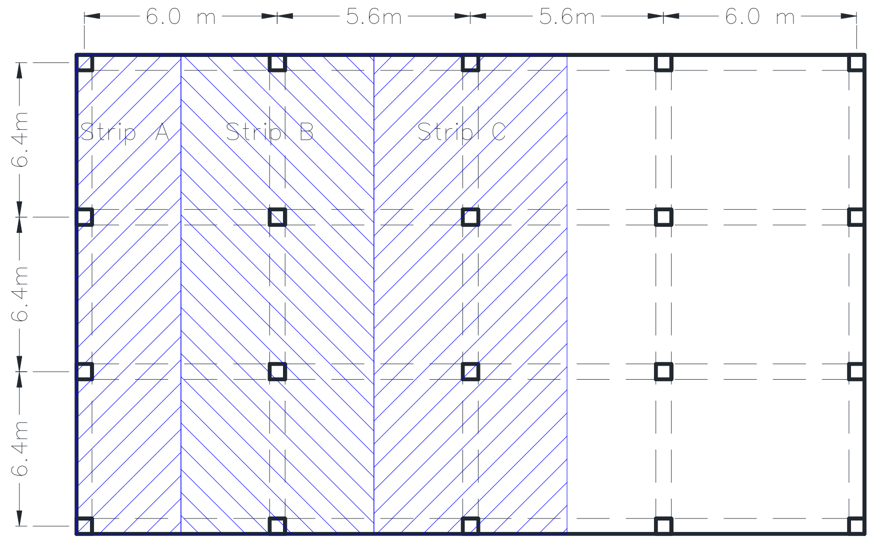
SOLUTION

- $slab.s.w. = (0.15)(24) = 3.6 kN/m^2 \Rightarrow D.L. = 3.6 + 3.8 = 7.4 kN/m^2$
- $q_u = (1.2)(7.4) + (1.6)(3.4) = 14.32 kN/m^2$

For N-S direction, the slab shall be divided to design strips as shown in Figure below,

For strip A,

$$M_o = \frac{q_u l_2 l_n^2}{8} \frac{(14.32)(\frac{6}{2} + 0.15)(6.4 - 0.3)^2}{8} \cong 209.8 kN.m$$



- Flange projection:

For edge beam : $4h_f$ or h_w whichever is smaller.

=either (4x150=600mm) or (600-150=450mm)

- Moment of inertia of beams: (**Home work**)

I for exterior beam = $(8.01)(10)^9$ mm^4

- Calculating α values for edge beam:

$$I_s = \frac{(\frac{6000}{2} + 150)(150)^3}{12} \cong (0.886)(10)^9 mm^4$$

$$\alpha = \frac{(8.01)(10)^9}{(0.886)(10)^9} = 9.04$$

For int. span

Negative design moment: $(0.65)(209.8) = 136.4kN.m$

Positive design moment: $(0.35)(209.8) = 73.4kN.m$

For $\frac{l_2}{l_1} = \frac{6}{6.4} = 0.9375$, $\alpha_{f1}\frac{l_2}{l_1} = (9.04)(0.9375) = 8.475$ α_{f1} is in the direction of span

The portion of the **interior negative moment** to be resisted by the column strip, per Table 13.6.4.1 of ACI code, by interpolation is:

$(0.77)(136.4) = 105kN.m$

Beam moment = (0.85)(105)=89.3$kN.m$

& the slab moment is: (0.15)(105)=15.8$kN.m$

While the middle strip will resist:

$136.4-105=31.4kN.m$

The portion of the **positive moment** to be resisted by the column strip, per Table 13.6.4.4 of ACI code, by interpolation is:

$(0.77)(73.4)=56.5kN.m$

Beam moment = (0.85)(56.5)=48.0$kN.m$

& the slab moment is: (0.15)(56.5)=8.5$kN.m$

While the middle strip will resist:

$73.4-56.5=16.9kN.m$

For ext. span

Int. negative design moment: $(0.7)(209.8)=146.9kN.m$

Positive design moment: $(0.57)(209.8)=119.6kN.m$

Ext. negative design moment: $(0.16)(209.8)=33.6kN.m$

For $\frac{l_2}{l_1}=\frac{6}{6.4}=0.9375$, $\alpha_{f1}\frac{l_2}{l_1}=(9.04)(0.9375)=8.475$ α_{f1} is in the direction of span

The portion of the **interior negative moment** to be resisted by the column strip, per Table 13.6.4.1 of ACI code, by interpolation is:

$(0.77)(146.9)=113.1kN.m$

Beam moment = (0.85)(113.1)=96.1$kN.m$

& the slab moment is: (0.15)(113.1)=17.0$kN.m$

While the middle strip will resist:

$146.9-113.1=33.8kN.m$

The portion of the **positive moment** to be resisted by the column strip, per Table 13.6.4.4 of ACI code, by interpolation is:

$(0.77)(119.6)=92.1kN.m$

Beam moment = (0.85)(92.1)=78.3$kN.m$

& the slab moment is: (0.15)(92.1)=13.8$kN.m$

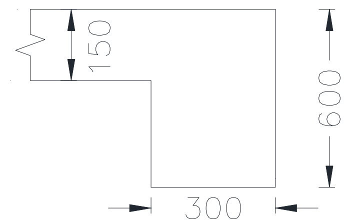
While the middle strip will resist:

$119.6-92.1=27.5kN.m$

While the portion of the **exterior negative moment** to be resisted by the column strip is calculated through:

$$\beta_t = \frac{E_{cb}C}{2E_c l_s}$$

$$C = \sum (1 - 0.63\frac{x}{y})(\frac{x^3 y}{3})$$



$$C = \left[1-(0.63)(\frac{300}{600})(\frac{(300)^3(600)}{3})\right]$$
$$+\left[1-(0.63)(\frac{150}{450})(\frac{(150)^3(450)}{3})\right] = (4.1)(10)^9 mm^4$$

$$I_s = \frac{(6000)(150)^3}{12} \cong (1.6875)(10)^9 mm^4$$

$$\beta_t = \frac{E_{cb}C}{2E_c l_s} = \frac{(4.1)(10)^9}{(2)(1.6875)(10)^9} = 1.215$$

Hence, from Table 13.6.4.2 :

(0.89)(33.6)=29.9$kN.m$

Beam moment = (0.85)(29.9)=25.4$kN.m$

Slab moment = (0.15)(29.9)=4.5$kN.m$

Middle strip moment =33.6-29.9=3.7 $kN.m$

For strip B

$$M_o = \frac{q_u l_2 l_n^2}{8} \frac{(14.32)(5.8)(6.4-0.3)^2}{8} \cong 386.3kN.m$$

- Moment of inertia of beam: (**Home work**)

  I for interior girders = $(9.56)(10)^9$ mm^4

  - Calculating α value for beam:

$$I_s = \frac{(5800)(150)^3}{12} \cong (1.63)(10)^9 mm^4$$

$$\alpha = \frac{(9.56)(10)^9}{(1.63)(10)^9} = 5.86$$

For int. span

Negative design moment: $(0.65)(386.3) = 251.1kN.m$

Positive design moment: $(0.35)(386.3) = 135.2 kN.m$

For $\frac{l_2}{l_1} = \frac{5.8}{6.4} = 0.906$, $\alpha_{f1}\frac{l_2}{l_1} = (5.86)(0.906) = 5.3$ α_{f1} is in the direction of span

The portion of the **interior negative moment** to be resisted by the column strip, per Table 13.6.4.1 of ACI code, by interpolation is:

$(0.78)(251.1) = 195.8 kN.m$

Beam moment = (0.85)(195.8)=166.4$kN.m$

& the slab moment is: (0.15)(195.8)=29.4$kN.m$

While the middle strip will resist:

$251.1 - 195.8 = 55.3 kN.m$

The portion of the **positive moment** to be resisted by the column strip, per Table 13.6.4.4 of ACI code, by interpolation is:

$(0.78)(135.2) = 105.5 kN.m$

Beam moment = (0.85)(105.5)=89.6$kN.m$

& the slab moment is: (0.15)(105.5)=15.86$kN.m$

While the middle strip will resist:

$135.2 - 105.5 = 29.7 kN.m$

For ext. span

Int. negative design moment: $(0.7)(386.3) = 270.4 kN.m$

Positive design moment: $(0.57)(386.3) = 220.2 kN.m$

Ext. negative design moment: $(0.16)(386.3) = 61.8 kN.m$

For $\frac{l_2}{l_1} = \frac{6}{6.4} = 0.9375$, $\alpha_{f1}\frac{l_2}{l_1} = (9.04)(0.9375) = 8.475$ α_{f1} is in the direction of span

The portion of the **interior negative moment** to be resisted by the column strip, per Table 13.6.4.1 of ACI code, by interpolation is:

$(0.78)(270.4) = 210.9 kN.m$

Beam moment = (0.85)(210.9)=179.2$kN.m$

& the slab moment is: (0.15)(210.9)=31.6$kN.m$

While the middle strip will resist:

$270.4 - 210.9 = 59.5 kN.m$