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3

هندسة الاسس

CHAPTER 3

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BEARING CAPACITY OF SHALLOW FOUNDATIONS

3.1 MODES OF FAILURE

Failure is defined as mobilizing the full value of soil shear strength accompanied with excessive settlements. For shallow foundations it depends on soil type, particularly its compressibility, and type of loading. Modes of failure in soil at ultimate load are of three types; these are (see **Fig. 1.5***):*

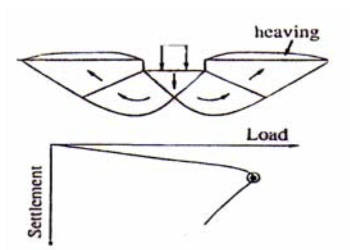
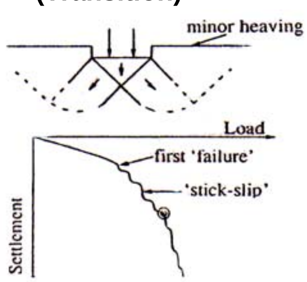
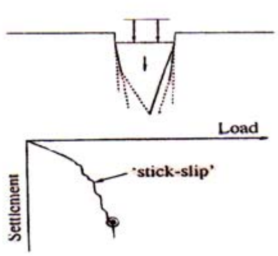
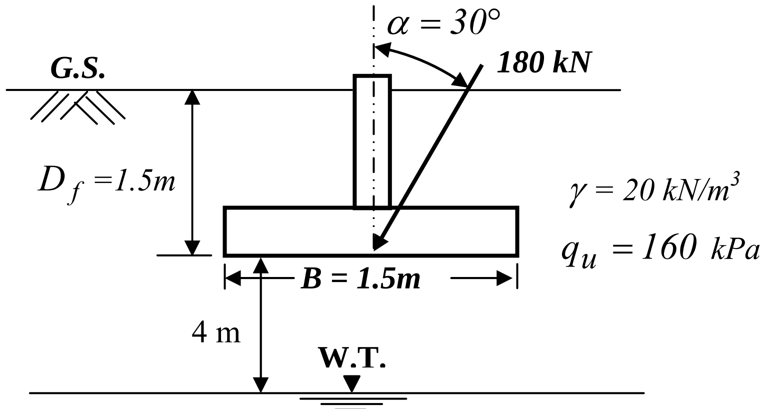
Mode of Failure	Characteristics	Typical Soils
1. General Shear failure  	• Well defined continuous slip surface up to ground level, • Heaving occurs on both sides with final collapse and tilting on one side, • Failure is sudden and catastrophic, • Ultimate value is peak value.	• Low compressibility soils • Very dense sands, • Saturated clays (NC and OC), • Undrained shear (fast loading).
2. local Shear failure (Transition)  	• Well defined slip surfaces only below the foundation, discontinuous either side, • Large vertical displacements required before slip surfaces appear at ground level, • Some heaving occurs on both sides with no tilting and no catastrophic failure, • No peak value, ultimate value not defined.	• Moderate compressibility soils • Medium dense sands,
3. Punching Shear failure  	• Well defined slip surfaces only below the foundation, non either side, • Large vertical displacements produced by soil compressibility, • No heaving, no tilting or catastrophic failure, no ultimate value.	• High compressibility soils • Very loose sands, • Partially saturated clays, • NC clay in drained shear (very slow loading), • Peats.

Fig. (3.1): Modes of failure.

Example (4): **A square footing of 1.5x1.5m is subjected to an inclined load as shown in figure below. What is the factor of safety against bearing capacity (use Terzaghi's equation).**



Solution:

By Terzaghi's equation: $q_{ult.} = cN_c.S_c + qN_q + \frac{1}{2}.B.\gamma.N\gamma.S_\gamma$

Shape factors: *from table (3.2) for square footing* $S_c = 1.3; S\gamma = 0.8$, $c = q_u / 2 = 80$ kPa

Bearing capacity factors: *from table (3.3) for* $\phi_u = 0$: $N_c = 5.7,..N_q = 1.0,..N_\gamma = 0$

$q_{ult.(vertical.load)} = 80(5.7)(1.3)+20(1.5)(1.0) + 0.5(1.5)(20)(0)(0.8) = 622.8$ kN/m^2

From Fig.(3.7) *with* $\alpha = 30°$ *and cohesive soil, the reduction factor for inclined load is 0.42.*

$q_{ult.(inclined.load)} = 622.8(0.42) = 261.576$ kN/m^2

$Q_v = Q.\cos 30 = 180\ (0.866) = 155.88$ kN

Factor of safety (against bearing capacity failure) $= \frac{Q_{ult.}}{Q_v} = \frac{261.576(1.5)(1.5)}{155.88} = 3.77$

Check for sliding:

$Q_h = Q.\sin 30 = 180\ (0.5) = 90$ kN

$H_{max.} = A'_f.C_a + \sigma' \tan\delta = (1.5)(1.5)(80) + (180)(\cos 30)(\tan 0) = 180$ kN

Factor of safety (against sliding) $= \frac{H_{max.}}{Q_h} = \frac{180}{90} = 2.0$ **(O.K.)**

- **Note:** *Due to* *scale effects,* N_γ *and then the ultimate bearing capacity decreases with increase in size of foundation.* Therefore, Bowle's (1996) suggested that for (**B > 2m**), with any bearing capacity equation of **Table (3.2)**, the term $(0.5B\gamma.N_\gamma S_\gamma d_\gamma)$ must be multiplied by a reduction factor:

$r_\gamma = 1 - 0.25\log\left(\frac{B}{2}\right)$; i.e., $0.5B\gamma.N_\gamma S_\gamma d_\gamma r_\gamma$

B (m)	2	2.5	3	3.5	4	5	10	20	100
r_γ	1	0.97	0.95	0.93	0.92	0.90	0.82	0.75	0.57

Table (3.3): Bearing capacity factors for Terzaghi's equation.

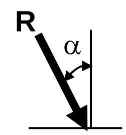
ϕ,..deg	N_c	N_q	N_γ	$K_{P\gamma}$
0	5.7 $^+$	1.0	0.0	10.8
5	7.3	1.6	0.5	12.2
10	9.6	2.7	1.2	14.7
15	12.9	4.4	2.5	18.6
20	17.7	7.4	5.0	25.0
25	25.1	12.7	9.7	35.0
30	37.2	22.5	19.7	52.0
34	52.6	36.5	36.0	
35	57.8	41.4	42.4	82.0
40	95.7	81.3	100.4	141.0
45	172.3	173.3	297.5	298.0
48	258.3	287.9	780.1	
50	347.5	415.1	1153.2	800.0

$^+ = 1.5\pi + 1$

Table (3.4): Shape, depth and inclination factors for Meyerhof's equation.

For	**Shape Factors**	**Depth Factors**	**Inclination Factors**
Any ϕ	$S_c = 1 + 0.2.K_P\frac{B}{L}$	$d_c = 1 + 0.2\sqrt{K_P}\frac{D_f}{B}$	$i_c = i_q = \left(1 - \frac{\alpha°}{90°}\right)^2$
$\phi \geq 10°$	$S_q = S_\gamma = 1 + 0.1.K_P\frac{B}{L}$	$d_q = d_\gamma = 1 + 0.1\sqrt{K_P}\frac{D_f}{B}$	$i_\gamma = \left(1 - \frac{\alpha°}{\phi°}\right)^2$
$\phi = 0$	$S_q = S_\gamma = 1.0$	$d_q = d_\gamma = 1.0$	$i_\gamma = 0$

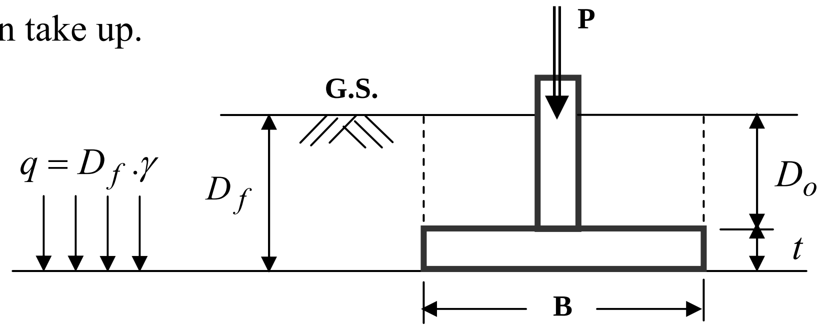
Where: $K_P = \tan^2(45 + \phi/2)$
$\alpha =$ angle of resultant measured from vertical without a sign.
B, L , D_f = width, length, and depth of footing.



Note:- When $\phi_{triaxial}$ is used for plan strain, adjust ϕ as: $\phi_{Ps} = (1.1 - 0.1\frac{B}{L})\phi_{triaxial}$

3.2 BEARING CAPACITY CLASSIFICATION *(According to column loads)*

- **Gross Bearing Capacity (q_{gross}):** It is the total unit pressure at the base of footing which the soil can take up.



q_{gross} = total pressure at the base of footing = $\sum P_{footing} / area.of.footing$.

where $\sum P_{footing} = p.(column.load) +$ *own wt. of footing + own wt. of earth fill over the footing.* $q_{gross} = (P + \gamma_s.D_o.B.L + \gamma_c.t.B.L)/B.L$

$$q_{gross} = \frac{P}{B.L} + \gamma_s.D_o + \gamma_c.t \quad \textbf{(3.1)}$$

- **Ultimate Bearing Capacity ($q_{ult.}$):** It is the maximum unit pressure or the maximum gross pressure that a soil can stand without shear failure.

- **Allowable Bearing Capacity ($q_{all.}$):** It is the ultimate bearing capacity divided by a reasonable factor of safety.

$$q_{all.} = \frac{q_{ult.}}{F.S} \quad \textbf{(3.2)}$$

- **Net Ultimate Bearing Capacity:** It is the ultimate bearing capacity minus the vertical pressure that is produced on horizontal plain at level of the base of the foundation by an adjacent surcharge.

$$q_{ult.-net} = q_{ult.} - D_f.\gamma \quad \textbf{(3.3)}$$

3.7 WHICH EQUATIONS TO USE?

Of the bearing capacity equations previously discussed, the most widely used equations are Meyerhof's and Hansen's. While Vesic's equation has not been much used (but is the suggested method in the American Petroleum Institute, RP2A Manual, 1984).

Table (3.6) : Which equations to use.

Use	***Best for***
Terzaghi	• *Very cohesive soils where D/B ≤ 1 or for a quick estimate of* $q_{ult.}$ *to compare with other methods,* • *Somewhat simpler than Meyerhof's, Hansen's or Vesic's equations; which need to compute the shape, depth, inclination, base and ground factors,* • *Suitable for a concentrically loaded horizontal footing,* • *Not applicable for columns with moment or tilted forces,* • *More conservative than other methods.*
Meyerhof, Hansen, Vesic	• *Any situation which applies depending on user preference with a particular method.*
Hansen, Vesic	• *When base is tilted; when footing is on a slope or when D/B >1.*

3.8 EFFECT OF SOIL COMPRESSIBILITY <u>(local shear failure)</u>

1. For clays sheared in drained conditions, Terzaghi (1943) suggested that the shear strength parameters c and ϕ should be reduced as:

$$c^* = 0.67c' \quad \text{and} \quad \phi^* = tan^{-1}(0.67\, tan\, \phi') \qquad \textbf{(3.6)}$$

2. For loose and medium dense sands (when $D_r \leq 0.67$), Vesic (1975) proposed:

$$\phi^* = tan^{-1}(0.67 + D_r - 0.75 D_r^2)\, tan\, \phi' \qquad \textbf{(3.7)}$$

where D_r is the relative density of the sand, recorded as a fraction.

<u>**Note:** For dense sands ($D_r > 0.67$) the strength parameters need not be reduced, since the general shear mode of failure is likely to apply.</u>

- **Net Allowable Bearing Capacity ($q_{all.-net}$):** It is the **net safe bearing capacity** or the ultimate bearing capacity divided by a reasonable factor of safety.

Approximate: $$q_{all.-net} = \frac{q_{ult.-net}}{F.S} = \frac{q_{ult.} - D_f \cdot \gamma}{F.S} \quad \text{...(3.4)}$$

Exact: $$q_{all.-net} = \frac{q_{ult.}}{F.S} - D_f \cdot \gamma \quad \text{...(3.5)}$$

3.3 FACTOR OF SAFETY IN DESIGN OF FOUNDATION

The general values of safety factor used in design of footings are 2.5 to 3.0, however, the choice of factor of safety (F.S.) depends on many factors such as:

1. the variation of shear strength of soil,

2. magnitude of damages,

3. reliability of soil data such as uncertainties in predicting the $q_{ult.}$,

4. changes in soil properties due to construction operations,

5. relative cost of increasing or decreasing F.S., and

6. the importance of the structure, differential settlements and soil strata underneath the structure.

3.4 BEARING CAPACITY REQUIREMENTS

Three requirements must be satisfied in determining bearing capacity of soil. These are:

(1) ***Adequate depth;*** the foundation must be deep enough with respect to environmental effects; such as: frost penetration, seasonal volume changes in the soil, to exclude the possibility of erosion and undermining of the supporting soil by water and wind currents, and to minimize the possibility of damage by construction operations,

Table (3.1): Bearing capacity values according to building codes.

Soil type	Description	Bearing pressure (kg/cm^2)		Notes
		Dry	submerged	
Rocks	1. bed rocks.	70		Unless they are affected by water.
	2. sedimentary layer rock (hard shale, sand stone, siltstone).	30		
	3. shest or erdwas.	20		
	4. soft rocks.	13		
Cohesionless soil	1. well compacted sand or sand mixed with gravel.	3.5-5.0	1.75-2.5	Footing width 1.0 ms.
	2. sand, loose and well graded or loose mixed sand and gravel.	1.5-3.0	0.5-1.5	
	3. compacted sand, well graded.	1.5-2.0	0.5-1.5	
	4. well graded loose sand.	0.5-1.5	0.25-0.5	
Cohesive soil	1. very stiff clay	2-4		It is subjected to settlement due to consolidation
	2. stiff clay	1-2		
	3. medium-stiff clay	0.5-1		
	4. low stiff clay	0.25-0.5		
	5. soft clay	up to 0.2		
	6. very soft clay	0.1-0.2		
	7. silt soil	1.0-1.5		

(b) Field Load Test

This test is fully explained in **(chapter 2)**.

(c) Bearing Capacity Equations

Several bearing capacity equations were developed *for the case of **general shear failure** by many researchers as presented in Table (3.2); see **Tables** (3.3, 3.4 and 3.5) for related factors.*

Table (3.2): Bearing capacity equations by the several authors indicated.

- Terzaghi (see **Table 3.3** for typical values for $K_{P\gamma}$ values)

$$q_{ult.} = cN_c.S_c + \bar{q}N_q + 0.5.B.\gamma.N\gamma.S_\gamma$$

$$N_q = \frac{e^{2[0.75\pi.-\frac{\phi}{2}(\frac{\pi}{180})].\tan\phi}}{2\cos^2(45+\phi/2)}; \qquad N_c = (N_q - 1).\cot\phi; \qquad N_\gamma = \frac{\tan\phi}{2}(\frac{k_{P\gamma}}{\cos^2\phi} - 1)$$

where a close approximation of $k_{P\gamma} \approx 3.\tan^2\left(45 + \frac{(\phi+33)}{2}\right)$.

		Strip	circular	square	rectangular
S_c	=	1.0	1.3	1.3	(1+ 0.3 B / L)
S_γ	=	1.0	0.6	0.8	(1- 0.2 B / L)

- Meyerhof (see **Table 3.4** for shape, depth, and inclination factors)

Vertical load: $q_{ult.} = c.N_c.S_c.d_c + q.N_q.S_q.d_q + 0.5.B.\gamma.N_\gamma.S_\gamma.d_\gamma$

Inclined load: $q_{ult.} = c.N_c.d_c.i_c + q.N_q.d_q.i_q + 0.5.B.\gamma.N_\gamma.d_\gamma.i_\gamma$

$$N_q = e^{\pi.\tan\phi}\tan^2(45+\phi/2); \qquad N_c = (N_q - 1).\cot\phi; \qquad N_\gamma = (N_q - 1).\tan(1.4\phi)$$

- Hansen (see **Table 3.5** for shape, depth, and inclination factors)

For..$\phi > 0$: $q_{ult.} = cN_cS_cd_ci_cg_cb_c + qN_qS_qd_qi_qg_qb_q + 0.5.B.\gamma.N_\gamma S_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$

For..$\phi = 0$: $q_{ult.} = 5.14S_u(1 + S'_c + d'_c - i'_c - b'_c - g'_c) + \bar{q}$

$$N_q = e^{\pi.\tan\phi}\tan^2(45+\phi/2); \qquad N_c = (N_q - 1).\cot\phi; \qquad N_\gamma = 1.5(N_q - 1).\tan\phi$$

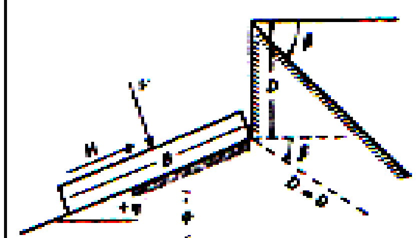
- Vesic (see **Table 3.5** for shape, depth, and inclination factors)

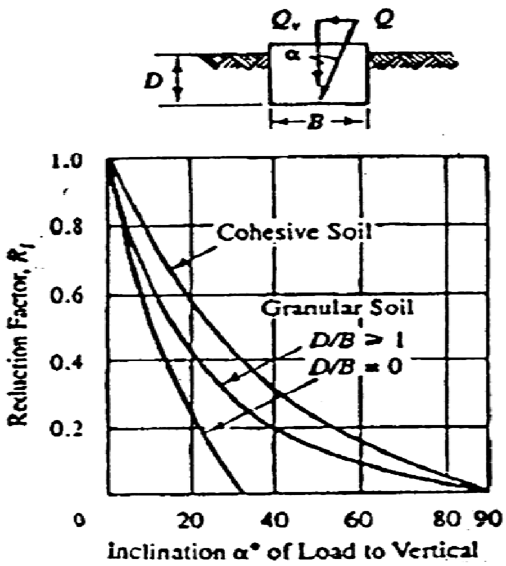
Use Hansen's equations above

$$N_q = e^{\pi.\tan\phi}\tan^2(45+\phi/2); \qquad N_c = (N_q - 1).\cot\phi; \qquad N_\gamma = 2(N_q + 1).\tan\phi$$

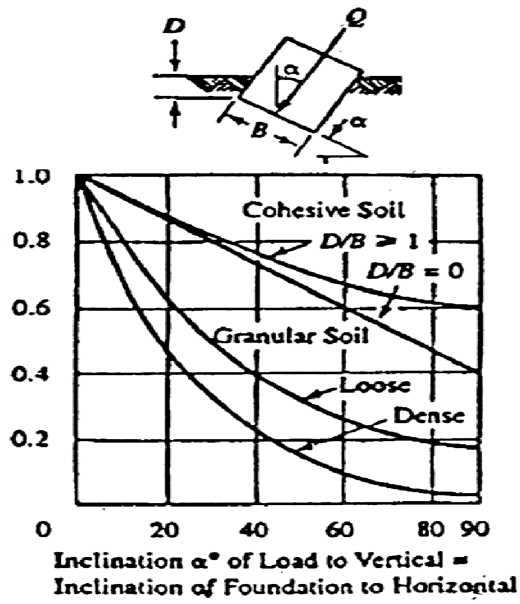
- *<u>All the bearing capacity equations above are based on general shear failure in soil.</u>*

Table (3.5): Shape, depth, inclination, ground and base factors for use in Hansen or Vesic bearing capacity equations of Table (3.2). (1) Factors apply to either method unless subscripted with (H) or (V). **(2)** Use primed factors when $\phi = 0$.

<table>
<tr><th>Shape factors</th><th>Depth factors</th><th>Inclination factors</th><th>Ground Factors (Base on slope)</th></tr>
<tr>
<td>$S'_c = 0.2\dfrac{B}{L}$
$S_c = 1 + \dfrac{N_q}{N_c}\cdot\dfrac{B}{L}$
S_c = 1.0 for strip
$S_q = 1 + \dfrac{B}{L}\tan\phi$
$S_\gamma = 1 - 0.4\dfrac{B}{L}$</td>
<td>$d'_c = 0.4k$
$d_c = 1 + 0.4k$
$d_q = 1 + 2\tan\phi(1-\sin\phi)^2 k$
d_γ = 1.0 for all ϕ
$k = \dfrac{D_f}{B}$ for $\dfrac{D_f}{B} \le 1$
$k = \tan^{-1}\dfrac{D_f}{B}$ **(rad)** for $\dfrac{D_f}{B} > 1$</td>
<td rowspan="2">$i'_{c(H)} = 0.5 - 0.5\sqrt{1 - \dfrac{H}{A_f C_a}}$
$i'_{c(V)} = 1 - \dfrac{mH}{A_f C_a N_c}$
$i_c = i_q - \dfrac{1 - i_q}{N_q - 1}$ (Hansen and Vesic)
$i_{q(H)} = \left[1 - \dfrac{0.5H}{V - A_f C_a \cot\phi}\right]^5$
$i_{q(V)} = \left[1 - \dfrac{H}{V - A_f C_a \cot\phi}\right]^m$
$i_{\gamma(H)} = \left[1 - \dfrac{0.7H}{V - A_f C_a \cot\phi}\right]^5$ for ($\eta = 0$)
$i_{\gamma(H)} = \left[1 - \dfrac{(0.7 - \eta^\circ/450)H}{V + A_f C_a \cot\phi}\right]^5$ for ($\eta > 0$)
$i_{\gamma(V)} = \left[1 - \dfrac{H}{V - A_f C_a \cot\phi}\right]^{m+1}$</td>
<td>$g'_c = \dfrac{\beta^\circ}{147^\circ}$
For Vesic use: $N_\gamma = -2\sin\beta$ for $\phi = 0$
$g_c = 1 - \dfrac{\beta^\circ}{147^\circ}$
$g_{q(H)} = g_{\gamma(H)} = (1 - 0.5\tan\beta)^5$
$g_{q(V)} = g_{\gamma(V)} = (1 - \tan\beta)^2$</td>
</tr>
<tr>
<td colspan="2" rowspan="2">Where
$e_B \ldots e_L$ = Eccentricity of load from center of footing area
A_f = Effective footing area $B' \times L'$
C_a = Adhesion to base = cohesion or a reduced value
D_f = Depth of footing (used with B and not B')
H = Horizontal component of load with $H \le C_a A_f + V\tan\delta$
V = Total vertical load on footing
β = Slope of ground away from base with downward = (−)
δ = Friction angle between base and soil: usually $\delta = \phi$ for concrete on soil
η = Tilt angle of base from horizontal with (−) upward as usual case
GENERAL NOTES
1. Do not use S_i in combination with i_i.
2. Can use S_i in combination with d_i, g_i, and b_i
3. For $L/B \le 2$ use ϕ_{tr}
For $L/B > 2$ use $\phi_{PS} = 1.5\phi_{tr} - 17$
For $\phi \le 34^\circ$ $\phi_{PS} = \phi_{tr}$</td>
<td>Base factors (Tilted base)
$b'_c = \dfrac{\eta^\circ}{147^\circ}$
$b_c = 1 - \dfrac{\eta^\circ}{147^\circ}$
$b_{q(H)} = \exp(-2\eta\pi\tan\phi/180)$
$b_{\gamma(H)} = \exp(-2.7\eta\pi\tan\phi/180)$
$b_{q(V)} = b_{\gamma(V)} = (1 - \eta\pi\tan\phi/180)^2$</td>
</tr>
<tr>
<td>$m = m_B = \dfrac{2 + B/L}{1 + B/L}$ for **H** parallel to **B**
$m = m_L = \dfrac{2 + L/B}{1 + L/B}$ for **H** parallel to **L**
Note: $i_q, i_\gamma > 0$</td>
<td>*Note:* $\beta + \eta < 90^\circ$ and $\beta < \phi$
</td>
</tr>
</table>



(a) horizontal foundation



(b) Inclined foundation

Figure (3.7): Inclined load reduction factors.

Important Notes:

- *Remember that in this case, Meyerhof's bearing capacity equation for inclined load (from Table 3.2) can be used directly:*

$$\boxed{q_{ult.(inclined..load)} = cN_c d_c i_c + \overline{q} N_q d_q i_q + 0.5\gamma'.B.N_\gamma d_\gamma i_\gamma} \text{..................}\textbf{(3.9)}$$

- The footings stability with regard to the inclined load's horizontal component also must be checked by calculating ***the factor of safety against sliding*** as follows:

$$Fs_{(slididing)} = \frac{H_{max.}}{H} \text{..}\textbf{(3.10)}$$

where:

H = *the inclined load's horizontal component,*

$\boxed{H_{max.} = the.\max imum.resisting.force = A'_f.C_a + \sigma' \tan\delta \text{ .... } for\ (c-\phi)\ soils;}$ *or*

$H_{max.} = A'_f.C_a$ .... *for the undrained case in clay* $(\phi_u = 0)$; *or*

BEARING CAPACITY EXAMPLES (1)

Example (1): **Determine the allowable bearing capacity of a strip footing shown below using Terzaghi and Hansen Equations if c = 0, $\phi = 30°$, D_f = 1.0m , B = 1.0m , $\gamma_{soil} = 19$ kN/m^3, the water table is at ground surface, and SF=3.**

Solution:

(a) By Terzaghi's equation:

$$q_{ult.} = cN_c.S_c + qN_q + \frac{1}{2}.B.\gamma.N\gamma.S_\gamma$$

Shape factors: from table (3.2), for strip footing $S_c = S\gamma = 1.0$

Bearing capacity factors: from table (3.3), for $\phi = 30°$, $N_q = 22.5, ..N_\gamma = 19.7$

$q_{ult.} = 0 + 1.0\ (19\text{-}9.81)22.5 + 0.5x1(19\text{-}9.81)19.7x1.0 = 297\ kN/m^2$

$q_{all.}$=297/3 = $\boxed{99\ kN/m^2}$

(b) By Hansen's equation:

$for..\phi > 0:$

$$q_{ult.} = cN_cS_cd_ci_cg_cb_c + qN_qS_qd_qi_qg_qb_q + 0.5\gamma.B.N_\gamma S_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$$

Since c = 0, any factors with subscript c do not need computing. Also, all $g_i..and..b_i$ *factors are 1.0; with these factors identified the Hansen's equation simplifies to:*

$q_{ult.} = \bar{q}N_qS_qd_q + 0.5\gamma'.B.N_\gamma S_\gamma d_\gamma$

From table (3.5): $\begin{cases} for...\phi \le 34°\ ..use..\phi_{ps} = \phi_{tr} \\ for\ L/B > 2\ ..use..\phi_{ps} = 1.5\phi_{tr} - 17 \end{cases}$, $\therefore .use.\phi_{ps} = 1.5\phi_{tr} - 17$

$$\therefore .use.\phi_{ps} = 1.5\phi_{tr} - 17,\quad 1.5\ x\ 30 - 17 = 28°\ ,$$

Bearing capacity factors: from table (3.4), for $\phi = 28°$, $N_q = 14.7,..N_\gamma = 10.9$

Shape factors: from table (3.5), $S_\gamma = S_q = 1.0$,

Depth factors: from table (3.5),

$$d_q = 1 + 2\tan\phi(1-\sin\phi)^2 \frac{D}{B},$$

$$d_q = 1 + 2.\tan 28(1-\sin 28)^2 \frac{1}{1} = 1.29, \quad and \quad d_\gamma = 1.0$$

$q_{ult.} = 1.0\ (19\text{-}9.81)14.7x1.29 + 0.5x1(19\text{-}9.81)10.9x1.0 = 224.355\ kN/m^2$

$q_{all.}$=224.355/3 = $\boxed{74.785\ kN/m^2}$

Example (2): A footing load test produced the following data:

D_f = **0.5m, B = 0.5m, L = 2.0m,** $\gamma'_{soil} = 9.31$ **kN/m^3,** $\phi_{tr} = 42.5°$, **c = 0,**

$P_{ult.}(measured) = 1863.kN$, $q_{ult.}(measured) = 1863/0.5x2 = 1863$ **kN/m^2.**

Required: compute $q_{ult.}$ by Hansen's and Meyerhof's equations and compare computed with measured values.

Solution:

(a) By Hansen's equation:

Since c = 0, and all $g_i..and..b_i$ factors are 1.0; the Hansen's equation simplifies to:

$$q_{ult.} = \bar{q}N_qS_qd_q + 0.5\gamma'.B.N_\gamma S_\gamma d_\gamma$$

From table (3.5): L/B = 2/0.5 = 4 > 2 $\therefore ..use..\phi_{ps} = 1.5\phi_{tr} - 17$,

1.5 x 42.5 - 17= 46.75° $\longrightarrow$ *take...$\phi = 47°$*

Bearing capacity factors: from table (3.2)

$$N_q = e^{\pi.\tan\phi}..\tan^2(45+\phi/2), \quad N_\gamma = 1.5(N_q - 1)\tan\phi$$

for $\phi = 47°$: $N_q = 187.2$, $N_\gamma = 299.5$

Shape factors: from table (3.5),

$$S_q = 1 + \frac{B}{L}\tan\phi = 1 + \frac{0.5}{2.0}\tan 47 = 1.27, \qquad S_\gamma = 1 - 0.4\frac{B}{L} = 1 - 0.4\frac{0.5}{2.0} = 0.9$$

Depth factors: from table (3.5),

$$d_q = 1 + 2\tan\phi(1-\sin\phi)^2\frac{D}{B}, d_q = 1 + 2\tan 47(1-\sin 47)^2\frac{0.5}{0.5} = 1.155, d_\gamma = 1.0$$

$$q_{ult.} = 0.5\,(9.31)187.2x1.27x1.155 + 0.5x0.5(9.31)299.5x0.9x1.0 = \boxed{1905.6\ kN/m^2}$$

versus 1863 kN/m^2 measured.

(b) By Meyerhof's equation:

From table (3.2) for vertical load with c = 0:

$$q_{ult.} = \bar{q}N_qS_qd_q + 0.5\gamma'.B.N_\gamma S_\gamma d_\gamma$$

From table (3.4): $\phi_{ps} = (1.1 - 0.1\frac{B}{L})\phi_{tr}$, $(1.1 - 0.1\frac{0.5}{2.0})42.5 = 45.7$, take...$\phi = 46°$

Bearing capacity factors: from table (3.2)

$$N_q = e^{\pi.\tan\phi}..\tan^2(45+\phi/2),\ \ N_\gamma = (N_q - 1)\tan(1.4\phi)$$

for $\phi = 46°$: $N_q = 158.5$, $N_\gamma = 328.7$

Shape factors: from table (3.4)

$$K_p = \tan^2(45+\phi/2) = 6.13, \qquad S_q = S_\gamma = 1 + 0.1.K_p\frac{B}{L} = 1 + 0.1(6.13)\frac{0.5}{2.0} = 1.15$$

Depth factors: from table (3.4)

$$\sqrt{K_p} = 2.47, \qquad d_q = d_\gamma = 1 + 0.1.\sqrt{K_p}\frac{D}{B} = 1 + 0.1(2.47)\frac{0.5}{0.5} = 1.25$$

$$q_{ult.} = 0.5(9.31)158.5x1.15x1.25 + 0.5x0.5(9.31)328.7x1.15x1.25 = 2160.4\ \ kN/m^2$$

versus 1863 kN/m^2 measured

$\therefore$ **Both Hansen's and Meyerhof's eqs. give over-estimated $q_{ult.}$ compared with measured.**

Inclination factors from table (3.5):

$$i_q = (1 - \frac{0.5H}{V + A_f.c.\cot\phi})^5 = (1 - \frac{0.5x200}{600 + 4x25x\cot 25})^5 = 0.52,$$

$$i_c = i_q - \frac{(1 - i_q)}{(N_q - 1)} = 0.52 - \frac{1 - 0.52}{10.7 - 1} = 0.47,$$

$$for..\eta > 0 : i_\gamma = (1 - \frac{(0.7 - \eta^\circ / 450)H}{V + A_f.c.\cot\phi})^5 = (1 - \frac{(0.7 - 10/450)200}{600 + 4x25x\cot 25})^5 = 0.40$$

The base factors $for..\eta = 10^\circ (0.175..radians)$ from table (3.5):

$$b_c = 1 - \frac{\eta^\circ}{147^\circ} = 1 - \frac{10}{147} = 0.93,$$

$$b_q = e^{(-2\eta\tan\phi)} = e^{(-2(0.175)\tan 25)} = 0.85, \quad b_\gamma = e^{(-2.7\eta\tan\phi)} = e^{(-2.7(0.175)\tan 25)} = 0.80$$

$$q_{ult.} = 25(20.7)(1.06)(0.47)(0.93) + 0.3(17.5)(10.7)(1.05)(0.52)(0.85)$$
$$+ 0.5(17.5)(2.0)(6.8)(1)(0.40)(0.80) = 304\ kN/m^2$$

$$q_{all.} = 304 / 3 = 101.3\ kN/m^2$$

$P_{all.} = q_{all.}.A_f$ =101.3(4) = 405.2 kN < 600 kN (the given load), ∴ B=2m is not adequate and, therefore it must be increased and $P_{all.}$ recomputed and checked.

3.9 FOOTINGS WITH INCLINED OR ECCENTRIC LOADS

- **INCLINED LOAD:**

If a footing is subjected to an inclined load (see Fig.3.7), the inclined load Q can be resolved into vertical and horizontal components. The vertical component Q_v can then be used for bearing capacity analysis in the same manner as described previously (Table 3.2). After the bearing capacity has been computed by the normal procedure, it must be corrected by an R_i factor using Fig.(3.7) as:

$$\therefore \quad \boxed{q_{ult.(inclined..load)} = q_{ult.(vertical..load)}.x.R_i} \quad \textbf{...(3.8)}$$

$H_{max.} = \sigma' \tan\delta$ .... *for a sand and the drained case in clay* $(c' = 0)$.

$A'_f = effective..area = B'.L'$

$C_a = adhesion = \alpha.C_u$

$where...\alpha = 1.0....for.soft.to.medium.clays.;\ and$

$.\alpha = 0.5....for.stiff.clays.$

σ' = the net vertical effective load = $Q_v - D_f.\gamma$; *or*

$\sigma' = (Q_v - D_f.\gamma) - u.A'_f$ *(if the water table lies above foundation level)*

δ = *the skin friction angle, which can be taken as equal to* (ϕ')*,and*

u = the pore water pressure at foundation level.

- **ECCENTRIC LOAD:**

Eccentric load result from loads applied somewhere other than the footing's centroid or from applied moments, such as those resulting at the base of a tall column from wind loads or earthquakes on the structure.

To provide adequate $SF_{(against.lifting)}$ *of the footing edge, it is recommended that the eccentricity* $(e \le B/6)$. Footings with eccentric loads may be analyzed for bearing capacity by two methods: (1) the concept of useful width and (2) application of reduction factors.

(1) Concept of Useful Width:

In this method, only that part of the footing that is symmetrical with regard to the load is used to determine bearing capacity by the usual method, with the remainder of the footing being ignored.

- *First, computes eccentricity and adjusted dimensions:*

$$e_x = \frac{M_y}{V}; \quad L' = L - 2e_x; \quad e_y = \frac{M_x}{V}; \quad B' = B - 2e_y; \quad A'_f = A' = B'.L'$$

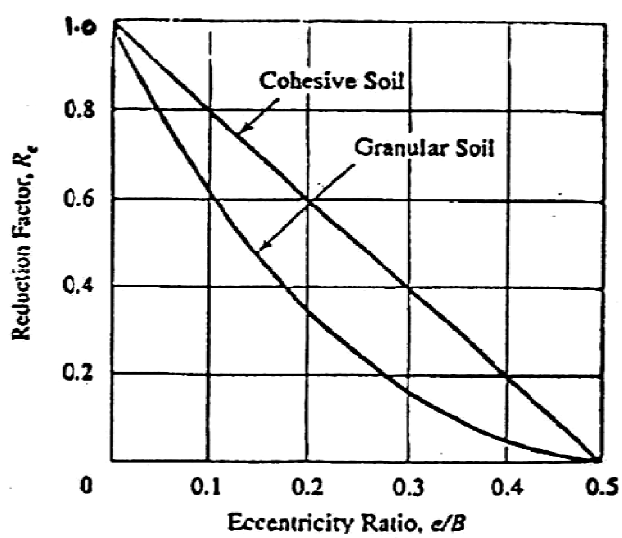
- Second, calculates $q_{ult.}$ from Meyerhof's, or Hansen's, or Vesic's equations (Table 3.2) using B' in the $(\frac{1}{2}B.\gamma.N_{\gamma})$ term and B' or/and L' in computing the shape factors and not in computing depth factors.

(2) Application of Reduction Factors:

First, computes bearing capacity by the normal procedure (using equations of Table 3.2), assuming that the load is applied at the centroid of the footing. Then, the computed value is corrected for eccentricity by a reduction factor (R_e) obtained from Figure (3.8) or from Meyerhof's reduction equations as:

$$\left.\begin{aligned} R_e &= 1 - 2(e/B) \text{ ....for.. cohesive ..soil} \\ R_e &= 1 - (e/B)^{1/2} \text{ ....for.cohesionless.soil} \end{aligned}\right\} \text{.......................(3.11)}$$

$$\therefore \quad \boxed{q_{ult.(eccentric)} = q_{ult.(concentric)} .x.R_e} \text{..................................(3.12)}$$

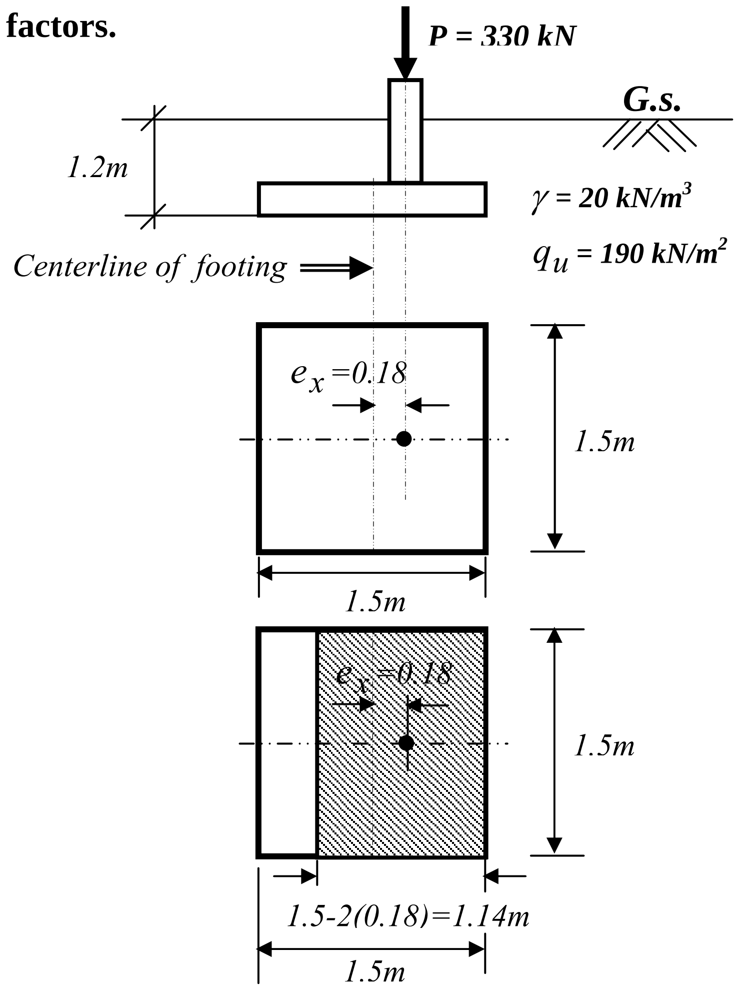


Figure(3.8): Eccentric load reduction factors.

Example (5): **A 1.5x1.5m square footing is subjected to eccentric load as shown below. What is the safety factor against bearing capacity failure (use Terzaghi's equation):**

(a) By the concept of useful width, and

(b) Using Meyerhof's reduction factors.



Solution:

(1) Using concept of useful width:

from Terzaghi's equation:

$$q_{ult.} = cN_c.S_c + qN_q + \frac{1}{2}.B'.\gamma.N\gamma.S_\gamma$$

Shape factors: from table (3.2) for square footing $S_c = 1.3; S\gamma = 0.8$, $c = q_u / 2 = 95\ kPa$

Bearing capacity factors: from table (3.3) for $\phi_u = 0$: $N_c = 5.7, N_q = 1.0, N_\gamma = 0$

The useful width is: $B' = B - 2e_x = 1.5 - 2(0.18) = 1.14m$

$q_{ult.} = 95(5.7)(1.3)+20(1.2)(1.0) + 0.5(1.14)(20)(0)(0.8) = 727.95\ kN/m^2$

Factor of safety (against bearing capacity failure) $= \dfrac{Q_{ult.}}{Q_v} = \dfrac{727.95(1.14)(1.5)}{330} = 3.77$

(2) *Using Meyerhof's reduction factors:*

In this case, $q_{ult.}$ *is computed based on the actual width: B = 1.5m*

from Terzaghi's equation:

$$q_{ult.} = 1.3cN_c + qN_q + 0.4B.\gamma.N\gamma$$

$q_{ult.(concentric.load)} = 1.3(95)(5.7) + 20(1.2)(1.0) + 0.4(1.5)(20)(0) = 727.95\ kN/m^2$

For eccentric load from figure (3.8):

with Eccentricity ratio $= \dfrac{e_x}{B} = \dfrac{0.18}{1.5} = 0.12$ *; and cohesive soil* $R_e = 0.76$

$\therefore q_{ult.(eccentric.load)} = 727.95\ (0.76) = 553.242\ kN/m^2$

Factor of safety (against bearing capacity failure) $= \dfrac{Q_{ult.}}{Q_v} = \dfrac{553.242(1.5)(1.5)}{330} = 3.77$

Example (6): A square footing of 1.8x1.8m is loaded with axial load of 1780 kN and subjected to M_x = 267 kN-m and M_y = 160.2 kN-m moments. Undrained triaxial tests of unsaturated soil samples give $\phi = 36°$ and $c = 9.4$ kN/m^2. If D_f = 1.8m, the water table is at 6m below the G.S. and $\gamma = 18.1$ kN/m^3, what is the allowable soil pressure if SF=3.0 using (a) Hansen bearing capacity and (b) Meyerhof's reduction factors.

Solution:

$e_y = \dfrac{267}{1780} = 0.15m$ **;** $e_x = \dfrac{160.2}{1780} = 0.09m$

$B' = B - 2e_y = 1.8 - 2(0.15) = 1.5m$ **;** $L' = L - 2e_x = 1.8 - 2(0.09) = 1.62m$

(a) Using Hansen's equation:

$(with...all...i_i, g_i..and...b_i..factors...are...1.0)$

$$q_{ult.} = cN_c.S_c.d_c + \overline{q}N_q.S_q.d_q + 0.5\gamma.B'.N_\gamma.S_\gamma.d_\gamma$$

Bearing capacity factors from table (3.2):

$N_c = (N_q - 1).\cot\phi$, $\quad N_q = e^{\pi.\tan\phi}..\tan^2(45 + \phi/2)$, $\quad N_\gamma = 1.5(N_q - 1)\tan\phi$

$for\ \phi = 36°: \quad N_c = 50.6, \quad N_q = 37.8, \quad N_\gamma = 40$

Shape factors from table (3.5):

$$S_c = 1 + \frac{N_q}{N_c}\frac{B'}{L'} = 1 + \frac{37.8}{50.6}\frac{1.5}{1.62} = 1.692, \qquad S_q = 1 + \frac{B'}{L'}\tan\phi = 1 + \frac{1.5}{1.62}\tan 36 = 1.673$$

$$S_\gamma = 1 - 0.4\frac{B'}{L'} = 1 - 0.4\frac{1.5}{1.62} = 0.629$$

Depth factors from table (3.5):

for D =1.8m, and B = 1.8m, D/B = 1.0 (shallow footing)

$$d_c = 1 + 0.4\frac{D}{B} = 1 + 0.4(1.0) = 1.4,$$

$$d_q = 1 + 2\tan\phi(1 - \sin\phi)^2\frac{D}{B} = 1 + 2\tan 36(1 - \sin 36)^2(1.0) = 1.246, \quad d_\gamma = 1.0$$

$q_{ult.}$ *= 9.4(50.6)(1.692)(1.4) + 1.8(18.1)(37.7)(1.673)(1.246)*

+ 0.5(18.1)(1.5)(40)(0.629)(1)= 4028.635 kN/m²

$q_{all.} = 4028.635 / 3 = 1342.878\ kN/m^2$

Actual soil pressure ($q_{act.}$*) = 1780/(1.5)(1.62)= 732.510 < 1342.878* **(O.K.)**

(b) Using Meyerhof's reduction:

$$R_{ex} = 1 - (\frac{e_x}{L})^{1/2} = 1 - (\frac{0.09}{1.8})^{0.5} = 0.78; \qquad R_{ey} = 1 - (\frac{e_y}{B})^{1/2} = 1 - (\frac{0.15}{1.8})^{0.5} = 0.72$$

Recompute $q_{ult.}$ *as for a centrally loaded footing, since the depth factors are unchanged.*

The revised Shape factors from table (3.5) are:

$$S_c = 1 + \frac{N_q}{N_c}\frac{B}{L} = 1 + \frac{37.8}{50.6}\frac{1.8}{1.8} = 1.75; \qquad S_q = 1 + \frac{B}{L}\tan\phi = 1 + \frac{1.8}{1.8}\tan 36 = 1.73$$

$$S_\gamma = 1 - 0.4\frac{B}{L} = 1 - 0.4\frac{1.8}{1.8} = 0.60$$

$$q_{ult.} = cN_c.S_c.d_c + \bar{q}N_q.S_q.d_q + 0.5\gamma.B.N_\gamma.S_\gamma.d_\gamma$$

$q_{ult.} = 9.4(50.6)(1.75)(1.4) + 1.8(18.1)(37.7)(1.73)(1.246)$

$+ 0.5(18.1)(1.8)(40)(0.60)(1) = 4212.403\ kN/m^2$

$q_{all.centrally.loaded.footing} = 4212.403 / 3 = 1404.134\ kN/m^2$

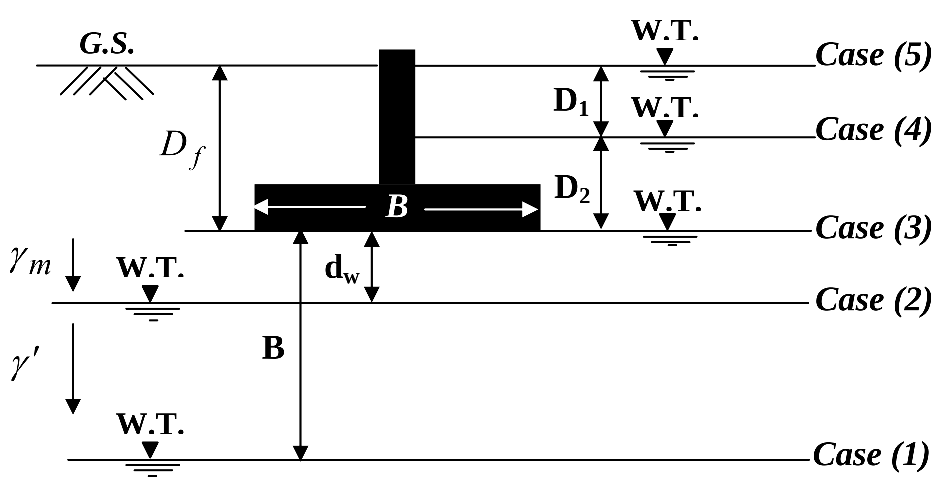
$q_{all.eccentric.loaded..footing} = q_{all.centrally.loaded.footing}(R_{ex})(R_{ey})$

$= 1404.134(0.78)(0.72) = 788.35\ kN/m^2$ *(very high)*

Actual soil pressure ($q_{act.}$) $= 1780/(1.8)(1.8) = 549.383 < 788.35$ **(O.K.)**

3.10 EFFECT OF WATER TABLE ON BEARING CAPACITY

Generally the submergence of soils will cause loss of all apparent cohesion, coming from capillary stresses or from weak cementation bonds. At the same time, the effective unit weight of submerged soils will be reduced to about one-half the weight of the same soils above the water table. Thus, through submergence, all the three terms of the bearing capacity (B.C.) equations may be considerably reduced. Therefore, it is essential that the B.C. analysis be made assuming the highest possible groundwater level at the particular location for the expected life time of the structure.



Case (1):

If the water table (W.T.) lies at B or more below the foundation base; no W.T. effect.

Case (2):

- *(from Ref.;Foundation Engg. Hanbook):* *if the water table (W.T.) lies within the depth (d_w<B) ; (i.e., between the base and the depth B), use $\gamma_{av.}$ in the term $\frac{1}{2}\gamma.B.N_\gamma$ as:*

$$\gamma_{av.} = \gamma' + (d_w / B)(\gamma_m - \gamma') \text{……..………..……….(from Meyerhof)}$$

- *(from Ref.;Foundation Analysis and Design):* *if the water table (W.T.) lies within the wedge zone $\{H = 0.5B.tan(45 + \phi / 2)\}$; use $\gamma_{av.}$ in the term $\frac{1}{2}\gamma.B.N_\gamma$ as:*

$$\gamma_{av.} = (2H - d_w)\frac{d_w}{H^2}.\gamma_{wet} + \frac{\gamma'}{H^2}(H - d_w)^2 \text{……….(from ,Bowles)}$$

where:

$H = 0.5B.tan(45 + \phi / 2)$.
γ' = *submerged unit weight* =($\gamma_{sat.} - \gamma_w$),
d_w = *depth to W.T. below the base of footing,*
$\gamma_m = \gamma_{wet}$ = *moist or wet unit weight of soil in depth* (d_w) , *and*

- *Snice in many cases of practical purposes, the term $\frac{1}{2}\gamma.B.N_\gamma$ can be ignored for conservative results, it is recommended for this case to use $\gamma = \gamma'$ in the term $\frac{1}{2}\gamma.B.N_\gamma$ instead of $\gamma_{av.}$*

$$(\gamma' < \gamma_{av.}(from..Meyerhof) < \gamma_{av.}(from..Bowles))$$

Case (3): *if d_w = 0 ; the water table (W.T.) lies at the base of the foundation; use $\gamma = \gamma'$*

Case (4): *if the water table (W.T.) lies above the base of the foundation; use:*

$$q = \gamma_t.D_{1(above..W.T.)} + \gamma'.D_{2(below..W.T.)} \text{ and } \gamma = \gamma' \text{ in } \frac{1}{2}\gamma.B.N_\gamma \text{ term.}$$

Case (5): *if the water table (W.T.) lies at ground surface (G.S.); use:* $q = \gamma'.D_f$ *and*

$\gamma = \gamma'$ *in* $\frac{1}{2}\gamma.B.N_\gamma$ *term.*

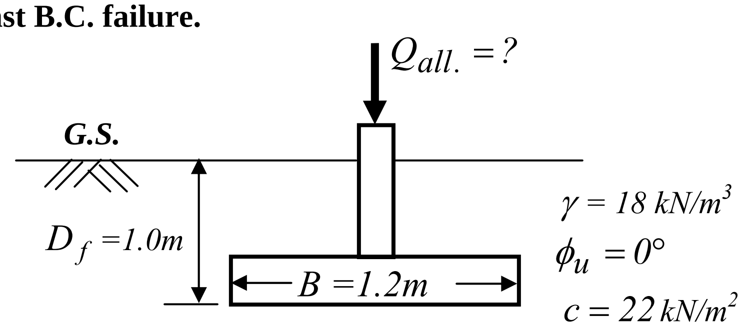
Note: *All the preceding considerations are based on the assumption that the seepage forces acting on soil skeleton are negligible. The seepage force adds a component to the body forces caused by gravity. This component acting in the direction of stream lines is equal to* $(i.\gamma_w)$*, where* i *is the hydraulic gradient causing seepage.*

Example (7): **A (1.2x4.2)m rectangular footing is placed at a depth of (**D_f **=1m) below the G.S. in clay soil with** $\phi_u = 0°$**,** $\gamma = 18$ **kN/m³,** $C_u = 22$ **kN/m². Find the allowable maximum load which can be applied under the following conditions:**

(a) W.T. at base of footing with $\gamma_{sat} = 20$ **kN/m³,**

(b) W.T. at 0.5m below the surface and $\gamma_{sat} = 20$ **kN/m³,**

(c) If the applied load is 400kN and the W.T. at the surface what will be the factor of safety of the footing against B.C. failure.



Solution:

L/B = 4.2/1.2 = 3.5 < 5 $\therefore$ *rectangular footing,*

D/B= 1/1.2 = 0.833 < 1.0 $\therefore$ *shallow footing; therefore Terzaghi's equation is suitable.*

By Terzaghi's equation: $q_{ult.} = cN_c.S_c + qN_q + \frac{1}{2}.B.\gamma.N\gamma.S_\gamma$

Shape factors: from table (3.2), for rectangular footing $S_c = (1+0.3\frac{B}{L})$; $S_\gamma = (1-0.2\frac{B}{L})$

Bearing capacity factors: from table (3.3), for $\phi = 0°$, $N_c = 5.7$, $N_q = 1.0$,..$N_\gamma = 0$

(a) for W.T. at base of footing:

$$q_{ult.} = (22)(5.7)\ (1+0.30\ \frac{1.2}{4.2}) + 1.0(18)(1)$$

$$+\ 0.5(1.2)(20\text{-}10)(0)(1\text{-}0.20\frac{1.2}{4.2}) = 154.148\ kN/m^2$$

$q_{all.}$ = 154.148 /3 = 51.388 kN/m^2

$Q_{all.}$ = 51.388(1.2x4.2) = $\boxed{258.970\ kN}$

(b) for W.T. at 0.5m below the surface:

$$q = \gamma_t . D_{1(above..W.T.)} + \gamma' . D_{2(below..W.T.)}$$

$D_1 = 0.5$ and $D_2 = 0.5$; $q = 18(0.5) + (20 - 10)(0.5) = 14\ kN/m^2$

$$q_{ult.} = (22)(5.7)\ (1+0.30\ \frac{1.2}{4.2}) + 1.0(14)(1)$$

$$+\ 0.5(1.2)(20\text{-}10)(0)(1\text{-}0.20\frac{1.2}{4.2}) = 150.148\ kN/m^2$$

$q_{all.}$ = 150.148 /3 = 50.049 kN/m^2

$Q_{all.}$ = 50.049(1.2x4.2) = $\boxed{252.249\ kN}$

(c) If the applied load is 400kN and the W.T. at the surface what will be the factor of safety of the footing against B.C. failure?.

$Q_{all.}$ = 400 kN; $q_{all.}$ = 400/(1.2(4.2)= 79.36 kN/m^2; $q = D_f . \gamma'$ =(1)(20-10)=10 kN/m^2

$$q_{ult.} = (22)(5.7)\ (1+0.30\ \frac{1.2}{4.2}) + 10(1) + 0.5(1.2)(20\text{-}10)(0)(1\text{-}0.20\frac{1.2}{4.2}) = 146.14\ kN/m^2$$

$$SF = \frac{q_{ult.}}{q_{all.}} = \frac{146.14}{79.36} = \boxed{1.8}$$

3.11 Bearing Capacity For Footings On Layered Soils

Stratified soil deposits are of common occurrence. It was found that when a footing is placed on stratified soils and the thickness of the top stratum form the base of the footing (d_1 or H) is less than the depth of penetration [$H_{crit.} = 0.5B\,tan(45+\phi/2)$]; in this case the rupture zone will extend into the lower layer (s) depending on their thickness and therefore require some modification of ultimate bearing capacity (qult.).

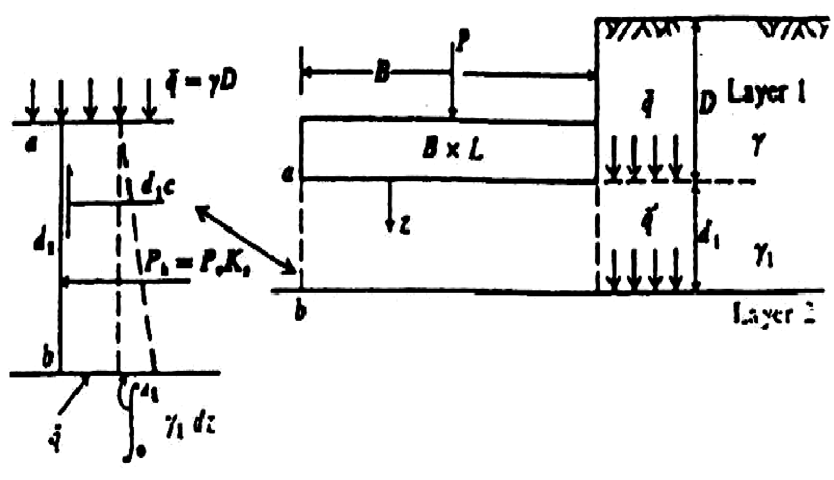
Several solutions have been proposed to estimate the bearing capacity of footings on layered soils, however, they are limited for the following three general cases:

Case (1): Footing on layered clays (all ϕ = 0):

(a) Top layer stronger than lower layer ($C_2/C_1 \leq 1$).

(b) Top layer weaker than lower layer ($C_2/C_1 > 1$).

For clays in undrained condition (ϕ_u = 0), the undrained shear strength (S_u or c_u) can be determined from unconfined compressive (q_u) tests. So that assuming a circular slip surface of the soil shear failure pattern, may give reasonably reliable results (***see figure (3.9)***).



$q' = q + \gamma_1 d_1$

Figure (3.9): Footings on layered clays.

The first situation occurs when the footing is placed on a stiff clay or dense sand stratum followed by a relatively soft normally consolidated clay. The failure in this case is basically a punching failure. While, the second situation is often found when the footing is placed on a relatively thin layer of soft clay overlying stiff clay or rock. The failure in this case occurs, at least in part by lateral plastic flow ***(see Fig.(3.10))***.

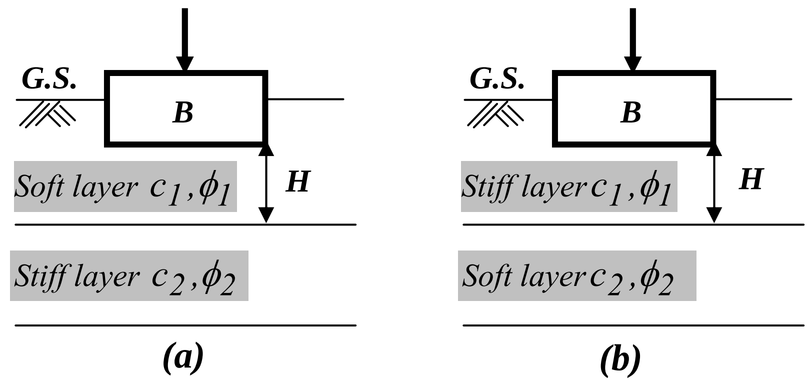


Figure (3.10): Typical two-layer soil profiles.

- **Hansen Equation** (Ref., Bowles's Book, 1996)

For both cases (a and b), the ultimate bearing capacity is calculated from Table (3.2) for (ϕ = 0) as:

$$q_{ult.} = S_u N_c (1 + S'_c + d'_c - i'_c - b'_c - g'_c) + q' \qquad \textbf{(3.25)}$$

If the inclination, base and ground effects are neglected, then equation (3.25) will be:-

$$q_{ult.} = S_u N_c (1 + S'_c + d'_c) + q' \qquad \textbf{(3.26)}$$

*where: S_u and N_c can be calculated by the following method (From **Bowles's** Book, 1996)**:***

In this method, S_u is calculated as an average value $C_{avg.}$ depending on the depth of penetration ($H_{crit.} = 0.5B\tan(45 + \phi/2)$, while $N_c = 5.14$. So that, equation (3.26) is written as:

$$\boxed{q_{ult.} = 5.14 C_{avg.} (1 + S'_c + d'_c) + q'} \qquad \textbf{(3.26b)}$$

where: $S_u = C_{avg.} = \dfrac{C_1 H + C_2\ [Hcrit - H]}{Hcrit}$;

$$S'c = 0.2\frac{B}{L}; \quad d'c = 0.4\frac{Df}{B}\ for\ \frac{Df}{B} \le 1; \quad and \quad d'_c = 0.4\tan^{-1}\frac{D}{B}\ for\ (D > B)$$

Case (2): Footing on layered $c-\phi$ soils as in Fig.(3.11):

(a) Top layer stronger than lower layer ($C_2/C_1 \le 1$).

(b) Top layer weaker than lower layer ($C_2/C_1 > 1$).

Figure (3.18) shows a foundation of any shape resting on an upper layer having strength parameters c_1, ϕ_1 and underlain by a lower layer with c_2, ϕ_2.

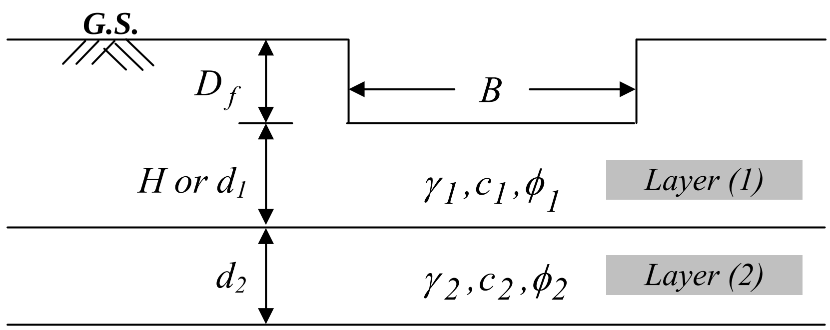


Figure (3.11): Footing on layered $c-\phi$ soils.

- **Hansen Equation** (Ref., Bowles's Book, 1996)

(1) *Compute $H_{crit.} = 0.5B\tan(45 + \phi_1/2)$ using ϕ_1 for the top layer.*

(2) *If $H_{crit.} > H$ compute the modified values of c and ϕ as:*

$$c^* = \frac{Hc_1 + (H_{crit.} - H)c_2}{H_{crit.}}; \qquad \phi^* = \frac{H\phi_1 + (H_{crit.} - H)\phi_2}{H_{crit.}}$$

Note: *A possible alternative for $c-\phi$ soils with a number of thin layers is to use average values of c and ϕ in bearing capacity equations of Table (3.2) as:*

$$c_{av.} = \frac{c_1H_1 + c_2H_2 + ... + c_nH_n}{\sum H_i}; \qquad \phi_{av.} = tan^{-1}\frac{H_1\,tan\phi_1 + H_2\,tan\phi_2 + ... + H_n\,tan\phi_n}{\sum H_i}$$

(3) *Use Hansen's equation from Table (3.2) for* $q_{ult.}$ *with* c^* *and* ϕ^* *as:*

$$q_{ult.} = c^* N_c S_c d_c i_c g_c b_c + qN_q S_q d_q i_q g_q b_q + 0.5\gamma BN_\gamma S_\gamma d_\gamma i_\gamma g_\gamma b_\gamma \text{......(3.27)}$$

If the effects of inclination, ground and base factors are neglected, then equation (3.27) will takes the form:

$$\boxed{q_{ult.} = c^* N_c S_c d_c + qN_q S_q d_q + 0.5\gamma BN_\gamma S_\gamma d_\gamma} \text{...(3.28)}$$

where:

Bearing capacity factors: from table (3.2)

$$N_q = e^{\pi.\tan\phi^*}\tan^2(45 + \phi^*/2),\, N_c = (N_q - 1)\cot\phi^*, \quad N_\gamma = 1.5(N_q - 1)\tan\phi^*$$

Shape factors from table (3.6): $S_c = 1 + \frac{N_q}{N_c}\frac{B}{L}, \quad S_q = 1 + \frac{B}{L}\tan\phi^*, \quad S_\gamma = 1 - 0.4\frac{B}{L}$

Depth factors: from table (3.6)

$$d_c = 1 + 0.4k, \qquad d_q = 1 + 2\tan\phi^*(1 - \sin\phi^*)^2 k, \qquad d_\gamma = 1.0$$

where: $k = \frac{D}{B}$ *for* $\frac{Df}{B} \le 1$ *or* $k = tan^{-1}\frac{D}{B}(radian)$ *for* $\frac{Df}{B} > 1$

Case (3): Footing in layered sand and clay soils:

(a) Sand overlying clay. *(b) Clay overlying sand.*

- **Hansen Equation** (Ref., Bowles's Book, 1996)

(1) *Compute* $H_{crit.} = 0.5B\tan(45 + \phi_1/2)$ *using* ϕ_1 *for the top layer.*

(2) *If* $H_{crit.} > H$*, for both cases; sand overlying clay or clay overlying sand, estimate* $q_{ult.}$ *as follows:*

$$\boxed{q_{ult.} = q_b + \frac{p.Pv.K_s.\tan\phi_1}{A_f} + \frac{p.d_1c_1}{A_f} \le q_t} \text{.................(3.29)}$$

where: q_t, q_b *= ultimate bearing capacities of footing with respect to top and bottom soils ,*

for $\phi > 0$ ***(sand or clay)***

$$q_t = c_1 N_{c1} S_{c1} d_{c1} + \gamma_1 D_f N_{q1} S_{q1} d_{q1} + 0.5 B \gamma_1 N_{\gamma 1} S_{\gamma 1} d_{\gamma 1} \quad \textbf{(3.29a)}$$

$$q_b = c_2 N_{c2} S_{c2} d_{c2} + \gamma_1 (D_f + H) N_{q2} S_{q2} d_{q2} + 0.5 B \gamma_2 N_{\gamma 2} S_{\gamma 2} d_{\gamma 2} \quad \textbf{(3.29b)}$$

for $\phi_u = 0$ ***(clay in undrained condition)***

$$q_t = 5.14 S_u (1 + S'_c + d'_c) + \gamma_1 D_f \quad \textbf{(3.29c)}$$

$$q_b = 5.14 S_u (1 + S'_c + d'_c) + \gamma_1 (D_f + H) \quad \textbf{(3.29d)}$$

Hansen's bearing capacity factors from table (3.2) with $(\phi = \phi_i)$:

$$N_q = e^{\pi . \tan\phi} \tan^2(45 + \phi / 2), \quad N_c = (N_q - 1)\cot\phi, \qquad N_\gamma = 1.5(N_q - 1)\tan\phi$$

Shape factors from table (3.5): $S_c = 1 + \frac{N_q}{N_c}\frac{B}{L}$, $S_q = 1 + \frac{B}{L}\tan\phi$, $S_\gamma = 1 - 0.4\frac{B}{L}$

Depth factors from table (3.5): $d_c = 1 + 0.4k$, $d_q = 1 + 2\tan\phi(1 - \sin\phi)^2 k$, $d_\gamma = 1.0$

where: $k = \frac{D}{B}$ for $\frac{Df}{B} \le 1$ or $k = \tan^{-1}\frac{D}{B}(radian)$ for $\frac{Df}{B} > 1$

p = total perimeter for punching = 2 (B+L) or $\pi.D$ *(diameter),*

P_v *= total vertical pressure from footing base to lower soil computed as:*

$$\int_0^{d_1} \gamma_1 h.dh + \bar{q} d_1 = \gamma_1 \frac{d_1^2}{2} + \gamma_1 D_f . d_1$$

K_s *= lateral earth pressure coefficient, which may range from* $\tan^2(45 \pm \phi / 2)$ *or use* $K_o = 1 - \sin\phi$,

$\tan\phi$ *= coefficient of friction between* P_v K_s *and perimeter shear zone wall,*

$p d_1 c_1$ *= cohesion on perimeter as a force,* A_f *= area of footing.*

(3) *Otherwise, if* $(H / B)_{crit.} \le (H / B)$*, then* $q_{ult.}$ *is estimated as the bearing capacity of the first soil layer whether it is sand or clay.*

Footings on layered soils

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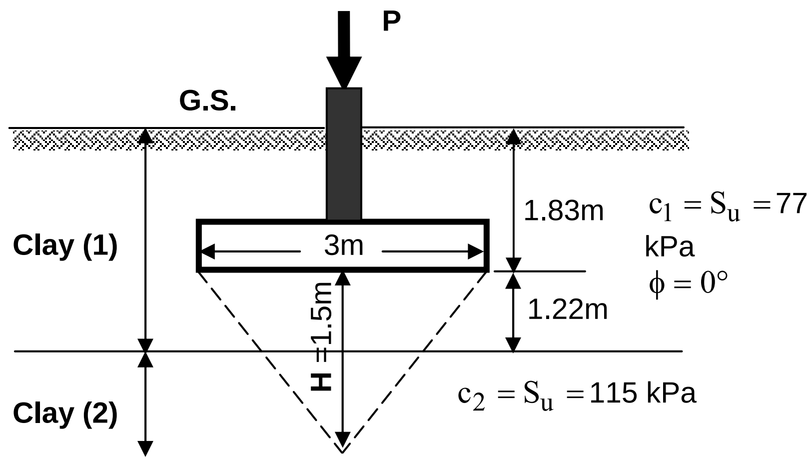
Example (8): *(footing on layered clay)*

A rectangular footing of 3.0x6.0m is to be placed on a two-layer clay deposit as shown in figure below. Estimate the ultimate bearing capacity.



Solution:

$$H_{crit.} = 0.5B\tan(45+\phi/2) = 0.5(3)\tan 45 = 1.5m > 1.22m$$

$\therefore$ *the critical depth penetrated into the* $2^{nd.}$ *layer of soil.*

For case(1); clay on clay layers using Hansen's equation:

- ***F**rom Bowles's Book, 1996:*

$$q_{ult.} = 5.14.C_{avg.}(1 + S'_c + d'_c) + q'$$

where:

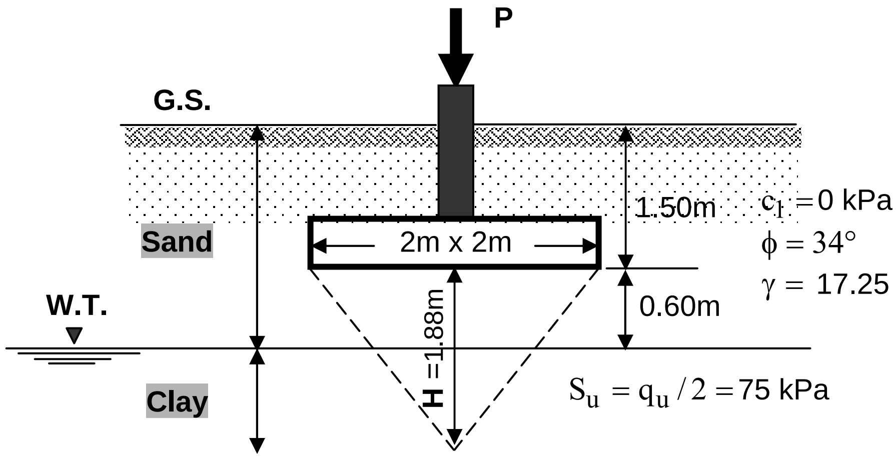
$$S_u = C_{avg.} = \frac{C_1H + C_2\,[Hcrit - H]}{Hcrit} = \frac{77(1.22) + 115(1.5 - 1.22)}{1.5} = 84.093$$

$$S'_c = 0.2B/L = 0.2(3/6) = 0.1; \quad for\ Df/B \le 1: \quad d'_c = 0.4D/B = 0.4(1.83/3) = 0.24$$

$$\therefore \quad q_{ult.} = 5.14(84.093)(1+0.1+0.24) + 1.83(17.26) = \boxed{610.784\ kPa}$$

Example (9): *(footing on sand overlying clay)*

A 2.0x2.0m square footing is to be placed on sand overlying clay as shown in figure below. Estimate the allowable bearing capacity of soil?.



Solution:

$$H_{crit.} = 0.5B\tan(45+\phi_1/2) = 0.5(2)\tan(45+34/2) = 1.88\,m > 0.6m$$

$\therefore$ *the critical depth* $H_{crit.} > H$ *penetrated into the* $2^{nd.}$ *layer of soil.*

For case (3); sand overlying clay using Hansen's equation:

$$q_{ult.} = q_b + \frac{p.Pv.K_s.\tan\phi_1}{A_f} + \frac{p.d_1 c_1}{A_f} \le q_t$$

where:

- ***for sand layer:***

$$q_t = \gamma_1 D_f N_{q1} S_{q1} d_{q1} + 0.5B\gamma_1 N_{\gamma 1} S_{\gamma 1} d_{\gamma 1}$$

Hansen's bearing capacity factors from Table (3.2) with $(\phi = 34°)$:

$$N_q = e^{\pi\tan 34}\tan^2(45+34/2) = 29.4\,, \qquad N_\gamma = 1.5(29.4-1)\tan 34 = 28.7$$

Shape factors from Table (3.5): $S_q = 1 + \frac{B}{L}\tan\phi = 1.67\,,\quad S_\gamma = 1 - 0.4\frac{B}{L} = 0.6$

<u>Depth factors from Table (3.5):</u>

$$d_q = 1 + 2\tan\phi(1-\sin\phi)^2 \frac{D_f}{B} = 1 + 2\tan 34(1-\sin 34)^2 \frac{1.5}{2} = 1.2,$$

$$d_\gamma = 1.0$$

$$\therefore \quad q_t = 17.25(1.5)(29.4)(1.67)(1.2) + 0.5(2)17.25)(28.7)(0.6)(1.0) = \boxed{1821.5\ kPa}$$

<u>for clay layer:</u>

$$q_b = 5.14 S_u (1 + S'_c + d'_c) + q'$$

$$S'_c = 0.2\frac{B}{L} = 0.2\frac{2}{2} = 0.2;$$

$$for\ \frac{Df}{B} > 1: \quad d'_c = 0.4\tan^{-1}\frac{D_f}{B} = 0.4\tan^{-1}(\frac{1.5+0.6}{2}) = 0.32\ ;$$

$$S_q = d_q = 1$$

$$\therefore \quad qb = 5.14(75)(1+0.2+0.32)+(1.5+0.6)(17.25) = \boxed{622\ kPa}$$

Now, obtain the punching contribution:

$$P_v = \int_0^{d_1} \gamma_1 h.dh + \overline{q}d_1 = \gamma_1 \left.\frac{d_1^2}{2}\right]_0^{0.6} + \gamma_1 D_f d_1 = 17.25\frac{0.6^2}{2} + 17.25(1.5)(0.6) = 18.6\ kN/m$$

$$K_o = 1 - \sin\phi = 1 - \sin 34 = 0.44,$$

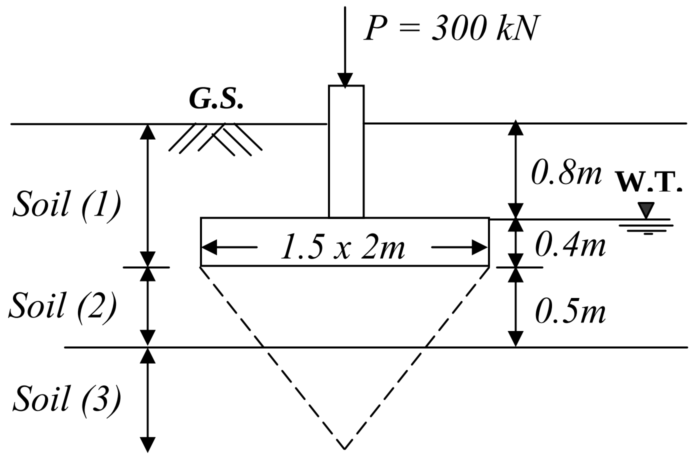
$$\therefore q_{ult.} = 622 + \frac{2(2+2)(18.6)(0.44)\tan 34}{2x2} + \frac{2(2+2)(0.6)(0)}{2x2} = 633\ kPa < q_{ult.} = 1821.5\ kPa$$

$$q_{all.} = 633/3 = \boxed{211\ kPa}$$

Example (10): *(footing on $c-\phi$ soils)*

Check the adequacy of the rectangular footing 1.5x2.0m shown in figure below against shear failure (use F.S.= 3.0), γ_w =10 kN/m^3 .

parameter	Soil (1)	Soil (2)	Soil (3)
Gs	2.70	2.65	2.75
e	0.8	0.9	0.85
c (kPa)	10	60	80
$\phi°$	35	0	0



Solution:

$$\gamma_{d1} = \frac{G_s \cdot \gamma_w}{1+e} = \frac{2.70(10)}{1+0.8} = 15 \ kN/m^3$$

$$\gamma_{sat\,1} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.70+0.8)10}{1+0.8} = 19.4 \ kN/m^3$$

$$\gamma_{d2} = \frac{G_s \cdot \gamma_w}{1+e} = \frac{2.65(10)}{1+0.9} = 18.7 \ kN/m^3$$

$$\gamma_{sat2} = \frac{(2.75+0.85)10}{1+0.85} = 19.45 \ kN/m^3$$

$$H_{crit.} = 0.5B\tan(45+\phi/2) = 0.5(1.5)\tan 45 = 0.75m > 0.50m$$

$\therefore$ *the critical depth penetrated into the soil layer (3).*

Since soils (2) and (3) are of clay layers, therefore; by using Hansen's equation:

- *From Bowles's Book, 1996:*

$$q_{ult.} = 5.14C_{avg.}(1 + S'_c + d'_c) + q'$$

where:

$$C_{avg.} = \frac{C_1H + C_2\ [Hcrit - H]}{Hcrit} = \frac{60(0.5) + 80\,(0.75 - 0.50)}{0.75} = 66.67$$

$S'_c = 0.2B / L = 0.2(1.5/2) = 0.15$;

for $Df / B \le 1\ \ d'_c = 0.4D/B = 0.4(1.2/1.5) = 0.32$

$\therefore\quad q_{ult.} = 5.14(66.67)(1+0.15+0.32)+0.8(15)+0.4(19.45\text{-}10) = 519.5\ kPa$

$$q_{all}(_{net}) = \frac{519.5}{3} - 15.78 = \boxed{157.4\ kPa}$$

$$q_{applied} = \frac{300}{1.5x2} = 100\ kPa < q_{all}(_{net}) = 157.4\ kPa\quad \therefore \textbf{(O.K.)}$$

Check for squeezing:

For no squeezing of soil beneath the footing: $(q_{ult.} > 4c_1 + \overline{q})$

$4c_1 + \overline{q} = 4(60)+ 0.8(15)+0.4(19.45\text{-}10)= 255.78\ kPa < 519.5\ kPa\ \ \therefore$ **(O.K.)**

3.12 Skempton's Bearing Capacity Equation

- **Footings on Clay and Plastic Silts:**

From Terzaghi's equation, the ultimate bearing capacity is:

$$q_{ult.} = cN_c.S_c + qN_q + \frac{1}{2}.B.\gamma.N\gamma.S_\gamma \quad \textbf{.....................(3.12)}$$

<u>*For saturated clay and plastic silts:*</u> ($\phi_u = 0$ and $N_c = 5.7, N_q = 1.0, .and. N_\gamma = 0$),

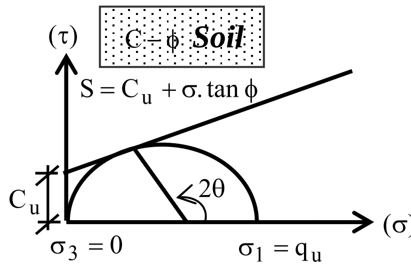
<u>*For strip footing:*</u> $S_c = S_\gamma = 1.0$

$$\boxed{q_{ult.} = cN_c + \overline{q}} \quad \textbf{.....................(3.30)}$$

$$q_{all.} = \frac{q_{ult.}}{3} \ \ and \qquad q_{all.(net)} = q_{all.} - \overline{q}$$

$$\therefore \qquad q_{all.(net)} = \frac{q_{ult.}}{3} - \overline{q} = \frac{cN_c + \overline{q}}{3} - \overline{q} = \frac{cN_c}{3} + (\frac{\overline{q}}{3} - \overline{q}) \quad \textbf{.....................(3.30a)}$$

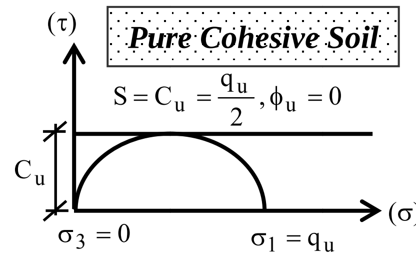
where: N_c = *bearing capacity factor obtained from* **figure (3.12)** *depending on shape of footing and* $\frac{D_f}{B} \cdot (\frac{\bar{q}}{3} - \bar{q})$ *is a small value can be neglected.*



for c − φ **soil**: $\boxed{\sigma_1 = \sigma_3 \tan^2(45+\phi/2) + 2c\tan(45+\phi/2)}$

for UCT: $\sigma_1 = q_u$ *and* $\sigma_3 = 0$; *then* $\boxed{q_u = 2c\tan(45+\phi/2)}$

or $\phi_u = 0$; $c = \frac{q_u}{2}$ *and equation (3.30a) will be:*



$$q_{all.(net)} = q_u \frac{N_c}{6} \quad \textbf{……...……………………...………….(3.30b)}$$

From **figure (3.12)** *for* $\frac{D_f}{B}$ *=0:* $N_c = 6.2$ *for square or circular footings; 5.14 for strip or continuous footings* *If* $N_c = 6.0$*, then:*

$$\boxed{q_{all.(net)} \approx q_u} \quad \textbf{…………...………………………...………………..(3.31)}$$

See figure (3.13) *for net allowable soil pressure for footings on clay and plastic silt.*

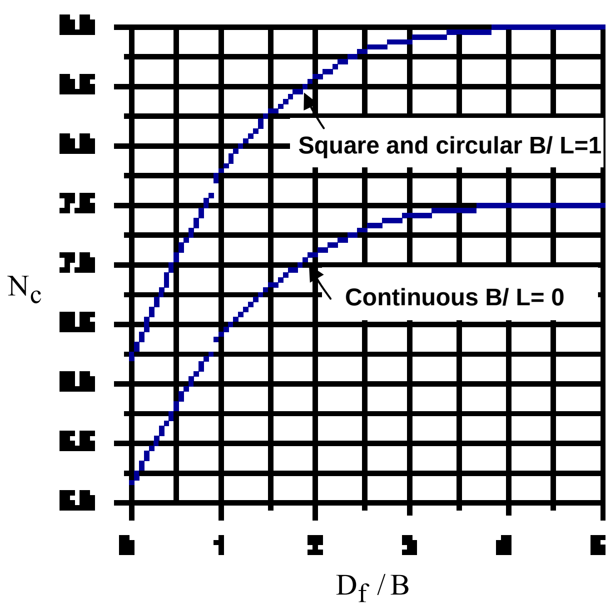


Figure (3.12): N_c **bearing capacity factor for Footings on clay under** $\phi = 0$ **conditions** (After Skempton, 1951).

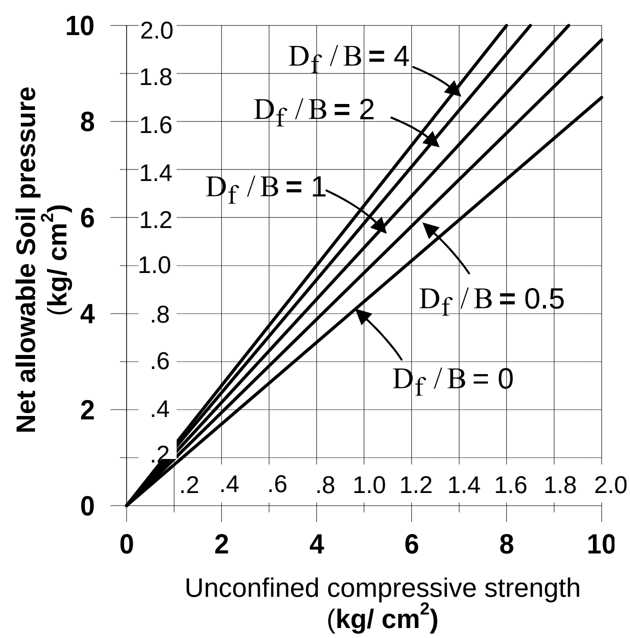
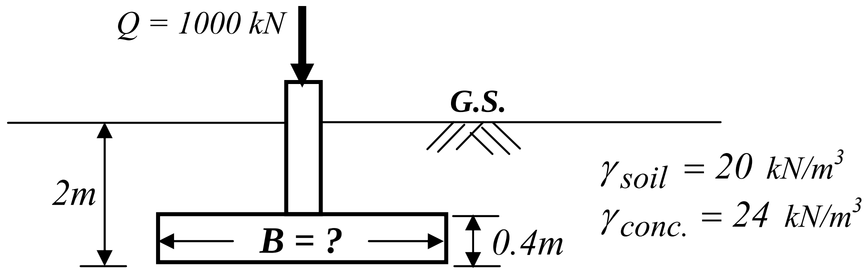


Figure (3.13): Net allowable soil pressure for footings on clay and plastic silt, determined for a factor of safety of 3 against bearing capacity failure (ϕ = 0 conditions). Chart values are for strip footings (B/L=0); and for other types of footings multiply values by (1+ 0.2B/L).

$$\boxed{N_{c(net)} = N_{c(strip)}(1 + 0.2\frac{B}{L}) \quad \underline{\textbf{or}} \quad N_{c(net)} = N_{c(square)}(0.84 + 0.16\frac{B}{L})}$$

Example (11): *(footing on clay)*

Determine the size of the square footing shown in figure below. If q_u = 100 kPa and F.S.= 3.0?



Solution:

Assume B =3.5m, D/B *= 2/3.5 = 0.57 then from* **figure (3.12)**: $N_c = 7.3$

$q_{ult.} = cN_c + \overline{q}$ = 50(7.3) + 2(20) = 405 kPa

$$q_{all.(net)} = \frac{q_{ult.}}{3} - \overline{q} = \frac{405}{3} - 20(1.6) - 24(0.4) = 93.4 \ kPa$$

Area=1000/93.4 = 10.71 m^2; for square footing: $B = \sqrt{10.71} = 3.27 < 3.5m$

$\therefore$ *take B =3.25m , and* D/B *= 2/3.25 = 0.61 then from figure (3.15):* $N_c = 7.5$

$q_{ult.} = cN_c + \overline{q}$ = 50(7.5) + 2(20) = 415 kPa

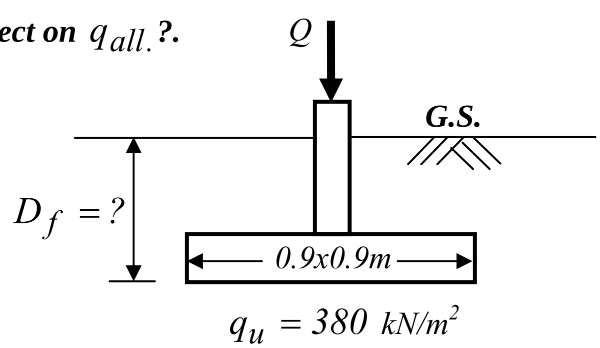
$$q_{all.(net)} = \frac{q_{ult.}}{3} - \overline{q} = \frac{415}{3} - 20(1.6) - 24(0.4) = 96.73 \ kPa$$

Area=1000/96.73 = 10.34 m^2; $B = \sqrt{10.34} = 3.21 \approx 3.25m$ **(O.K.)**

$\therefore$ <u>*use B x B = (3.25 x 3.25)m*</u>

Example (12): *(footing on clay)*

For the square footing shown in figure below. If q_u = 380 kPa and F.S.= 3.0, determine $q_{all.}$ and $D_f(min.)$ which gives the maximum effect on $q_{all.}$?.



Solution:

From Skempton's equation:

For strip footing: $q_{all.(net)} = \dfrac{cN_c}{3}$

For square footing: $q_{all.(net)} = \dfrac{cN_c}{3} x1.2$

From Skempton's **figure (3.12)** *at* $D_f / B = 4$ *and* $B/L=1$ *(square footing):* $N_c = 9$

$$\therefore\ q_{all.(net)} = \frac{\frac{380}{2}(9)}{3} = \boxed{570\ kPa} \quad and \quad D_f = 4(0.9) = \boxed{3.6m}$$

- ## Rafts on Clay:

If $q_b = \dfrac{\Sigma Q}{A} = \dfrac{Total.load(D.L.+L.L.)}{area} > q_{all.}$ *use pile or floating foundations.*

From Skempton's equation, the ultimate bearing capacity (for strip footing) is:

$$q_{ult.} = cN_c + \bar{q} \quad \text{..}\textbf{(3.30)}$$

$$q_{ult.(net)} = cN_c\ , \qquad q_{all.(net)} = \frac{cN_c}{F.S.} \qquad or \qquad F.S. = \frac{cN_c}{q_{all.(net)}}$$

Net soil pressure = $q_b - D_f.\gamma$

$$\therefore \quad \boxed{F.S. = \frac{cN_c}{q_b - Df.\gamma}} \quad \text{..}\textbf{(3.32)}$$

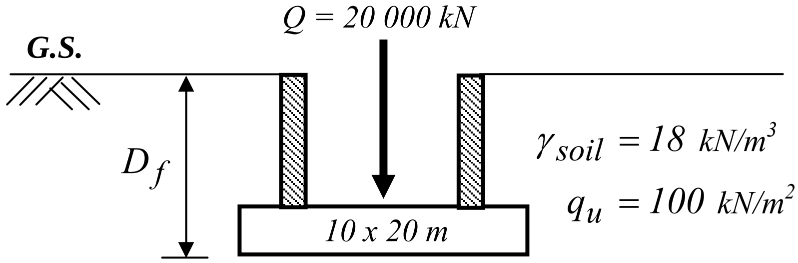
Notes:

(1) *If* $q_b = D_f.\gamma$ *(i.e.,* $F.S. = \infty$*) the raft is said to be fully compensated foundation (in this case, the weight of foundation (D.L.+ L.L.) = the weight of excavated soil).*

(2) *If* $q_b > D_f.\gamma$ *(i.e.,* $F.S. = certain.value$*) the raft is said to be partially compensated foundation such as the case of storage tanks.*

Example (13): *(raft on clay)*

Determine the F.S. for the raft shown in figure for the following depths: D_f =1m,2m, and 3m?.



Solution:

$$F.S. = \frac{cN_c}{q_b - Df.\gamma}$$

- For $D_f = 1m$:

From figure (3.12) $D_f / B = 1/10 = 0.1$ and $B / L = 0$:

$$N_{c\,strip} = 5.4 \text{ and } \boxed{N_{c\,rectangular} = N_{c\,strip}(1 + 0.2B/L)} = 5.4\,(1 + 0.2\frac{10}{20}) = 5.94$$

$$\therefore\ F.S. = \frac{cN_c}{q_b - Df.\gamma} = \frac{(100/2)5.94}{\frac{20000}{10x20} - 1(18)} = \frac{50(5.94)}{100 - 18} = 3.62$$

- For $D_f = 2m$:

From figure (3.12) $D_f / B = 2/10 = 0.2$ and $B / L = 0$:

$$N_{c\,strip} = 5.5 \quad \text{and} \quad N_{c\,rectangular} = 5.5\,(1 + 0.2\frac{10}{20}) = 6.05$$

$$\therefore\ F.S. = \frac{cN_c}{q_b - Df.\gamma} = \frac{(100/2)6.05}{\frac{20000}{10x20} - 2(18)} = \frac{50(6.05)}{100 - 36} = 4.72$$

- For $D_f = 3m$:

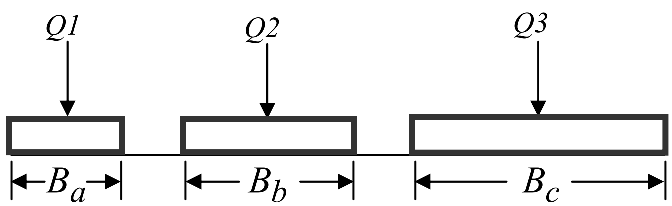
From figure (3.12) $D_f / B = 3/10 = 0.3$ and $B / L = 0$:

$$N_{c\,strip} = 5.7 \quad \text{and} \quad N_{c\,rectangular} = 5.7\,(1 + 0.2\frac{10}{20}) = 6.27$$

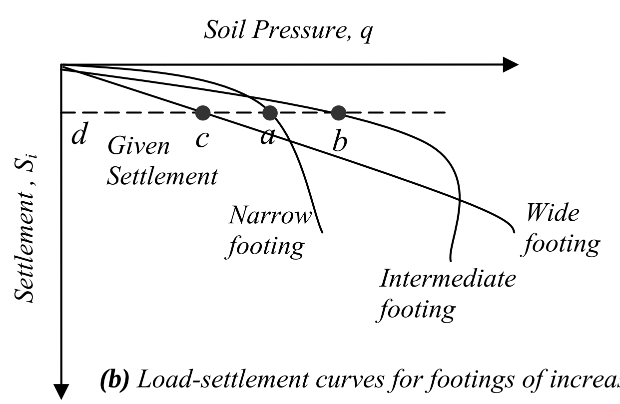
$$\therefore\ F.S. = \frac{cN_c}{q_b - Df.\gamma} = \frac{(100/2)6.27}{\frac{20000}{10x20} - 3(18)} = \frac{50(6.27)}{100 - 54} = 6.81$$

*(3) the wider the footing, the greater $q_{ult.}$/unit area. However, for a given settlement S_i such as (1 inch or 25mm), the soil pressure is greater for a footing of intermediate width B_b than for a large footing with a width B_c or for a narrow footing with width B_a (**see figure 3.14a**).*

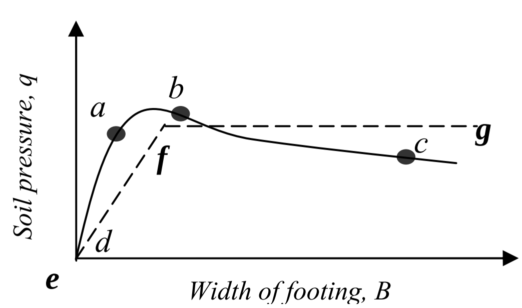
*(4) for $\frac{D_f}{B}$ = constant and a given settlement on sand, there is an actual relationship between $q_{all.}$ and B represented by (solid line) **(see figure 3.14b).** However, as basis for design a substitute relation (dashed lines) can be used as shown in **(figure 3.14c).** The error for footings of usual dimensions is less than ± 10%. The position of the broken line efg is differs for different sands.*



***(a)** Footings of different widths.*



***(b)** Load-settlement curves for footings of increasing widths.*



***(c)** Variation of soil pressure with B for given settlement, Si.*

Figure (3.14): Footings on sand.

(5) <u>the design charts for proportioning shallow footings on sand and nonplastic silts are shown in **Figures (3.15, 3.16 and 3.17).**</u>

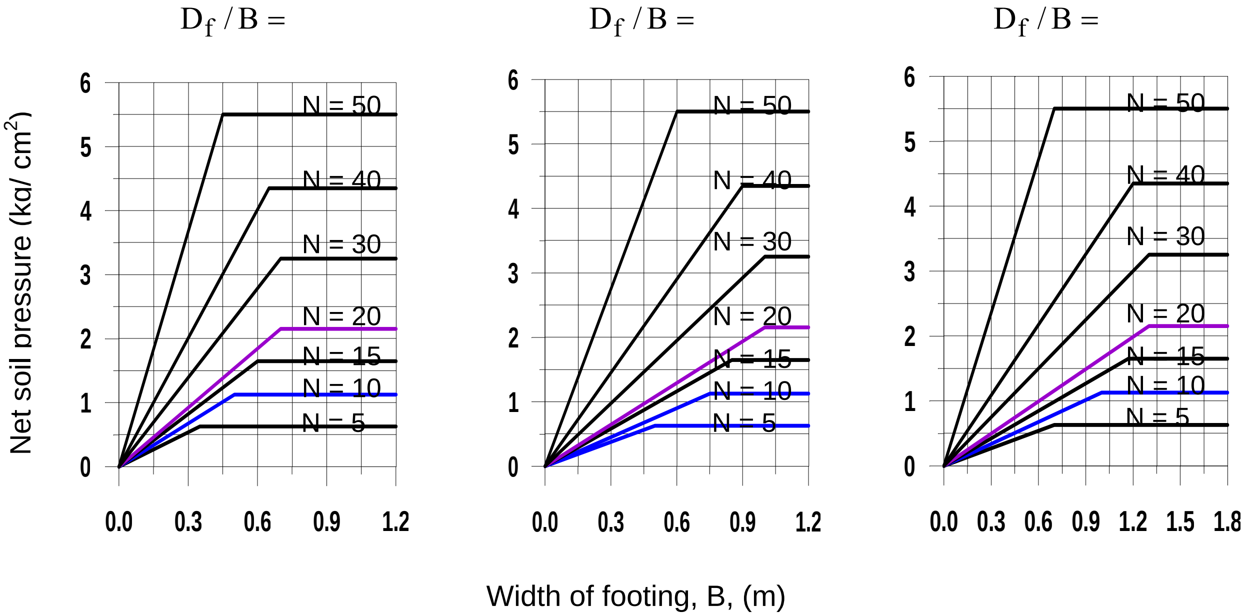


Fig.(3.15): Design charts for proportioning shallow footings on sand.

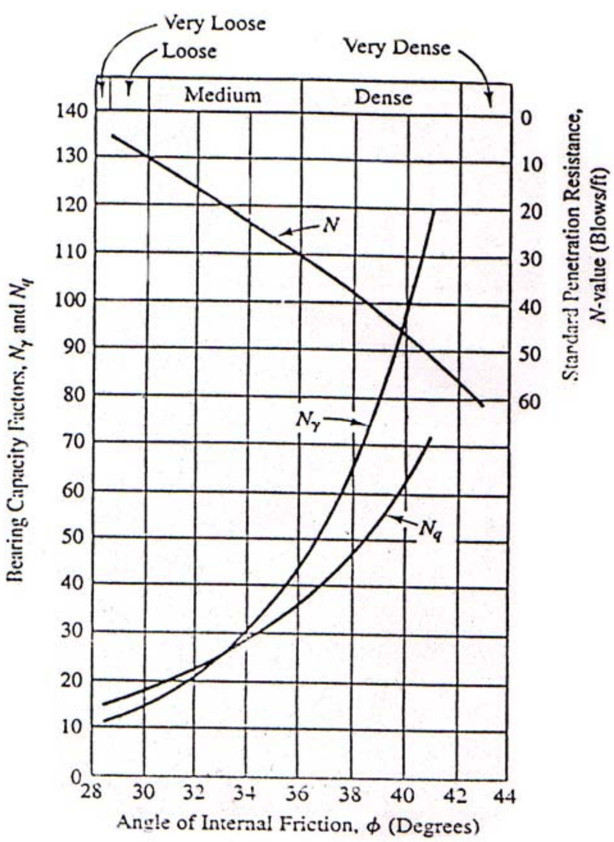


Fig.(3.16): Relationship between bearing capacity factors and ϕ.

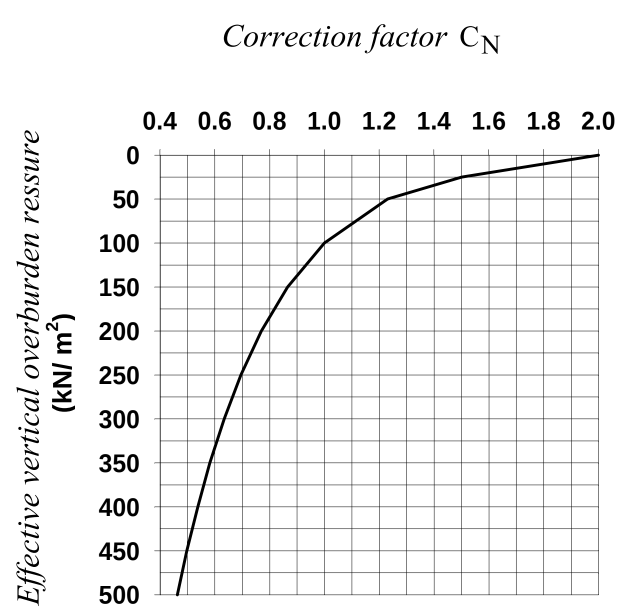


Fig.(3.17): Chart for correction of N-values in sand for overburden pressure.

Limitations of using charts (3.15, 3.16 and 3.17):

- *These charts are for strip footing, while for other types of footings multiply $q_{all.}$ by (1+ 0.2 B/L).*
- *The charts are derived for shallow footings ($D_f / B \le 1$); $\gamma = 100$ Ib/ft^3; settlement = 1.0 (inch); F.S. = 2.0; no water table (far below the footing); and corrected N-values.*
- *N-values must be corrected for:*

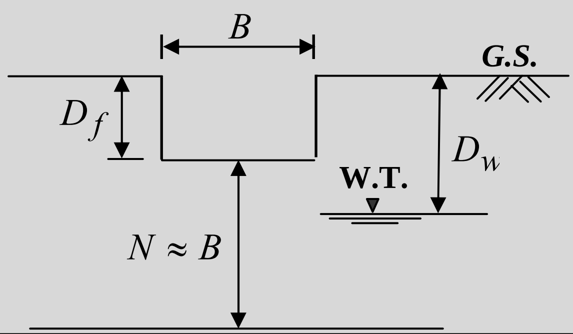
  (i) overburden pressure effect using figure (3.17) or the following formulas:

$$C_N = 0.77 \log \frac{20}{\overline{P}_o(Tsf)} \qquad or \qquad C_N = 0.77 \log \frac{2000}{\overline{P}_o(kPa)}$$

  If $\overline{p}_o < 0.25(Tsf)$ or $< 25(kPa)$, (no need for overburden pressure correction).

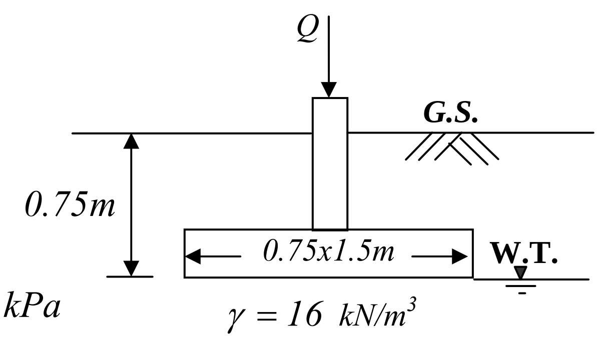
  (ii) and water table effect:

$$C_w = 0.5 + 0.5 \frac{D_w}{B + D_f}$$



Example (14): *(footing on sand)*

Determine the gross bearing capacity and the expected settlement of the rectangular footing shown in figure below. If $N_{avg.}$ (not corrected) =22 and the depth for correction = 6m?.



Solution:

$P'_o = 0.75(16) + 5.25(16\text{-}9.81) = 44.5$ kPa >25 kPa

$$C_N = 0.77\log\frac{2000}{\overline{P}_o(kPa)} = 0.77\log\frac{2000}{44.5} = 1.266$$

$$C_w = 0.5 + 0.5\frac{D_w}{B + D_f} = 0.5 + 0.5\frac{0.75}{0.75 + 0.75} = 0.75$$

$N_{corr.}$=22(1.266)(0.75)= 20.8 (use N = 20)

From **figure (3.15)** *for footings on sand: at D_f / B = 1 and B = 0.75m (2.5ft) and N 20 <u>for strip footing:</u>* $q_{all.(net)}$ = 2.2(Tsf)x105.594 = 232.307 kPa

<u>for rectangular footing:</u> $q_{all.(net)}$ = 232.307 x (1+0.2B/L) = 255.538 kPa

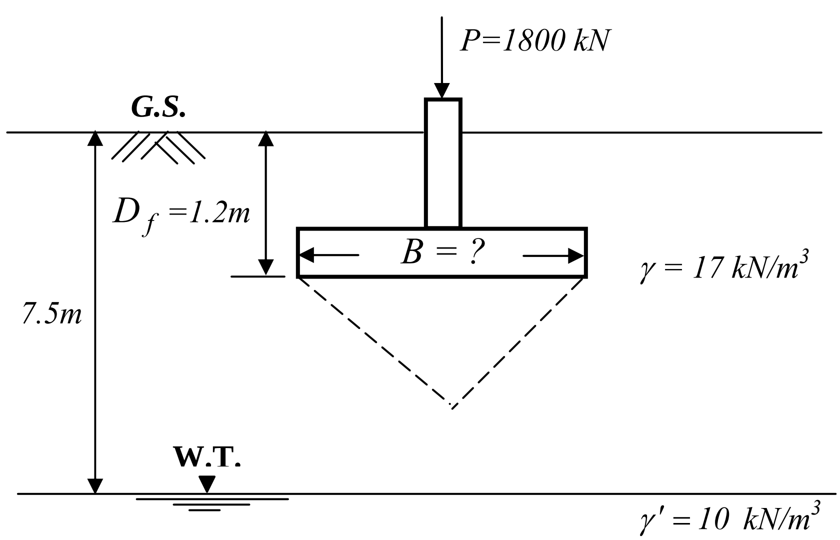
$q_{gross} = q_{all.(net)} + D_f .\gamma$ = 255.538 + 0.75(16) = $\boxed{267.538\ kPa}$

And the maximum settlement is not more than (1 inch or 25mm).

<u>Example (15):</u> *(bearing capacity from field tests)*

SPT results from a soil boring located adjacent to a planned foundation for a proposed warehouse are shown below. If spread footings for the project are to be found (1.2m) below surface grade, what foundation size should be provided to support (1800 kN) column load? Assume that 25mm settlement is tolerable, W.T. encountered at (7.5m).

SPT sample depth (m)	N_{field}
0.3	9
1.2	10
2.4	15
3.6	22
4.8	19
6	29
7.5	33
10	27



<u>Solution:</u>

Find σ'_o at each depth and correct N_{field} values. Assume B = 2.4 m

At depth B below the base of footing (1.2+2.4) = 3.6m; $N'_{avg.} = (15+19+25)/3 = 20$

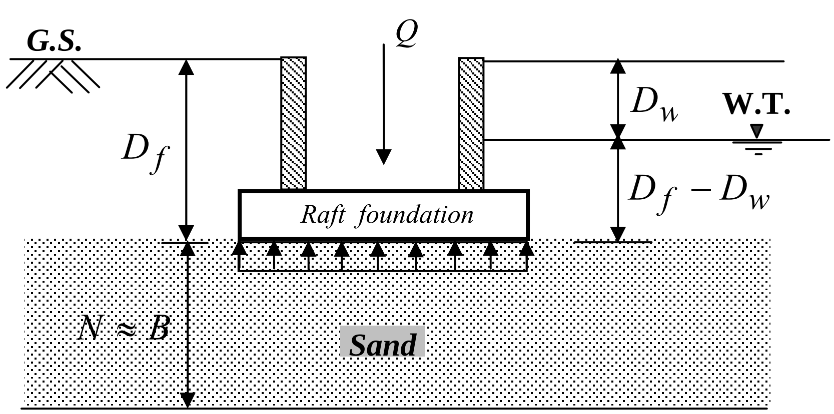
For $N'_{avg.} = 20$*, and* $D_f / B = 0.5$*;* $q_{all.}$ *=2.2 T/ft² = 232.31 kPa from Fig.***(3.15).**

SPT sample depth (m)	N_{field}	σ'_o (kN/m²)	σ'_o (T/ft²)	C_N **(Fig.3.17)**	$N' = C_N . N_{field}$
0.3	9				
1.2	10	20.4	0.21	1.55	15
2.4	15	40.8	0.43	1.28	19
3.6	22	61.2	0.64	1.15	25
4.8	19	81.6	0.85	1.05	20
6	29	102	1.07	0.95	27
7.5	33	127.5	1.33	0.90	30
10	27	152.5	1.59	0.85	23

Say B = 2.5 m, $q_{all.} = \frac{P}{B.x.L}$, $L = \frac{1800}{232.31 X 2.5} = 3.10m$, $\therefore$*use* **(2.5 x 3.25)m** *footing.*

- ## Rafts on Sand:

For allowable settlement = 2 (inch) and differential settlement >3/4 (inch) provided that $D_f \geq (8\,ft).or.(2.4m)\,min.$ *the allowable net soil pressure is given by:*



$$\boxed{q_{all.(net)} = C_w \frac{S_{all.}(N)}{9}} \quad \ldots\ldots\ldots\ldots\ldots \; for\, 5 \leq N \leq 50 \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots \textbf{(3.35)}$$

If C_w*=1 and* $S_{all.} = 2''$*; then* $\boxed{q_{all.(net)}} = 1.0\frac{2.0(N)}{9} = \boxed{0.22N(Tsf) = 23.23N(kPa)}$

and $\quad q_{gross} = q_{all.(net)} + D_f.\gamma = \dfrac{\Sigma Q}{Area}$

where: $D_f.\gamma = D_w\gamma + (D_f - D_w)(\gamma - \gamma_w) + (D_f - D_w)\gamma_w$

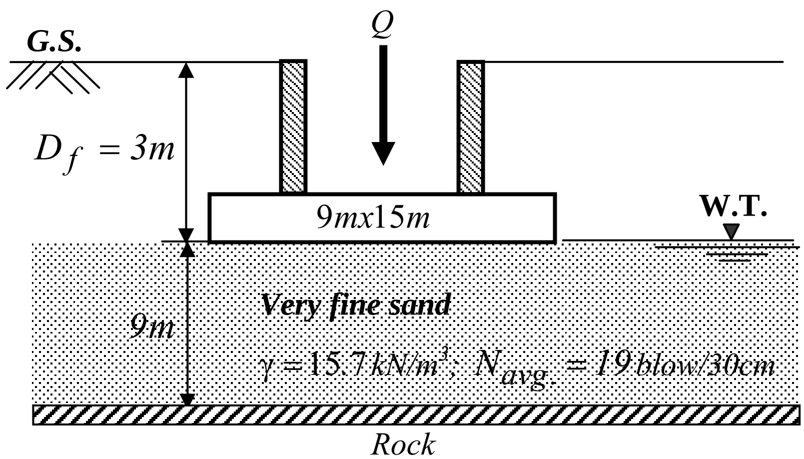
$$C_w = 0.5 + 0.5\frac{D_w}{B + D_f} = \text{(correction for water table)}$$

N = SPT number (corrected for both W.T. and overburden pressure).

> ***Hint:*** *A raft-supported building with a basement extending below water table is acted on by hydroustatic uplift pressure or buoyancy equal to* $(D_f - D_w)\gamma_w$ *per unit area.*

Example (16): *(raft on sand)*

Determine the maximum soil pressure that should be allowed at the base of the raft shown in figure below If $N_{avg.}$ ***(corrected) =19?.***



Solution:

For raft on sand: $\boxed{q_{all.(net)} = 23.23N(kPa)} = 23.23(19) = 441.37\ kPa$

Correction for water table: $C_w = 0.5 + 0.5\dfrac{D_w}{B + D_f} = 0.5 + 0.5\dfrac{3}{9+3} = 0.625$

$\therefore\ q_{all.(net)} = 441.37(0.625) = 275.856\ \ kPa$

The surcharge = $D_f.\gamma = 3(15.7) = 47.1\ kPa$

and $\quad q_{gross} = q_{all.(net)} + D_f.\gamma = 275.856 + 47.1 = 323\ kPa$

3.14 Bearing Capacity of Footings on Slopes

If footings are on slopes, their bearing capacities are less than if the footings were on level ground. In fact, bearing capacity of a footing is inversely proportional to ground slope.

- **Meyerhof's Method:**

In this method, the ultimate bearing capacity of footings on slopes is computed using the following equations:

$$(q_{ult.})_{continuous.footing.on.slope} = cN_{cq} + \frac{1}{2}\gamma.B.N_{\gamma q} \quad \textbf{(3.36)}$$

$$(q_{ult.})_{c.or.s.footing.on.slope} = (q_{ult.})_{continuous.footing.on.slope}\left[\frac{(q_{ult.})_{c.or.s.footing.on.level.ground}}{(q_{ult.})_{continuous.footing.on.level.ground}}\right] \quad \textbf{(3.37)}$$

where:

N_{cq} *and* $N_{\gamma q}$ *are bearing capacity factors for footings on or adjacent to a slope; determined from* ***figure (3.18).***

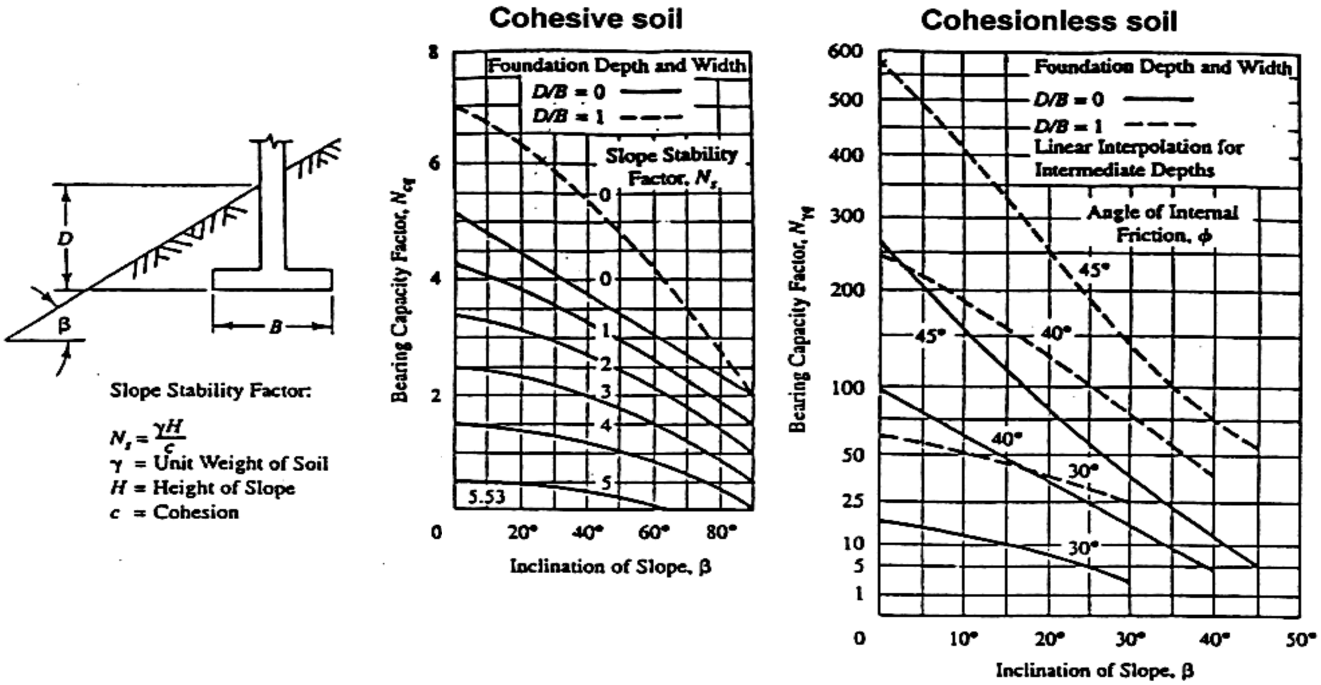
c or s footing denotes either circular or square footing, and

$(q_{ult.})$ *of footing on level ground is calculated from Terzaghi's equation.*

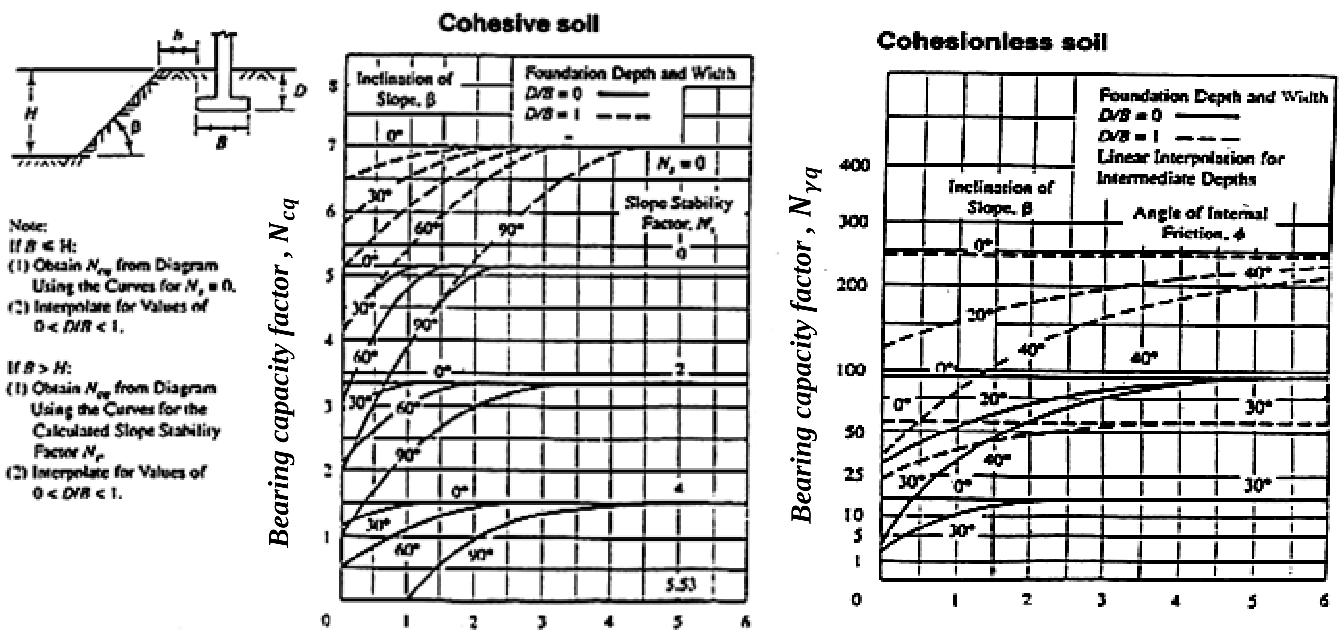
Notes:

(1) A $\phi_{triaxial}$ should not be adjusted to ϕ_{ps}, since the slope edge distorts the failure pattern such that plane-strain conditions may not develop except for large b/B ratios.

(2) For footings on or adjacent to a slope, the overall slope stability should be checked for the footing load using a slope-stability program or other methods *such as* ***method of slices by Bishop's***.



(a) on face of slope.



Distance of foundation from edge of slope b/B (for Ns = 0) or b/H (for Ns > 0).

Distance of foundation from edge of slope, b/B

(b) on top of slope.

***Figure (3.18): bearing capacity factors for continuous footing** (after Meyerhof).*

Solution:

$$(q_{ult.})_{c.or.s.footing.on.slope} = (q_{ult.})_{continuous.footing.on.slope}\left[\frac{(q_{ult.})_{c.or.s.footing.on.level.ground}}{(q_{ult.})_{continuous.footing.on.level.ground}}\right] \quad \text{.....(3.37)}$$

$$(q_{ult.})_{continuous.footing.on.slope} = (0)N_{cq} + \frac{1}{2}(19.5)(1.0)(25) = \boxed{243.75 kN/m^2}$$

$(q_{ult.})$ *of square or strip footing on level ground is calculated from Terzaghi's equation:*

$$q_{ult.} = cN_cS_c + qN_q + \frac{1}{2}.B.\gamma.N\gamma.S_\gamma$$

Bearing capacity factors from table (3.3): *for* $\phi = 30°$; $N_c = 37.2,..N_q = 22.5,..N_\gamma = 19.7$

Shape factors table (3.2): *for square footing* $S_c = 1.3, S\gamma = 0.8$; *strip footing* $S_c = S_\gamma = 1.0$

$$(q_{ult.})_{square.footing.on.level.ground} = 0 + 1.0\,(19.5)(22.5) + 0.5(1.0)(19.5)(19.7)(0.8) = 592.4\ kN/m^2$$

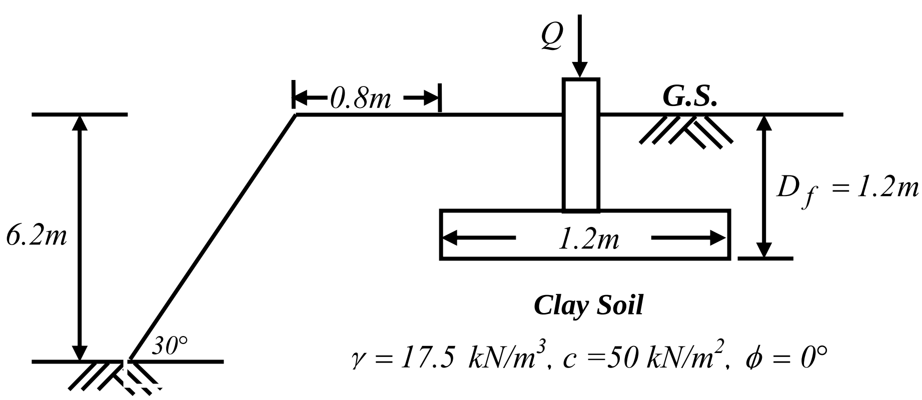
$$(q_{ult.})_{continuous.footing.on.level.ground} = 0 + 1.0\,(19.5)(22.5) + 0.5(1.0)(19.5)(19.7)(1.0) = 630.8\ kN/m^2$$

$$\therefore \quad (q_{ult.})_{square.footing.on.slope} = 243.75\frac{592.4}{630.8} = 228.912\ kN/m^2$$

and $(q_{all.})_{square.footing.on.slope} = \dfrac{228.912}{3} = \boxed{76\ kN/m^2}$

Example (19): *(footing on top of a slope)*

A shallow continuous footing in clay is to be located close to a slope as shown in figure. The ground water table is located at a great depth. Determine the gross allowable bearing capacity using F.S. = 4



Solution:

Since B<H assume the stability number $N_s = 0$ *and for* ***purely cohesive soil****,* $\phi = 0$

$$\boxed{(q_{ult.})_{continuous.footing.on.slope} = cN_{cq}}$$

From figure (3.18-b) for cohesive soil: with $\phi = 30°$, $N_s = 0$, $\frac{b}{B} = \frac{0.8}{1.2} = 0.67$, *and*

$\frac{D_f}{B} = \frac{1.2}{1.2} = 1.0$ *(use the dashed line)* $\longrightarrow$ $N_{cq} = 6.3$

$(q_{ult.})_{continuous.footing.on.slope} = (50)(6.3) = 315\ kN/m^2$

$q_{all.} = 315/4 = \boxed{78.8\ kN/m^2.}$

3.15 Foundation on Rock

It is common to use the building code values for the allowable bearing capacity of rocks (***see Table 3.8*****).** However, there are several significant parameters which should be taken into consideration together with the recommended code value; such as site geology, rock type and quality (as RQD).

Usually, the shear strength parameters c and ϕ of rocks are obtained from high Pressure Triaxial Tests. However, for most rocks $\phi = 45°$ except for limestone or shale $\phi = (38° - 45°)$ can be used. Similarly in most cases we could estimate $c = 5$ MPa with a conservative value.

Table (3.8): Allowable contact pressure $q_{all.}$ ***of jointed rock.***

RQD %	$q_{all.}$ **(T/ft²)**	$q_{all.}$ **(kN/m²)**	**Quality**
100	*300*	*31678*	*Excelent*
90	*200*	*21119*	*Very good*
75	*120*	*12671*	*Good*
50	*65*	*6864*	*Medium*
25	*30*	*3168*	*Poor*
0	*10*	*1056*	*Very poor*
1.0 (T/ft²) = 105.594 (kN/m²)			

Example (20): *(RQD)*

A core advance of 1500mm produced a sample length of 1310mm consisting of dust, gravel and intact pieces of rock. The sum of pieces 100mm or larger in length is 890mm.

Solution:

$$\text{The recovery ratio } (L_r) = \frac{1310}{1500} = 0.87\text{ ; and } (RQD) = \frac{890}{1500} = 0.59$$

Example (21): *(foundation on rock)*

A pier with a base diameter of 0.9m drilled to a depth of 3m in a rock mass. If RQD = 0.5, $\phi = 45°$ ***and c = 3.5 MPa ,*** γ_{rock} ***= 25.14 kN/m³, estimate*** $q_{all.}$ ***of the pier using Terzaghi's equation.***

Solution:

By Terzaghi's equation: $q_{ult.} = cN_c.S_c + qN_q + \frac{1}{2}.B.\gamma.N\gamma.S_\gamma$

Shape factors: from table (3.2) for circular footing: $S_c = 1.3$; $S_\gamma = 0.6$

Bearing capacity factors: $N_c = 5\tan^4(45+\phi/2)$, $N_q = \tan^6(45+\phi/2)$, $N_\gamma = N_q + 1$

$$\text{for } \phi = 45°, \quad N_c = 170, \quad N_q = 198, \quad N_\gamma = 199$$

$$q_{ult.} = (3.5x10^3)(170)(1.3) + (3)(25.14)(198) + 0.5(25.14)(0.9)(199)(0.6) = 789.78 \ MPa$$

and

$$q_{all.} = \frac{q_{ult.}(RQD)^2}{F.S.} = \frac{789.78(0.5)^2}{3.0} = 65.815..MPa$$

Al-Mansour University College

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Civil Eng. Dept.

المرحلة الرابعة

4th. Stage

Foundation Engineering

2022 – 2023

4

هندسة الاسس

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CHAPTER 4

STRESSES IN SOIL MASS

4.1 DEFIFINTIONS

- **VERTTICAL STRESS**
  Occurs due to internal or external applied load such as, overburden pressure, weight of structure and earthquake loads.

- **HORIZONTAL STRESS**
  Occurs due to vertical stress or earth pressure, water pressure, wind loads or earthquake horizontal loads.

- **ISOBAR**
  It is a contour connecting all points below the ground surface of equal intensity of pressure.

- **PRESSURE BULB**
  The zone in a loaded soil mass bounded by an isobar of a given pressure intensity is called a pressure bulb for that intensity.

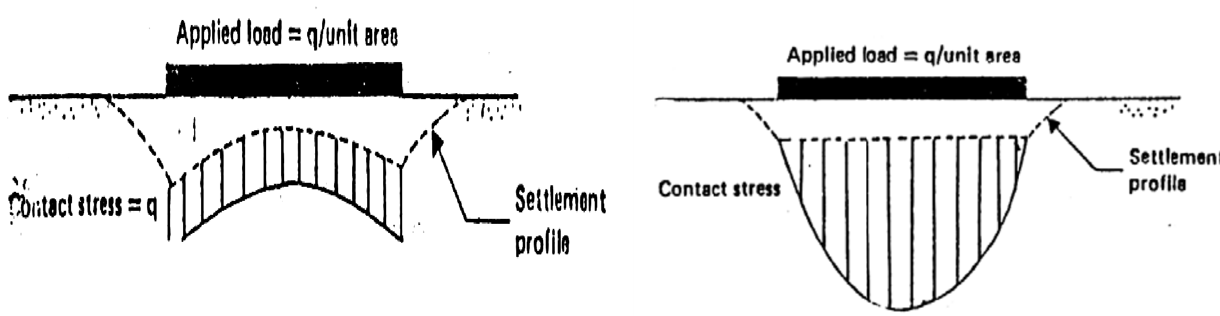
4.2 CONTACT PRESSURE

The analysis of Borowicka (1938) shows that the distribution of contact stress was dependent on a non-dimensional factor defined as:

$$k_r = \frac{1}{6}\left(\frac{1-\nu^2{}_s}{1-\nu^2{}_f}\right)\left(\frac{E_f}{E_s}\right)\left(\frac{T}{b}\right)^3 \quad \text{..}\textbf{(4.1)}$$

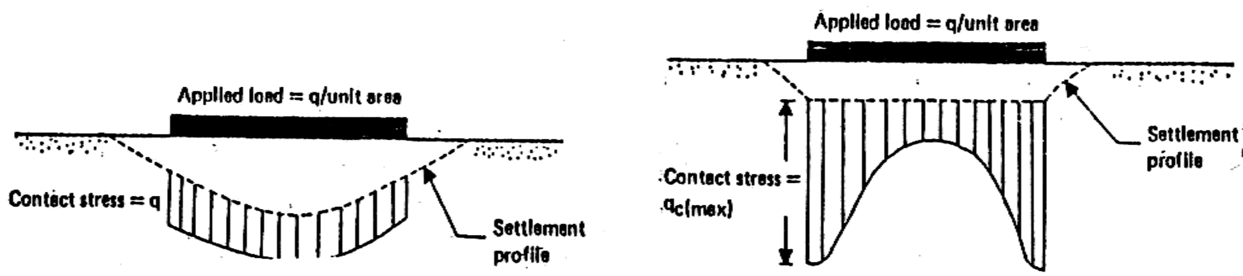
where: ν_s and ν_f = Poisson's ratio for soil and foundation materials, respectively,
E_s and E_f = Young's modulus for soil and foundation materials, respectively,
T = thickness of foundation,
B = half-width for strip footing; or radius for circular footing,
k_r = 0 indicates a perfectly flexible foundation; or
∞ means a perfectly rigid foundation.

The actual soil pressures distributions of rigid and flexible footings resting on sand and clay soils are shown in **Figures (4.1 and 4.2)**.



(a) Flexible **(b) Rigid**

Figure (4.1): Foundations on sand.



(a) Flexible **(b) Rigid**

Figure (4.2): Foundations on clay.

4.3 ASSUMPTIONS: The soil is assumed as:

(1) Semi-infinite in extent; x and y are infinite but the depth z has a limit value (Half-space),
(2) Isotropic; the soil has same properties in all directions,
(3) Homogeneous,
(4) Elastic and obeys Hook's law; the soil has linear relationship,
(5) Stresses at a point due to more than one surface load are obtained by superposition, and
(6) Negative values of loading can be used if the stresses due to excavations were required or the principle of superposition was used.

STRESS INCREASE DUE TO DIFFERENT LOADING

(1) POINT LOAD

- **BOUSSINESQ METHOD FOR HOMOGENEOUS SOIL:**

This method can be used for point loads acts directly at or outside the center. For a central load acting on the surface, its nature at depth z and radius r according to (simple radial stress distribution) is a cylinder in two-dimensional condition and a sphere in three-dimensional case.

Z = 1 m					
r (m)	r/z	A_b	z^2	Q/z^2	$\sigma_z \ (Q/m^2)$
0.0	0	0.47746	1.0	Q	0.47746Q
0.4	0.4	0.32946	1.0	Q	0.32946Q
0.8	0.8	0.13862	1.0	Q	0.13862Q
1.2	1.2	0.05134	1.0	Q	0.05134Q
1.6	1.6	0.01997	1.0	Q	0.01997Q
2.0	2.0	0.00854	1.0	Q	0.00854Q
2.4	2.4	0.00402	1.0	Q	0.00402Q
2.8	2.8	0.00206	1.0	Q	0.00206Q
3.6	3.6	0.00066	1.0	Q	0.00066Q
5.0	5.0	0.00014	1.0	Q	0.00014Q

Z = 2 m					
r (m)	r/z	A_b	z^2	Q/z^2	$\sigma_z \ (Q/m^2)$
0.0	0	0.47746	4.0	0.25Q	0.1194Q
0.4	0.2	0.43287	4.0	0.25Q	0.1082Q
0.8	0.4	0.32946	4.0	0.25Q	0.0824Q
1.2	0.6	0.22136	4.0	0.25Q	0.0553Q
1.6	0.8	0.13862	4.0	0.25Q	0.0347Q
2.0	1.0	0.08440	4.0	0.25Q	0.0211Q
2.4	1.2	0.05134	4.0	0.25Q	0.0129Q
2.8	1.4	0.03168	4.0	0.25Q	0.0079Q
3.6	1.8	0.01290	4.0	0.25Q	0.0032Q
5.0	2.5	0.00337	4.0	0.25Q	0.0008Q

Z = 3 m					
r (m)	r/z	A_b	z^2	Q/z^2	$\sigma_z \ (Q/m^2)$
0.0	0	0.47746	9.0	Q/9	0.0531Q
0.4	0.1333	0.45630	9.0	Q/9	0.0507Q
0.8	0.2666	0.40200	9.0	Q/9	0.0447Q
1.2	0.4000	0.32950	9.0	Q/9	0.0366Q
1.6	0.5333	0.25555	9.0	Q/9	0.0284Q
2.0	0.6666	0.19060	9.0	Q/9	0.0212Q
2.4	0.8000	0.13862	9.0	Q/9	0.0154Q
2.8	0.9333	0.09983	9.0	Q/9	0.0111Q
3.6	1.2000	0.05134	9.0	Q/9	0.0057Q
5.0	3.3333	0.01710	9.0	Q/9	0.0019Q

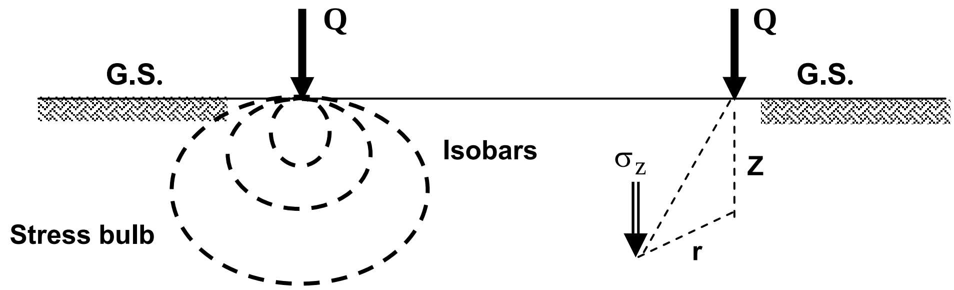


Figure (4.3): Vertical stress due to point load.

The vertical stress increase below or outside the point of load application is calculated as:

$$\sigma_z = \frac{Q}{.z^2} A_b \quad \text{...}\mathbf{(4.2)}$$

where: $A_b = \dfrac{3/2\pi.}{\left[1+(\frac{r}{z})^2\right]^{5/2}}$

- **WESTERGAARD METHOD FOR STRATIFIED SOIL:**

$$\sigma_z = \frac{Q}{2\pi.z^2} \frac{\sqrt{(1-2\mu)/(2-2\mu)}}{\left\{\left[(1-2\mu)/(2-2\mu)\right]+(\frac{r}{z})^2\right\}^{3/2}}$$

$\text{.............................}\mathbf{(4.3a)}$

where: $\mu = \text{Poisson's ratio.}$

when $\mu = 0$: $\sigma_z = \dfrac{Q}{.z^2} A_W$ ....**(4.3b)**

where: $A_W = \dfrac{1/\pi.}{\left[1+2(\frac{r}{z})^2\right]^{3/2}}$; Values of A_W can be tabulated for different values of μ as:

r/z	A_W		
	$\mu = 0$	$\mu = 0.4$	
0.0	0.3183	0.9549	**Note that :** At (r/z ≈ 1.8) both Boussinessq and Westergaard methods give equal values of σ_z.
0.2	0.2836	0.6916	
0.8	0.0925	0.0897	
1.0	0.0613	0.0516	
2.0	0.0118	0.0076	
3.0	0.0038	0.0023	
4.0	0.0017	0.0010	

Example (4.1): A concentrated point load Q acts vertically at the ground surface. Determine the vertical stress σ_z for each of the following cases:

a. Along the depth for r = 2m, and

b. At depth z = 2m.

Solution: From Boussinesq's equation: $\sigma_z = \frac{Q}{.z^2} A_b$ where: $A_b = \frac{3/2\pi.}{\left[1+(\frac{r}{z})^2\right]^{5/2}}$

(a) For r = 2m, the values of σ_z at various arbitrarily selected depths are given in the following table and the distribution of σ_z with depth is shown in **Figure (4.4 a)**.

z (m)	r/z	A_b	z^2	Q/z^2	σ_z (Q/m^2)
0.0	∞	0	0	∞	Indeterminate
0.4	5.0	0.00014	0.16	6.250Q	0.0009Q
0.8	2.5	0.00337	0.64	1.563Q	0.0053Q
1.2	1.67	0.01712	1.44	0.694Q	0.0119Q
1.6	1.25	0.04543	2.56	0.391Q	0.0178Q
2.0	1.00	0.08440	4.00	0.250Q	0.0211Q
2.4	0.83	0.12775	5.76	0.174Q	0.0222Q
2.8	0.71	0.17035	7.84	0.128Q	0.217Q
3.6	0.56	0.24372	12.96	0.0772Q	0.0188Q
5.0	0.40	0.32946	25.00	0.0400Q	0.0132Q
10.0	0.20	0.43287	100.00	0.0100Q	0.0043Q

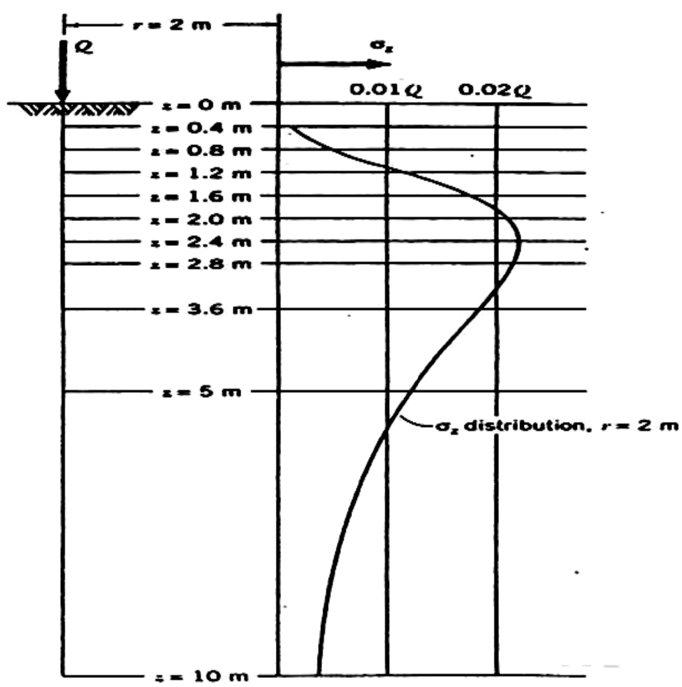


Figure (4.4 a): σ_z distribution with depth at a fixed radial distance from point of surface load.

(b) At depth z = 2m, the values of σ_z for various horizontal distances of r are given in the following table and the distribution of σ_z with r is shown in **Figure (4.4 b)**.

r (m)	r/z	A_b	z^2	Q/z^2	σ_z (Q/m^2)
0.0	0	0.47746	4.0	0.25Q	0.1194Q
0.4	0.2	0.43287	4.0	0.25Q	0.1082Q
0.8	0.4	0.32946	4.0	0.25Q	0.0824Q
1.2	0.6	0.22136	4.0	0.25Q	0.0553Q
1.6	0.8	0.13862	4.0	0.25Q	0.0347Q
2.0	1.0	0.08440	4.0	0.25Q	0.0211Q
2.4	1.2	0.05134	4.0	0.25Q	0.0129Q
2.8	1.4	0.03168	4.0	0.25Q	0.0079Q
3.6	1.8	0.01290	4.0	0.25Q	0.0032Q
5.0	2.5	0.00337	4.0	0.25Q	0.0008Q
10.0	5.0	0.00014	4.0	0.25Q	0.0001Q

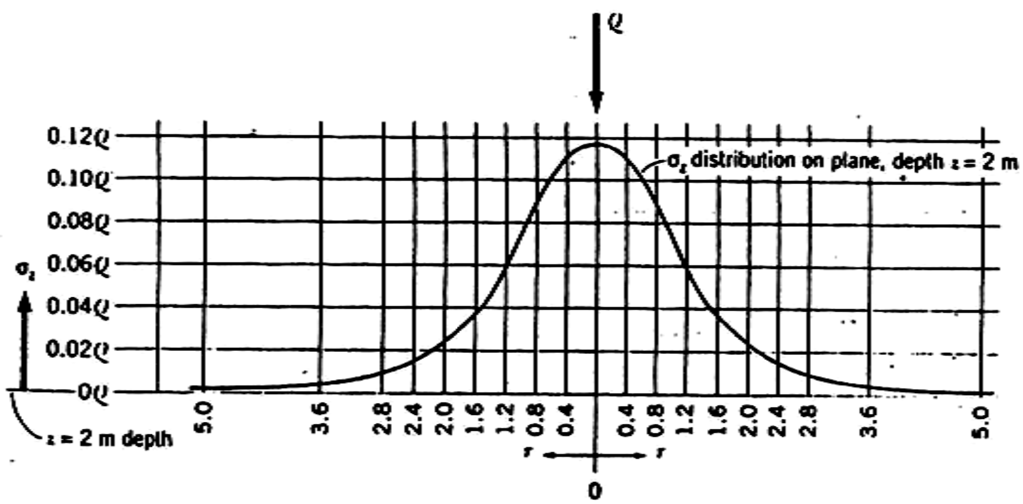


Figure (4.4 b): σ_z distribution with depth at a fixed radial distance from point of surface load.

Example (4.2): Q, is a concentrated point load acts vertically at the ground surface. Determine the vertical stress σ_z for various values of horizontal distances r and at z = 1, 2, 3, and 4m, then plot the σ_z distribution for all z depths.

Solution: From Boussinesq's equation: $\sigma_z = \frac{Q}{.z^2} A_b$ where: $A_b = \frac{3/2\pi.}{\left[1+(\frac{r}{z})^2\right]^{5/2}}$

σ_z for z = 1, 2, 3, and 4m depths is given in the following tables and their distributions with horizontal distances are shown in **Figure (4.5)**.

Z = 4 m					
r (m)	r/z	A_b	z^2	Q/z^2	σ_z (Q/m^2)
0.0	0	0.47746	16	Q/16	0.02984Q
0.4	0.1	0.46573	16	Q/16	0.02911Q
0.8	0.2	0.43287	16	Q/16	0.02705Q
1.2	0.3	0.38492	16	Q/16	0.02406Q
1.6	0.4	0.32946	16	Q/16	0.02059Q
2.0	0.5	0.27332	16	Q/16	0.01708Q
2.4	0.6	0.22136	16	Q/16	0.01384Q
2.8	0.7	0.17619	16	Q/16	0.01101Q
3.6	0.9	0.10833	16	Q/16	0.00677Q
5.0	1.25	0.04543	16	Q/16	0.00284Q

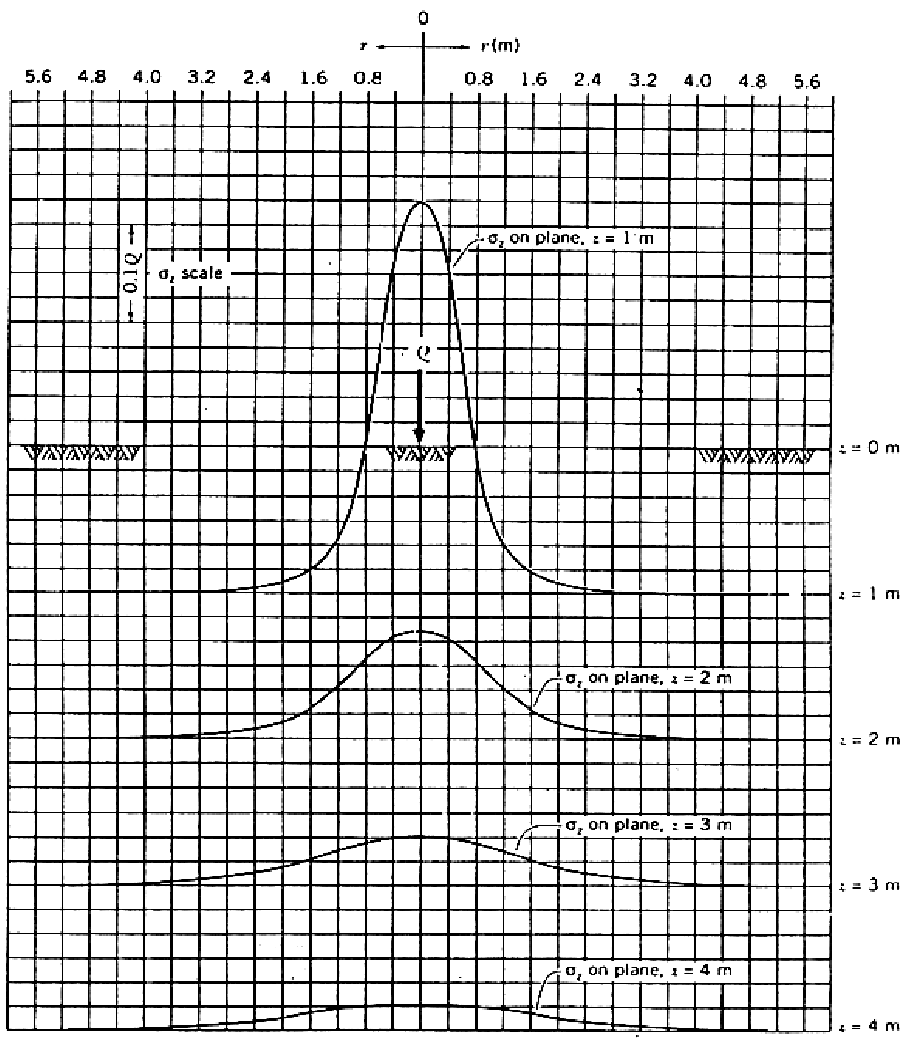
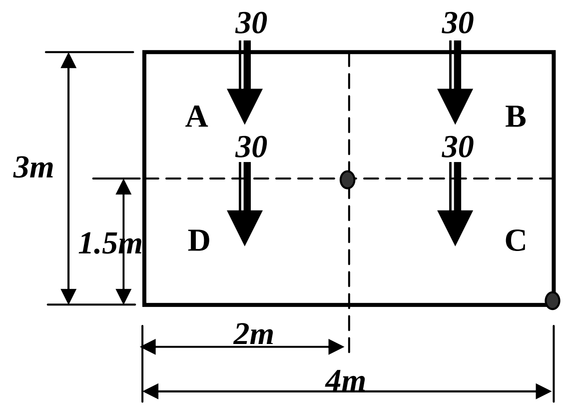


Figure (4.5): σ_z distribution with horizontal distance from point of surface load at several depths.

Example (4.3): An elastic soil medium of (4m x 3m) rectangular area is shown in figure. If the area is divided into 4 elementary areas of (2m x 1.5m) each that subjected at its surface a concentrated loads of (30 ton) at its centroid, use the Boussinesq's equation to find the vertical pressure at a depth of 6m below:

1. the center of the area,
2. one corner of the area.



Solution:

From Boussinesq's equation: $\sigma_z = \frac{Q}{.z^2} A_b$ where: $A_b = \frac{3/2\pi.}{\left[1+(r/z)^2\right]^{5/2}}$

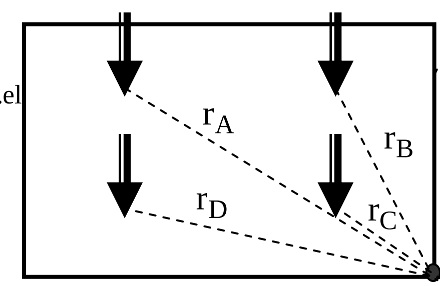
(a) At the center of the area:

r = 1.25m, r/z = 1.25/6 = 0.208

$A_b = 3/2\pi./.\left[1+(0.208)^2\right]^{5/2} = 0.4293$, $\sigma_{z\,(One.el}$ 5

T/m^2

$\sigma_{z\,(Total)} = (4).0.35775 = 1.43\,T/m^2$



At one corner of the area:

Elementary area	r (m)	r/z	A_b	$\sigma_z = \frac{Q}{.z^2} A_b$
A	3.750	0.625	0.209	0.174
B	2.462	0.410	0.324	0.270
C	1.250	0.208	0.429	0.358
D	3.092	0.515	0.265	0.221
$\sum$				1.023 T/m2

(5) TRIANGULAR LOADED STRIP AREA:

To calculate the vertical stress under triangular loaded strip area **(see Figure 4.12**).

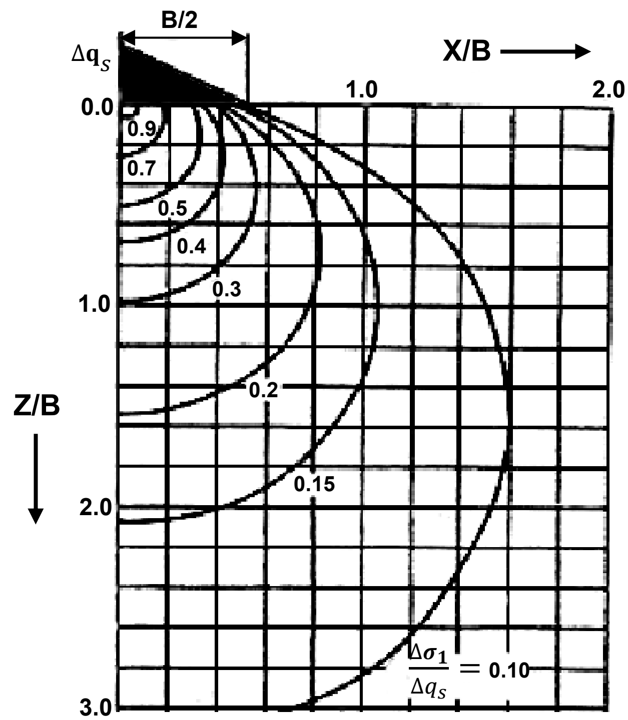
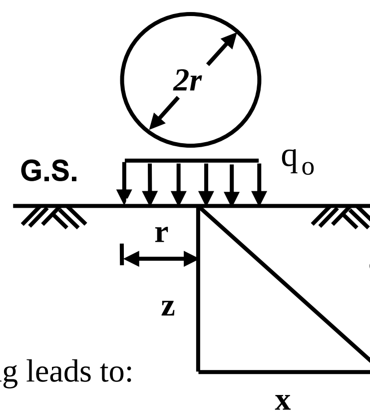


Figure (4.12): Pressure bulbs for vertical stresses under triangular strip load.

(6) UNIFORMLY LOADED CIRCULAR AREA:



$$\sigma_z = \int_{\theta=0}^{2\pi} \frac{3q}{.2\pi.z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2} dA$$

where, $dA = \frac{1}{2} r^2 .d\theta$; which after integrating and simplifying leads to:

$$\sigma_z = \frac{I.xq_o}{100} \quad \textbf{..........(4.5)}$$

where, I = Influence factor depends on (z/r and x/r); expressed in percentage of surface contact pressure, q_o, for vertical stress under uniformly loaded circular area (**see Figure 4.13**).

(2) 2:1 APPROXIMATION METHOD *for depths < 2.5 (width of loaded area):*

Total load on the surface = q.B.L; and Area at depth z = (L + z) (B + z)

$$\sigma_z = \frac{q.B.L}{(L+z)(B+z)} \quad \text{.......................................(4.4)}$$

Type of footing	Area, A_z
Square	$(B+z)^2$
Rectangular	$(B+z)(L+z)$
Circular	$\pi(D+z)^2/4$
Strip or wall	$(B+z).1$

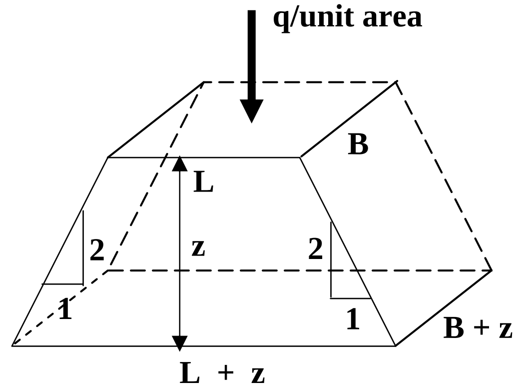


Figure (4.6): 2:1 Stress distribution method.

(3) UNIFORMLY LOADED LINE OF FINITE LENGTH:

Figure (4.7) shows a line load of equal intensity **q** applied at the surface. For an element selected at an arbitrary fixed point in the soil mass, an expression for σ_z could be derived by integrating Boussinesq's expression for point load as:

$$\sigma_z = \frac{Q}{.z^2} P_o \quad \text{.......................................(4.5)}$$

where:

$$P_o = \frac{1}{2\pi(m^2+1)^2}\left[\frac{3n}{\sqrt{n^2+1+m^2}} - \left(\frac{n}{\sqrt{n^2+1+m^2}}\right)^3\right]$$

$m = x/z$, and $n = y/z$

Values of P_o for various combinations of **m** and **n** are given in **Table (4.1).**

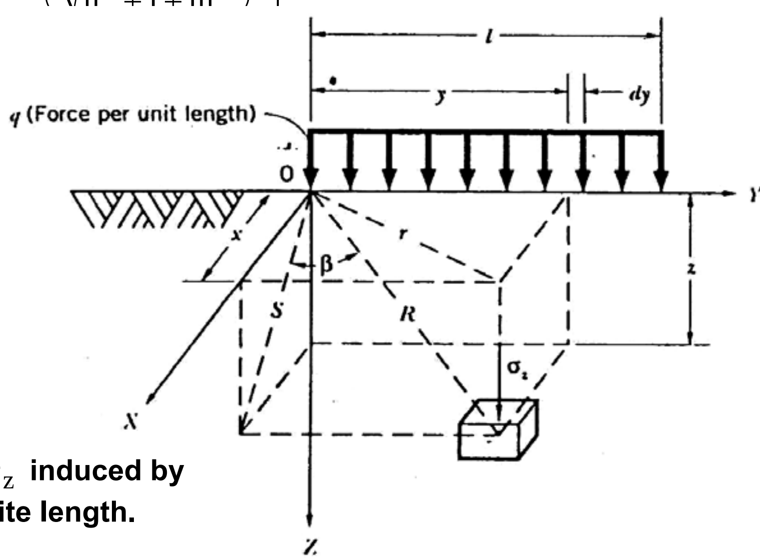


Figure (4.7): Vertical stress σ_z induced by Line loads of finite length.

Table (4.1): Influence values P_0 for case of uniform line load of finite length.

m	n = 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.0	0.04735	0.09244	0.13342	0.16917	0.19929	0.22398	0.24379	0.25947	0.27176	0.28135	0.29464	0.30277
0.1	0.04619	0.09020	0.13023	0.16520	0.19470	0.21892	0.23839	0.25382	0.26593	0.27539	0.28853	0.29659
0.2	0.04294	0.08391	0.12127	0.15403	0.18178	0.20466	0.22315	0.23787	0.24947	0.25857	0.27127	0.27911
0.3	0.03820	0.07472	0.10816	0.13764	0.16279	0.18367	0.20066	0.21429	0.22511	0.23365	0.24566	0.25315
0.4	0.03271	0.06406	0.09293	0.11855	0.14058	0.15905	0.17423	0.18651	0.19634	0.20418	0.21532	0.22235
0.5	0.02715	0.05325	0.07742	0.09904	0.11782	0.13373	0.14694	0.15775	0.16650	0.17354	0.18368	0.19018
0.6	0.02200	0.04322	0.06298	0.08081	0.09646	0.10986	0.12112	0.13045	0.13809	0.14430	0.15339	0.15931
0.7	0.01752	0.03447	0.05035	0.06481	0.07762	0.08872	0.09816	0.10608	0.11265	0.11805	0.12607	0.13140
0.8	0.01379	0.02717	0.03979	0.05136	0.06172	0.07080	0.07862	0.08525	0.09082	0.09546	0.10247	0.10722
0.9	0.01078	0.02128	0.03122	0.04041	0.04872	0.05608	0.06249	0.06800	0.07268	0.07663	0.08268	0.08687
1.0	0.00841	0.01661	0.02441	0.03169	0.03832	0.04425	0.04948	0.05402	0.05793	0.06126	0.06645	0.07012
1.2	0.00512	0.01013	0.01495	0.01949	0.02369	0.02752	0.03097	0.03403	0.03671	0.03905	0.04281	0.04558
1.4	0.00316	0.00626	0.00927	0.01213	0.01481	0.01730	0.01957	0.02162	0.02345	0.02508	0.02777	0.02983
1.6	0.00199	0.00396	0.00587	0.00770	0.00944	0.01107	0.01258	0.01396	0.01522	0.01635	0.01828	0.01979
1.8	0.00129	0.00256	0.00380	0.00500	0.00615	0.00724	0.00825	0.00920	0.01007	0.01086	0.01224	0.01336
2.0	0.00085	0.00170	0.00252	0.00333	0.00410	0.00484	0.00554	0.00619	0.00680	0.00736	0.00836	0.00918
2.5	0.00034	0.00067	0.00100	0.00133	0.00164	0.00194	0.00224	0.00252	0.00278	0.00303	0.00349	0.00389
3.0	0.00015	0.00030	0.00045	0.00060	0.00074	0.00088	0.00102	0.00115	0.00127	0.00140	0.00162	0.00183
4.0	0.00004	0.00008	0.00012	0.00016	0.00020	0.00024	0.00027	0.00031	0.00035	0.00038	0.00045	0.00051

m	n = 1.6	1.8	2.0	2.5	3.0	4.0	5.0	6.0	8.0	10.0	∞
0.0	0.30784	0.31107	0.31318	0.31593	0.31707	0.31789	0.31813	0.31822	0.31828	0.31830	0.31831
0.1	0.30161	0.30482	0.30692	0.30966	0.31080	0.31162	0.31186	0.31195	0.31201	0.31203	0.31204
0.2	0.28402	0.28716	0.28923	0.29193	0.29307	0.29388	0.29412	0.29421	0.29427	0.29428	0.29430
0.3	0.25788	0.26092	0.26293	0.26558	0.26670	0.26750	0.26774	0.26783	0.26789	0.26790	0.26792
0.4	0.22683	0.22975	0.23169	0.23426	0.23535	0.23614	0.23638	0.23647	0.23653	0.23654	0.23656
0.5	0.19438	0.19714	0.19899	0.20147	0.20253	0.20331	0.20354	0.20363	0.20369	0.20371	0.20372
0.6	0.16320	0.16578	0.16753	0.16990	0.17093	0.17169	0.17192	0.17201	0.17207	0.17208	0.17210
0.7	0.13496	0.13735	0.13899	0.14124	0.14224	0.14297	0.14320	0.14329	0.14335	0.14336	0.14338
0.8	0.11044	0.11264	0.11416	0.11628	0.11723	0.11795	0.11818	0.11826	0.11832	0.11834	0.11835
0.9	0.08977	0.09177	0.0318	0.09517	0.09608	0.09677	0.09699	0.09708	0.09713	0.9715	0.09716
1.0	0.07270	0.07452	0.07580	0.07766	0.07852	0.07919	0.07941	0.07949	0.07955	0.07957	0.07958
1.2	0.04759	0.04905	0.05012	0.05171	0.05248	0.05310	0.05330	0.05338	0.05344	0.05345	0.05347
1.4	0.03137	0.03253	0.03340	0.03474	0.03542	0.03598	0.03617	0.03625	0.03630	0.03632	0.03633
1.6	0.02097	0.02188	0.02257	0.02368	0.02427	0.02478	0.02496	0.02504	0.02509	0.02510	0.02512
1.8	0.01425	0.01496	0.01551	0.01643	0.01694	0.01739	0.01756	0.01765	0.01768	0.01769	0.01771
2.0	0.00986	0.01041	0.01085	0.01160	0.01203	0.01244	0.01259	0.01266	0.01271	0.01272	0.01273
2.5	0.00424	0.00453	0.00477	0.00523	0.00551	0.00581	0.00593	0.00599	0.00603	0.00605	0.00606
3.0	0.00201	0.00217	0.00231	0.00258	0.00277	0.00298	0.00307	0.00312	0.00316	0.00317	0.00318
4.0	0.00057	0.00063	0.00068	0.00078	0.00086	0.00096	0.00102	0.00105	0.00108	0.00109	0.00110

Problem (4.4): **Given:** q = 100 kN/m.
Find: The vertical stress σ_z at points 0, 1, and 2 shown in Fig. (4.8)?

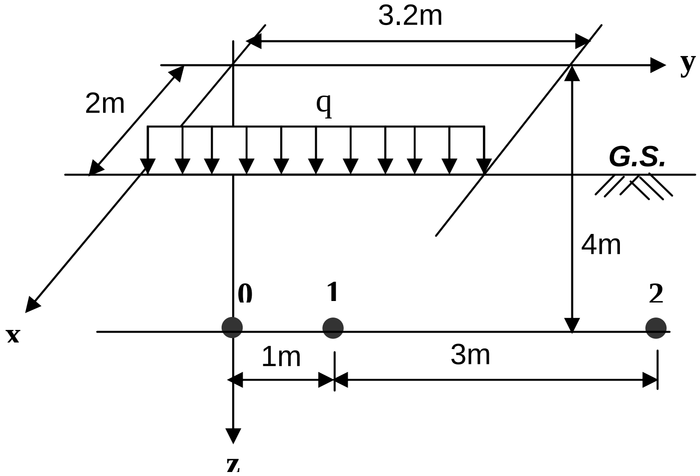


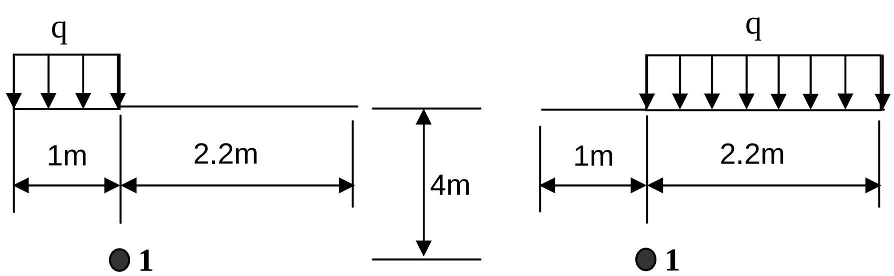
Figure (4.8): Uniformly line surface load of finite length.

Solution:

(a) σ_z at point (0):

For m = x/z = 2/ 4 = 0.5 and n = y/z = 3.2/ 4 = 0.8 from **Table (4.1)**: $P_0 = 0.15775$

$$\sigma_{z(0)} = \frac{Q}{z} P_0 = \frac{100}{4} 0.15775 = 3.944.kN/m^2$$



(a) Left load. **(b) Right load.**

Figure (4.9): σ_z **at point (1).**

(b) σ_z at point (1):

$$\sigma_{z(1)} = \sigma_{z(1L)} + \sigma_{z(1R)}$$

From Fig. (4.9-a)

For m = x/z = 2/ 4 = 0.5 and n = y/z = 1/ 4 = 0.25 from **Table (4.1)**: $P_0 = 0.06534$

$$\sigma_{z(1L)} = \frac{Q}{z} P_0 = \frac{100}{4} (0.06534) = 1.634.kN/m^2$$

From Fig. (4.9-b)

For $m = x/z = 2/4 = 0.5$ and $n = y/z = 2.2/4 = 0.55$ from **Table (4.1)**: $P_0 = 0.12578$

$$\sigma_{z(1R)} = \frac{Q}{z} P_0 = \frac{100}{4}(0.12578) = 3.144.kN/m^2$$

$$\sigma_{z(1)} = \sigma_{z(1L)} + \sigma_{z(1R)} = 1.634 + 3.144 = 4.778.kN/m^2$$

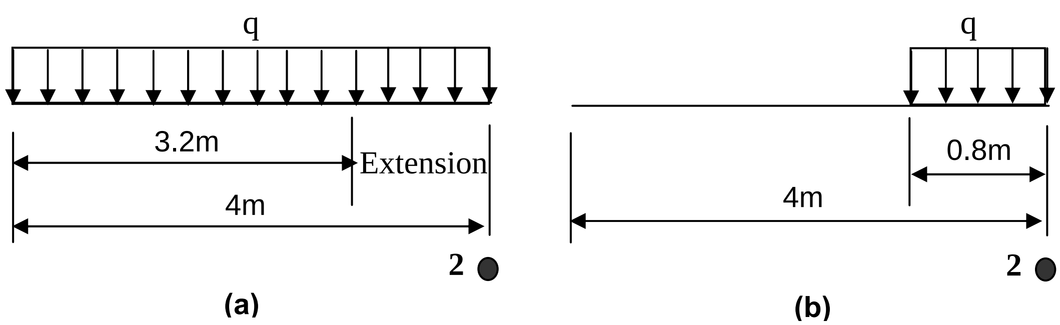


Figure (4.10): σ_z at point (2).

(c) σ_z at point (2):

$$\sigma_{z(2)} = \sigma_{z(2L)} - \sigma_{z(2R)}$$

From Fig. (4.10-a):

For $m = x/z = 2/4 = 0.5$ and $n = y/z = 4/4 = 1.0$ from **Table (4.1)**: $P_0 = 0.1735$

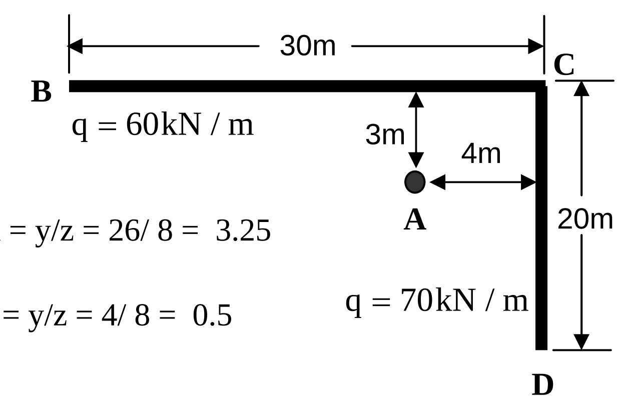
From Fig. (4.10-b):

For $m = x/z = 2/4 = 0.5$ and $n = y/z = 0.8/4 = 0.2$ from **Table (4.1)**: $P_0 = 0.0532$

$$\sigma_{z(2)} = \sigma_{z(2L)} - \sigma_{z(2R)} = \frac{100}{4}(0.1735 - 0.0532) = 3.01.kN/m^2$$

Problem (4.5): **Given:** Two walls loaded as shown in Fig..

Find: the vertical stress σ_z at z = 8m below point A?



Solution:

Wall BC:

26m: For $m = x/z = 3/8 = 0.375$ and $n = y/z = 26/8 = 3.25$

From **Table (4.1)**: $P_0 = 0.26$

4m: For $m = x/z = 3/8 = 0.375$ and $n = y/z = 4/8 = 0.5$

From **Table (4.1)**: $P_0 = 0.15$

$$\sigma_{z(BC)} = \frac{q}{z} P_0 = \frac{60}{8}(0.26 + 0.15) = 3.10.kN/m^2$$

Wall CD:

3**m:** For $m = x/z = 4/8 = 0.5$ and $n = y/z = 3/8 = 0.375$
From **Table (4.1)**: $P_0 = 0.09$

17m: For $m = x/z = 4/8 = 0.5$ and $n = y/z = 17/8 = 2.125$
From **Table (4.1)**: $P_0 = 0.20$

$$\sigma_{z(CD)} = \frac{q}{z} P_0 = \frac{70}{8}(009 + 0.20) = 2.50.kN/m^2$$

$$\sigma_{z(A)} = 3.10 + 2.50 = 5.6.kN/m^2$$

(4) UNIFORMLY LOADED STRIP AREA:

To calculate the vertical stress under uniformly loaded strip area **(see Figure 4.11)**.

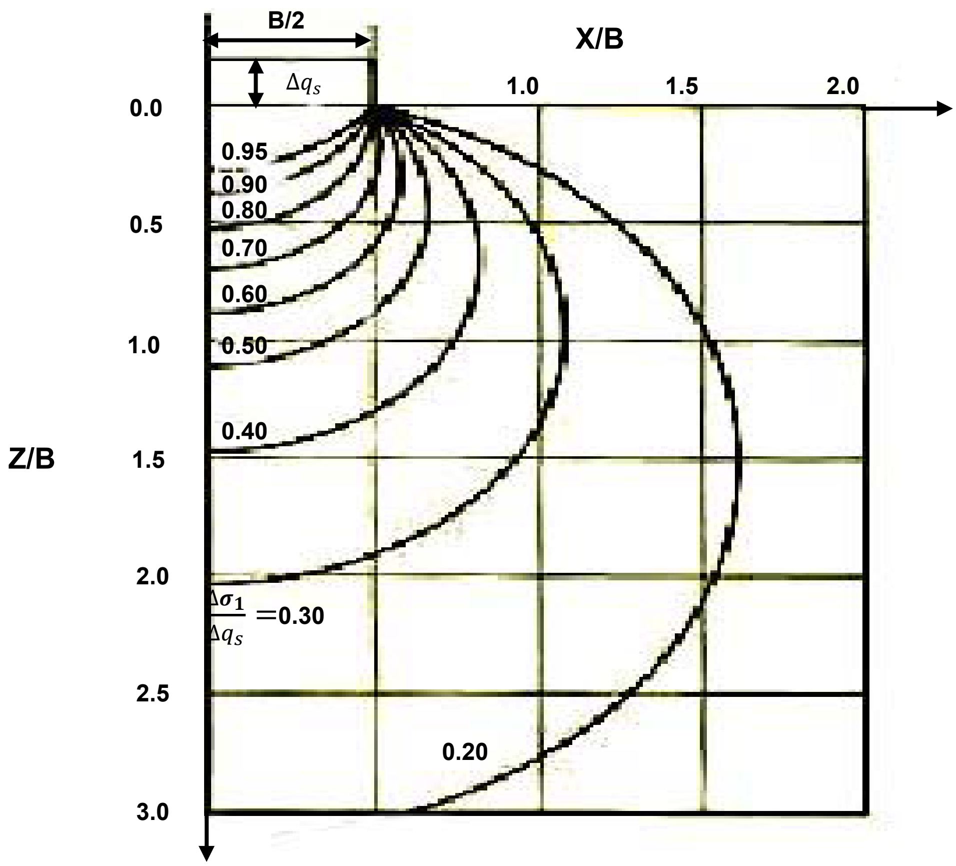


Figure (4.11): Pressure bulbs for vertical stresses under strip load.

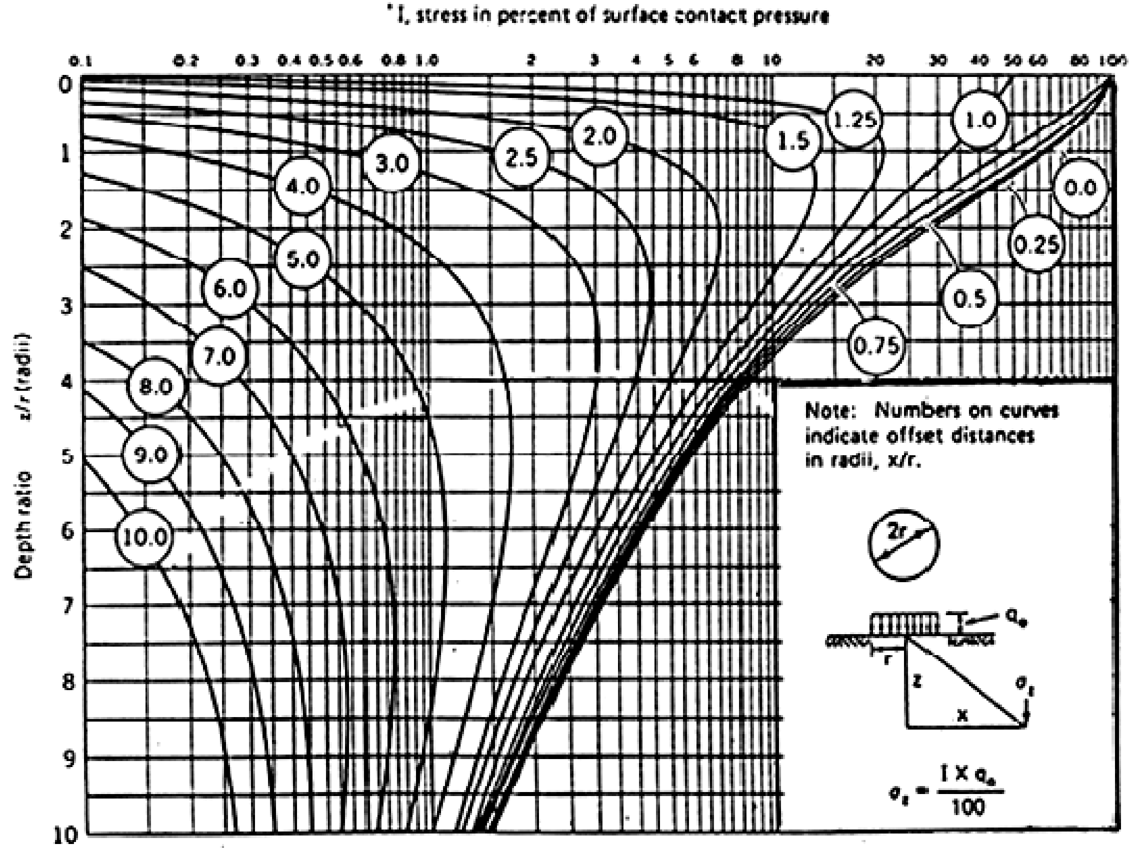


Figure (4.13): Influence values expressed in percentage of surface contact pressure for vertical stress under uniformly loaded circular area *(after Foster and Ahlvin, 1954, as cited by U.S. Navy, 1971).*

Problem (٤.6):

Given: A circular area, r = 1.6m, induces a soil pressure at the surface of 100 kN/m^2.

Find: the vertical stress σ_z at:

(a) z = 2m directly under the center of the circular area.

(b) z = 2m below and 2m away from the center of the circle.

Solution:

a. For z/r = 2/1.6 = 1.25 and x/r = 0; *from **Fig. (4.13):*** $I = 52$

$$\sigma_z = \frac{I.q_o}{100} = \frac{52.(100)}{100} = 52 \ \mathbf{kN/m^2.}$$

b. For z/r = 2/1.6 = 1.25 and x/r = 2/1.6 = 1.25; *from **Fig. (4.13):*** $I = 22$

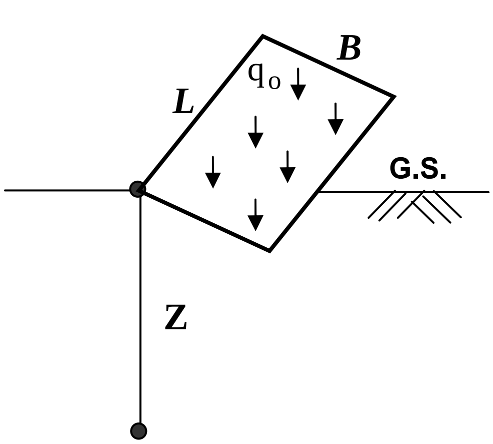
$$\sigma_z = \frac{I.q_o}{100} = \frac{22.(100)}{100} = 22 \ \text{kN/m}^2.$$

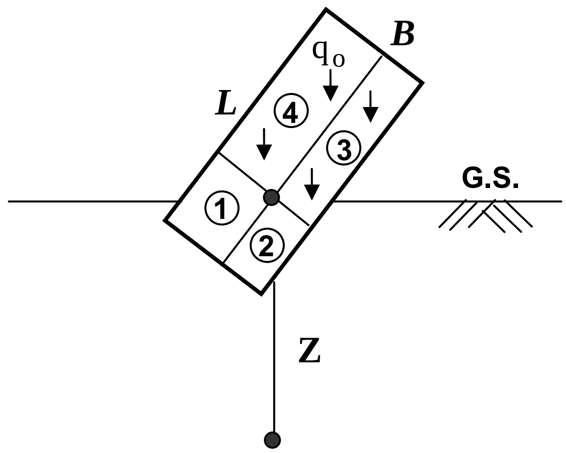
(6) UNIFORMLY LOADED RECTANGULAR OR SQUARE AREA:

The vertical stress increase below the corner of a flexible rectangular or square loaded area is calculated as:

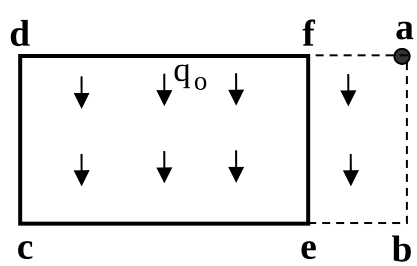
$$\sigma_z = I.q_o \quad \text{.....(4.6)}$$

where, I = influence factor, depends on (m = B/z, and n = L/z) obtained from **(Figure 4.14).**





Example (1): $\sigma_z = I.q_o$ **Example (2):** $\sigma_z = q_o[I_1 + I_2 + I_3 + I_4]$



Example (3): $\sigma_{z(a)} = q_o[I_{abcd} - I_{abef}]$

Examples for Vertical stress under the corner of a uniformly loaded rectangular area.

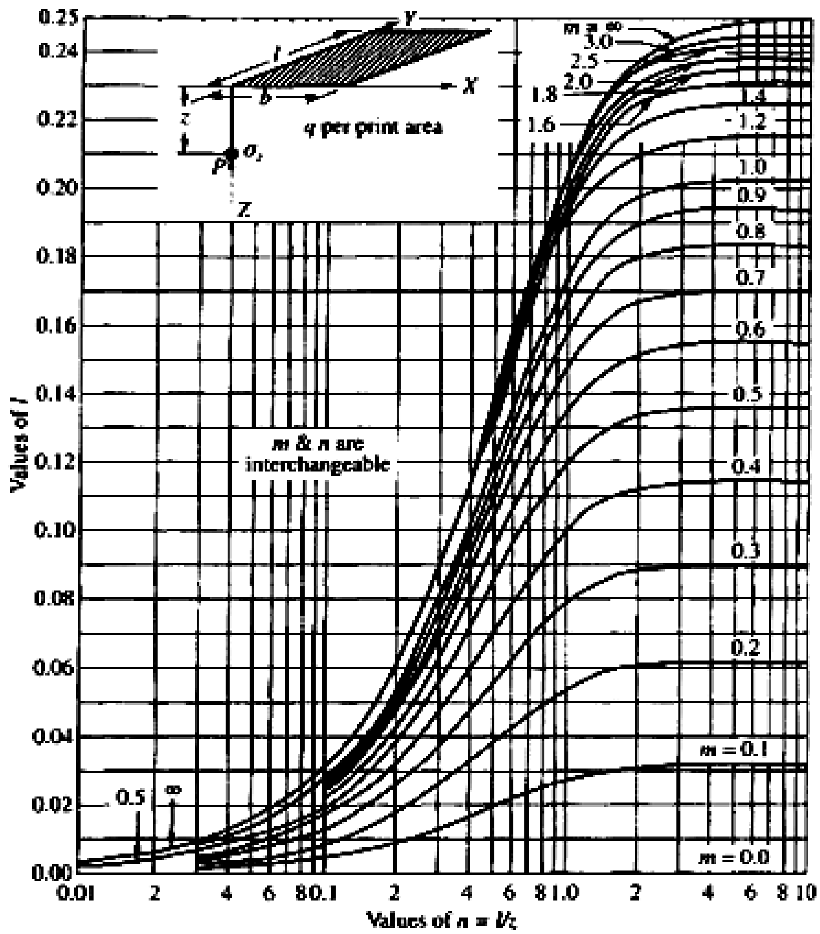
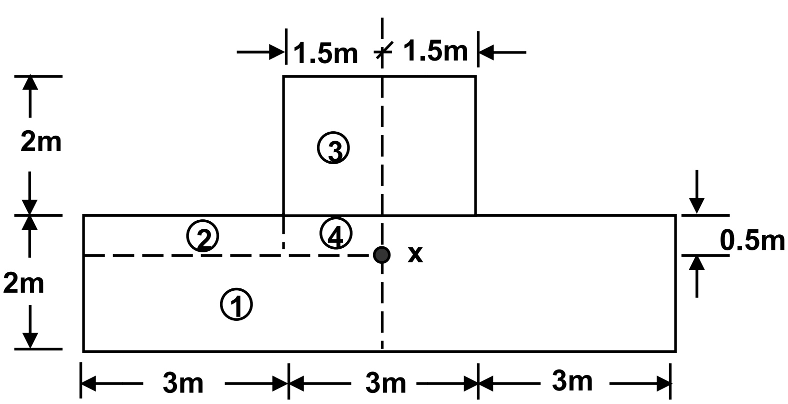


Figure (4.14): Values of I for vertical stress below the corner of a flexible rectangular area *(after Fadum, 1948).*

Problem (4.7): The plan of a foundation is given in the Fig. below. The uniform contact pressure is 40 kN/m^2. Determine the vertical stress increment due to the foundation at a depth of (5m) below the point (x).

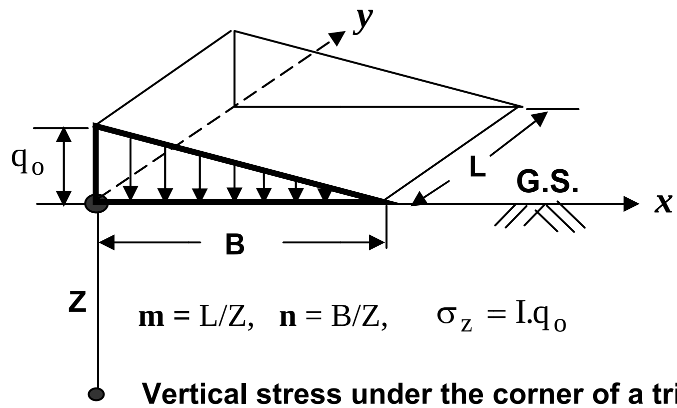


(7) TRIANGULAR LOAD OF LIMITED LENGTH:

The vertical stress under the corners of a triangular load of limited length is calculated as:

$$\sigma_z = I.q_o \quad \text{..(4.7)}$$

where, I = influence factor, depends on (m = L/z, and n = B/z) obtained from **(Figure 4.15).**



Vertical stress under the corner of a triangular load of limited length.

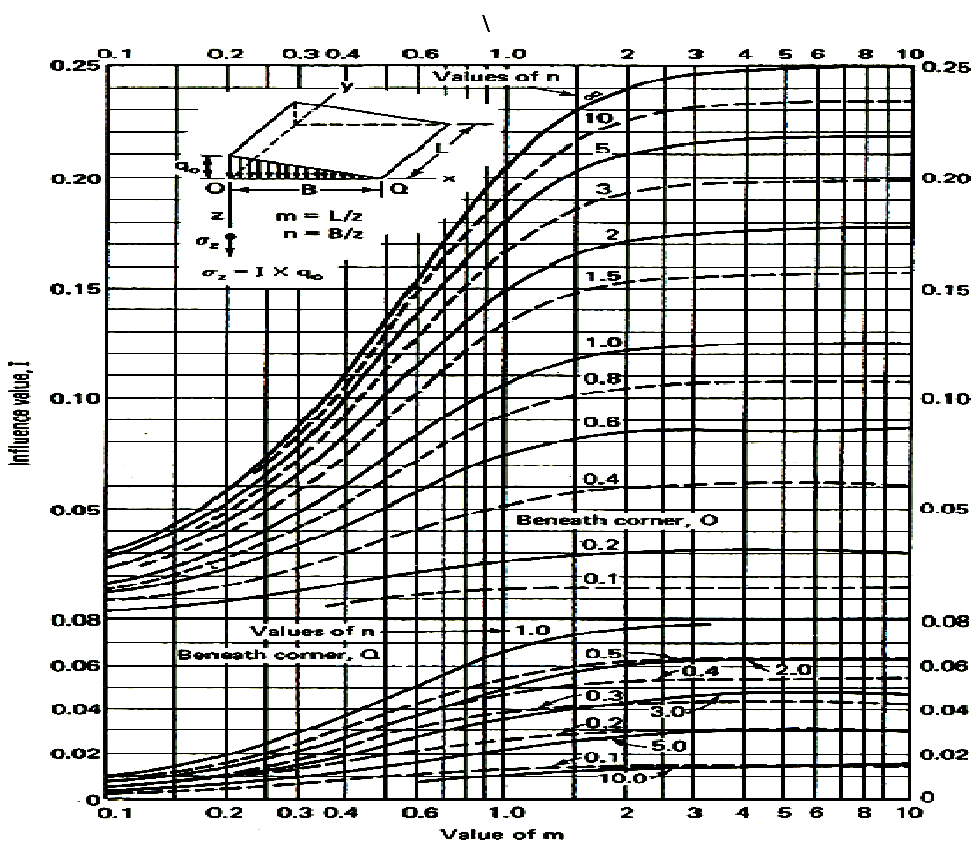


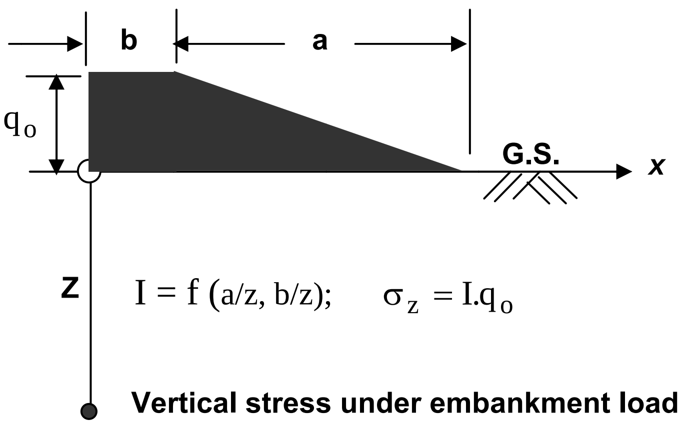
Figure (4.15): Influence values for vertical stress under the corners of a triangular load of limited length *(after U.S. Navy, 971).*

(8) EMBANKMENT LOADING:

The vertical stress under embankment loading is calculated as:

$$\sigma_z = I.q_o \quad \text{………………………………………………………….(4.8)}$$

where, I = influence factor depends on (a/z, and b/z) determined from **(Figure 4.16).**



Vertical stress under embankment loading.

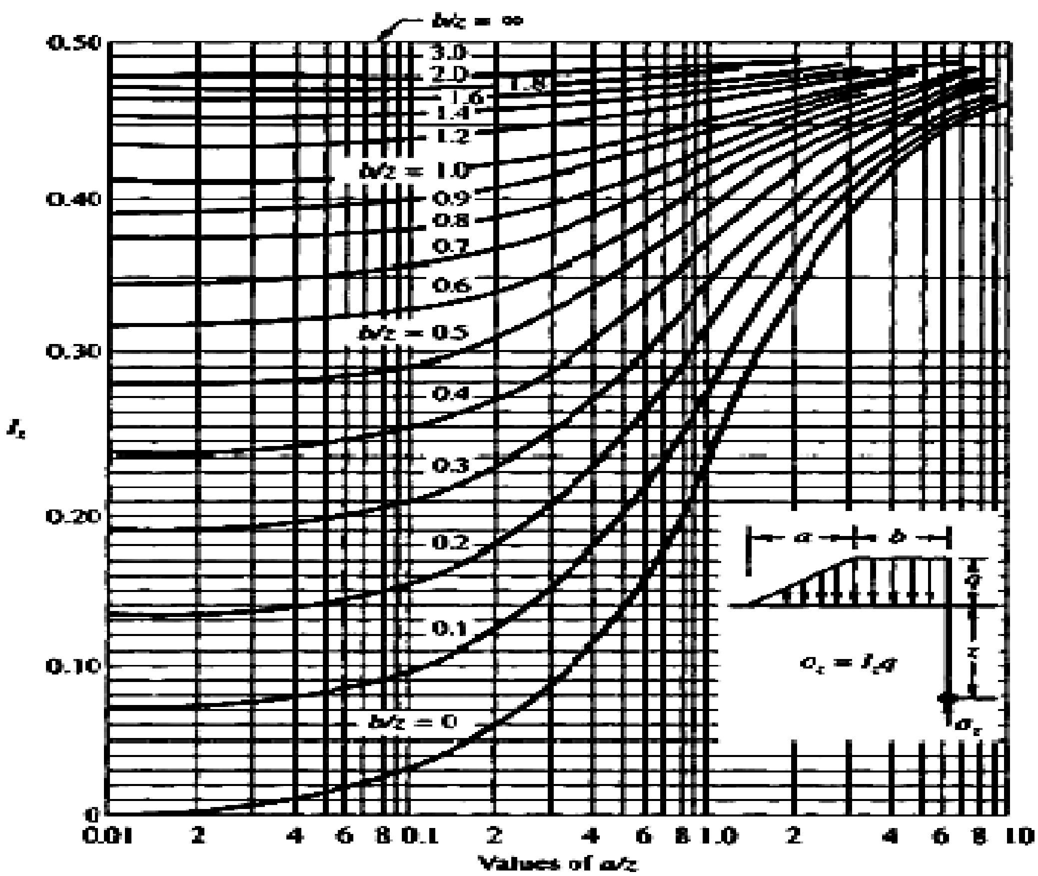
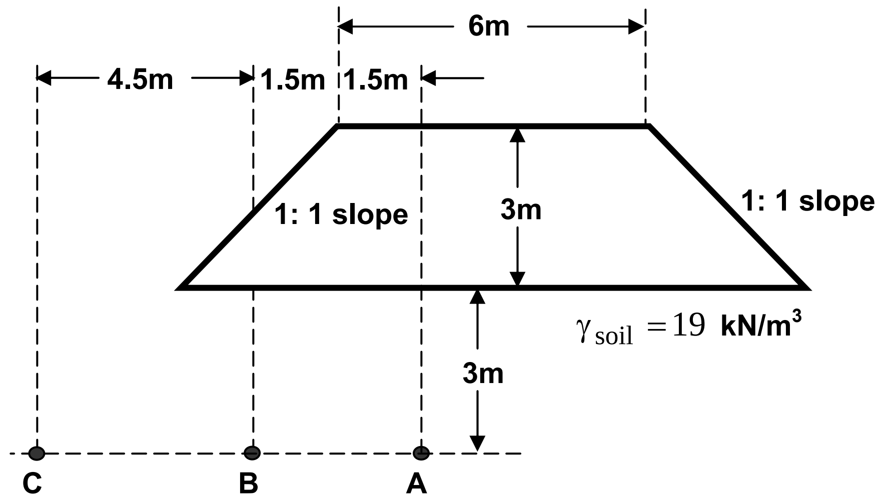


Figure (4.16): Influence factor for embankment loading *(after Osterberg, 1957).*

Problem (4.9): An embankment of (3m) high is to be constructed as shown in the figure below. If the unit weight of compacted soil is 19 kN/m^3, calculate the vertical stress due to the embankment loading at (A), (B), and (C) points.



Solution:

(1) Vertical stress at A:

From Fig. (4.17a): $\sigma_{zA} = \sigma_{z(1)} + \sigma_{z(2)}$

Left-hand section: b/z = 1.5/3 = 0.5 and a/z = 3/3 = 1.0, *from* ***Fig. (4.16);*** $I_1 = 0.396$

Right-hand section: b/z = 4.5/3 = 1.5 and a/z = 3/3 = 1.0, *from* ***Fig. (4.16);*** $I_2 = 0.477$

$\sigma_{zA} = (I_1 + I_2)q = [0.396 + 0.477](19)(3) = 49.761$ kN/m^2

(2) Vertical stress at B:

From Fig. (4.17b): $\sigma_{zB} = \sigma_{z(1)} + \sigma_{z(2)} - \sigma_{z(3)}$

Left-hand section: b/z = 0/3 = 0 and a/z = 1.5/3 = 0.5, *from* ***Fig. (4.16);*** $I_1 = 0.140$

Middle section: b/z =7.5/3 = 2.5 and a/z = 3/3 = 1.0, *from* ***Fig. (4.16);*** $I_2 = 0.493$

Right-hand section: b/z = 0/3 = 0 and a/z = 1.5/3 = 0.5, *from* ***Fig. (4.16);*** $I_3 = 0.140$

$\sigma_{zB} = (I_1.q_1) + (I_2.q_2) - (I_3.q_3)$

$= (0.14)(19)(1.5) + (0.493)(19)(3) - (0.14)(19)(1.5) = 28.101$ kN/m^2

(3) Vertical stress at C:

Using Fig. (4.17c): $\sigma_{zC} = \sigma_{z(1)} - \sigma_{z(2)}$

Left-hand section, b/z = 12/3 = 4 and a/z = 3/3 = 1.0, *from* ***Fig. (4.16);*** $I_1 = 0.498$.

Right-hand section, b/z = 3/3 = 1.0 and a/z = 3/3 = 1.0, *from* ***Fig. (4.16);*** $I_2 = 0.456$.

$\sigma_{zC} = (I_1 - I_2)q = (0.498 - 0.456)(19)(3) = 2.394$ kN/m^2

Table (5.4): Limiting values of deflection ratios
(The 1955 Soviet Code of Practice).

Building type	Deflection ratio (Δ / L)		Average maximum Settlement (cm)
	Sand	Clay	
Steel and concrete frames	0.0010	0.0013	10
Multistory buildings $L/H \le 3$ $L/H \ge 5$	 0.003 0.005	 0.004 0.007	 8 $L/H \ge 2.5$ 10 $L/H \le 1.5$
One-story building	0.001	0.001	-----------
Water towers, Ring foundations	0.004	0.004	-----------

L = length between two adjacent points under consideration, and
H = height of wall above foundation.

Table (5.5): Limiting angular distortion for various structures
(Bjerrum, 1963).

Category of potential damage	Angular distortion $\beta_{max.}$
Safe limit for flexible brick walls (L/H >4)	1/150
Danger for structural damage of general buildings	1/150
Cracking in panel and brick walls	1/150
Visible tilting of high rigid buildings	1/250
First cracking in panel walls	1/300
Safe limit of no cracking of building	1/500
Danger for frames with diagonals	1/600

where, $I_1 ..and.. I_2$ are influence factors = $f(H/B',..L/B)$ *obtained from **Table (5.9)***, and
H = depth of hard stratum

I_D = Depth factor (Fox, 1948) = $f(D_f / B,..\mu_s,..and..L/B)$ which can be approximated by:

$$I_D = 0.66\left(\frac{D_f}{B}\right)^{(-0.19)} + 0.025\left(\frac{L}{B} + 12\mu_s - 4.6\right)$$

Note: *when $D_f = 0$, the value of $I_D = 1$ in all cases.*

C_N = Number of contributing corners = **4** *for center*, **2** *for edges*, and **1** *for corners*.

Table (5.7): Typical values of μ_s.

Type of Soil	μ_s
Clay, saturated	*0.40 – 0.50*
Clay, unsaturated	*0.10 – 0.30*
Sandy clay	*0.20 – 0.30*
Silt	*0.30 – 0.35*
Sand (dense)	*0.20 – 0.40*
Coarse (void ratio = 0.4 - 0.7)	*0.15*
Fine-grained (void ratio = 0.4 - 0.7)	*0.25*
Rock	*0.10 – 0.40*
Loess	*0.10 – 0.30*
Concrete	*0.15*

Table (5.8): Typical values of E_s for selected soils

(filed values depend on stress history, water content, density, etc.).

Type of Soil	E_s *(MPa)*
Clay	
Very soft	*2-15*
Soft	*5-25*
Medium	*15-50*
Hard	*50-100*
Sandy	*25-250*
Glacial till	
Loose	*10-153*
Dense	*144-720*
Very Dense	*478-1440*
Loess	*14-57*
Sand	
Silty	*7-21*
Loose	*10-24*
Dense	*48-81*
Sand and gravel	
Loose	*48-144*
Dense	*96-192*
Shale	*144-14400*
Silt	*2-20*

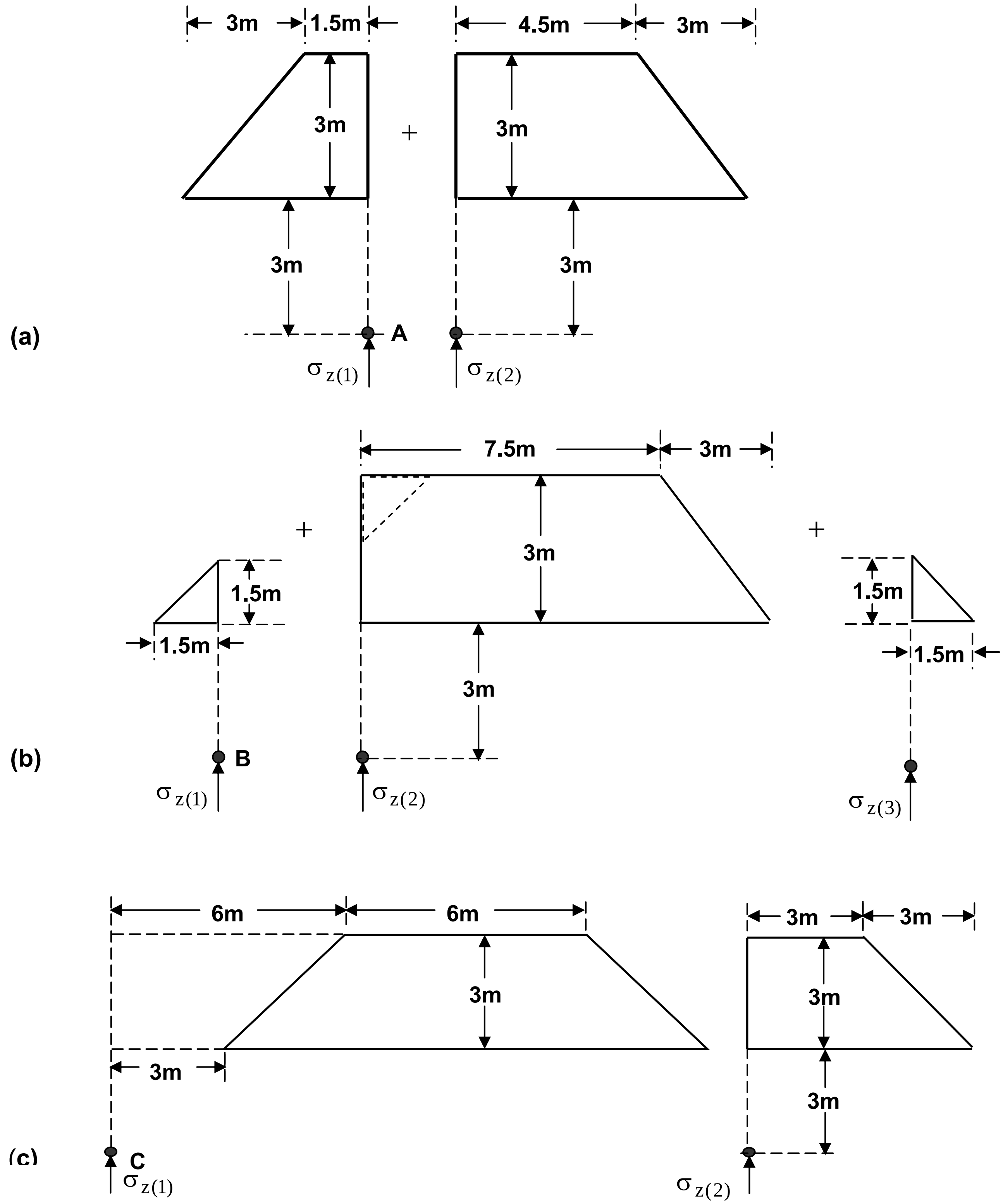


Figure (4.17): Solution of Example (4.9).

(9) ANY SHAPE LOADED AREA (NEWMARK CHART):

The stress on an elemental area dA of soil due to surface contact pressure q_o is calculated as:

$$dq = \frac{3q_o}{2\pi.z^2}\frac{1}{[1+(r/z)^2]^{5/2}}dA$$

but $dA = 2\pi.r.dr$ $\quad\therefore\quad q = \int_0^r \frac{3q_o}{2\pi.z^2}\frac{2\pi.r.dr}{[1+(r/z)^2]^{5/2}}$

or $\quad q = q_o\left\{1 - \frac{1}{[1+(r/z)^2]^{3/2}}\right\}$

$$(r/z) = \sqrt{(1-q/q_o)^{-2/3} - 1} \quad \text{..}\mathbf{(4.9)}$$

Prepare a chart on transparent paper with r_i circles as follows with $18°$ sectors:

q/q_o	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
(r/z)	0.270	0.400	0.518	0.637	0.766	0.918	1.110	1.387	1.908	∞

In this case, each circle of the chart is subdivided into 20 units, therefore the number of units for 10 circles = (20 units x 10 circles) = 200 and the influence value (I_V =1/200 = 0.005). If the scale distance (AB) is assumed = 5 cm, then:

r_i (cm)	1.35	2	2.59	3.18	3.83	4.59	5.55	6.94	9.54	∞

To estimate σ_z:

(1) Adopt a scale such that, the scale distance (AB) is equal to the required depth (z),

(2) Based on the scale adopted in (1), replot the plan of the loaded area,

(3) Place the plan plotted in (2) on the Newmark chart in such a way that the point (P) at which the vertical stress is required,

(4) Count the number of blocks, N of the chart which fall inside the plan, and

(5) calculate σ_z as:

$$\sigma_z = q.(I_V).(N) \quad \text{..}\mathbf{(4.10)}$$

where, I_v = Influence value of the chart **(see Figure 4.18).**

- ## Important Notes about Newmark Chart

a. If the stress is required at different depth, then the plan is drawn again to a different scale such that the new depth z is equal to the distance (AB) on the chart.

b. The use of Newmark's chart is based on a factor termed the influence value, determined from the number of units into which the chart is subdivided. For example; **Fig.(4.18)** is subdivided into 200 units (20 units x 10 circles), therefore the influence value is (1/200 = 0.005). But if the series of rings are subdivided into 400 units, then, $I_V = 1/400 = 0.0025$.

c. In making a chart, it is necessary that the sum of units between two concentric circles multiplied by I_V be equal to the change in q/q_o of the two rings. (i.e., if the change in two rings is 0.1 q/q_o, then I_V x number of units should equal to 0.1).

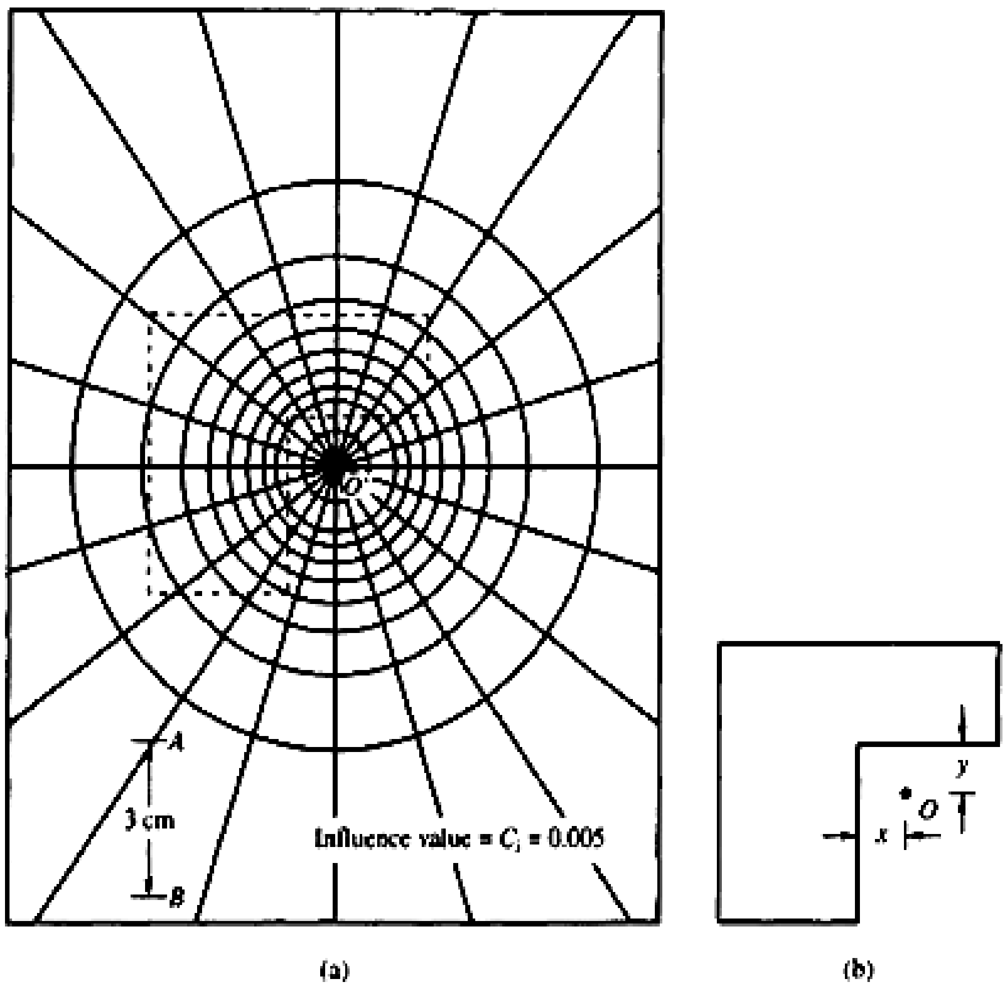


Figure (4.18): Influence chart for computation of vertical pressure *(after Newmark, 1942).*

CHAPTER 5

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SETTLEMENT OF BUILDINGS

5.1 TYPES OF SETTLEMENT

Settlement is a term that describes the vertical displacement of a structure, footing, road or embankment due to the downward movement of a point.

From structural point of view, settlement of structures may be of two types:

- **Equal or uniform settlement:** This type has no serious implication on the structure or civil engineering performance of the building. But it should have a maximum limit to prevent the failure of soil under the structure.

- **Differential settlement:** It means that one point of the structure settles more or less than the others, therefore, it may lead to damage of the superstructure. Usually it occurs due to one or more of the following:

1. Variation of soil stratum (the subsoil is not homogeneous).
2. Variation in loading condition.
3. Large loaded area on flexible footing.
4. Differential difference in time of construction, and
5. Ground condition, such as slopes.

5.2 TILTING OF FOUNDATIONS

The limiting values of foundation tilting are presented in Table (5.1) and can be calculated as:

$$\tan_{\omega_L} = \frac{M_L}{L^2\,B}\,\frac{1-\mu_s^2}{E_s}\,I_m \quad \text{...(5.1a)}$$

$$\tan_{\omega_B} = \frac{M_B}{B^2\,L}\,\frac{1-\mu_s^2}{E_s}\,I_m \quad \text{...(5.1b)}$$

where,

M_L = moment in L - direction = Q. e_L
M_B = moment in B - direction = Q. e_B
ω_L and ω_B = tilting angles in L and B directions, respectively, and
I_m = moment factor that depends on the footing size as given in Table (5.2).

Table (5.1): Effect of foundation tilting on structures.

ω (in radians)	Result to structure
1/150	Major damage
1/250	Tilting becomes visible
1/300	First cracks appear
1/500	No cracks (safe limit)

Table (5.2): Values of I_m for various footing shapes.

Footing type		I_m
Circular		6.00
Rectangular with L/B =	1.00 (Square)	3.70
	1.50	5.12
	1.25	5.00
	2.00	5.38
	2.50	5.71
	5.00	5.82
	10.0	5.93
	∞ (Strip)	5.10

5.3 LIMITING VALUES OF SETTLEMENT PARAMETERS

Many investigators and building codes recommended the allowable values for the various parameters of total and differential settlements as presented in Tables (5.3 - 5.6).

Table (5.3): Limiting values of maximum total settlement, maximum differential settlement, and maximum angular distortion for building purposes
(Skempton and MacDonald, 1956).

Settlement parameter	Settlement (mm)		
	Sand		Clay
	Ref.1	Ref.2	Rf.2
Maximum total settlement, $S_{T(max.)}$	20	32	45
Maximum differential settlement, $\Delta S_{T(max.)}$ • Isolated foundations. • Raft foundations.	 25 50	 51 51-76	 76 76 - 127
Maximum angular distortion, $\beta_{max.}$	1/300		

Ref. 1 - Terzaghi and Peck (1948), Ref. 2 - Skempton and MacDonald (1956) sampling disturbance. Therefore, some correlations which relate C_c with soil composition parameter have been published and two of them are as follows:

$$C_c = 0.009(LL - 10) \text{ .... (\textbf{Terzaghi and Peck}, 1948)}$$

$$C_c \approx 0.5\rho_s \frac{PI}{100} \text{ .... (\textbf{Wroth}, 1979)}$$

where, LL = liquid limit, PI = plasticity index, and ρ_s = particle density.

Method (A):

1. Calculate the effective pressure σ'_o at center of the clay layer before the application of load.

2. Calculate the ***weighted average pressure increase at mid of clay layer using Simpson's rule***:

$$\Delta\sigma_{avg.} = \frac{1}{6}(\Delta\sigma_t + 4\Delta\sigma_m + \Delta\sigma_b)$$

where, $\Delta\sigma_t$, $\Delta\sigma_m$, and $\Delta\sigma_b$ are respectively the pressure increase due to applied load at the top, middle and bottom of clay layer.

3. Using σ'_o and $\Delta\sigma_{avg.}$ calculated above, obtain Δe from equations below, whichever is applicable.

(i) If $\sigma'_p < \sigma'_o$, the soil is **under consolidated**:

$$\Delta e = C_c \log_{10} \frac{\sigma'_o + \Delta\sigma_{avg.}}{\sigma'_p} \text{ ....} \textbf{(5.8a)}$$

(ii) If $\sigma'_p = \sigma'_o$ (OCR = 1), the soil is **normally consolidated**:

$$\Delta e = C_c \log_{10} \frac{\sigma'_o + \Delta\sigma_{avg.}}{\sigma'_o} \text{ ....} \textbf{(5.8b)}$$

(iii) If $\sigma'_p > \sigma'_o$ (OCR > 1), the soil is **overconsolidated,** and

(a) If $\sigma'_p \geq \sigma'_o + \Delta\sigma_{avg.}$ then;

$$\Delta e = C_s \log_{10} \frac{\sigma'_o + \Delta\sigma_{avg.}}{\sigma'_o} \text{ ....} \textbf{(5.8c)}$$

(b) If $\sigma'_p < \sigma'_o + \Delta\sigma_{avg.}$ then;

$$\Delta e = C_c \log_{10} \frac{\sigma'_o + \Delta\sigma_{avg.}}{\sigma'_p} + C_s \log_{10} \frac{\sigma'_p}{\sigma'_o} \text{ ....} \textbf{(5.8d)}$$

5. Calculate the consolidation settlement by:

$$S_c = \frac{\Delta e}{1 + e_o} H_t \text{ ....} \textbf{(5.8e)}$$

where, $e_o = \omega_o . G_s$

Method (B):

Table (5.6): Recommendation of European Committee for Standardization (1994) **on differential settlement parameters**.

Item	*Parameter*	*Magnitude*	*Comments*
Limiting values for serviceability	S_T	*25 mm* *50 mm*	*Isolated shallow foundation* *Raft foundation*
	ΔS_T	*5 mm* *10 mm* *20 mm*	*Frames with rigid cladding* *Frames with flexible cladding* *Open frames*
	β	*1/500*	-----------
Maximum acceptable foundation movement	S_T	*50mm*	*Isolated shallow foundation*
	ΔS_T	*20mm*	*Isolated shallow foundation*
	β	$\approx$ *1/500*	-----------

5.4 COMPONENTS OF TOTAL SETTLEMENT

Foundation settlement mainly consists of three components (see **Fig. (5.1)**):

(i) **Immediate settlement (S_i):** occurs due to elastic deformation of soil particles upon load application with no change in water content.

(ii) **Primary consolidation settlement (S_C):** occurs as the result of volume change in saturated fine grained soils due to expulsion of water from the void spaces of the soil mass with time.

(iii) **Secondary consolidation settlement (S_{sc}):** occurs after the completion of the primary consolidation due to plastic deformation of soil (reorientation of the soil particles). It forms the major part of settlement in highly organic soils and peats.

$$\therefore \quad S_T = S_i + S_c + S_{sc} \quad \textbf{(5.2)}$$

These components occur in different types of soils with varying circumstances:

- **For clay:** $S_T = S_i$ (**minimum**) + S_c (**major**) + S_{sc} (**small, but present to certain extent**)
  Therefore, for clay these settlements must be calculated.
- **For sand:** $S_T = S_i$ (**major**) + S_c (**present but mixed with S_i**) + S_{sc} (**undefined**)
  Since sand is permeable, therefore, Terzaghi theory cannot be applicable.

$$S_{i(flexible)} = q_o . B' \frac{1-\mu_s^2}{E_s} I_s . I_D . C_N \quad \text{(5.3)}$$

$$S_{i(rigid)} \approx 0.93 . S_{i(flexible)} \quad \text{(5.4)}$$

$$S_{i(average)} = 0.85 . S_{i(center)} \quad \text{(5.5)}$$

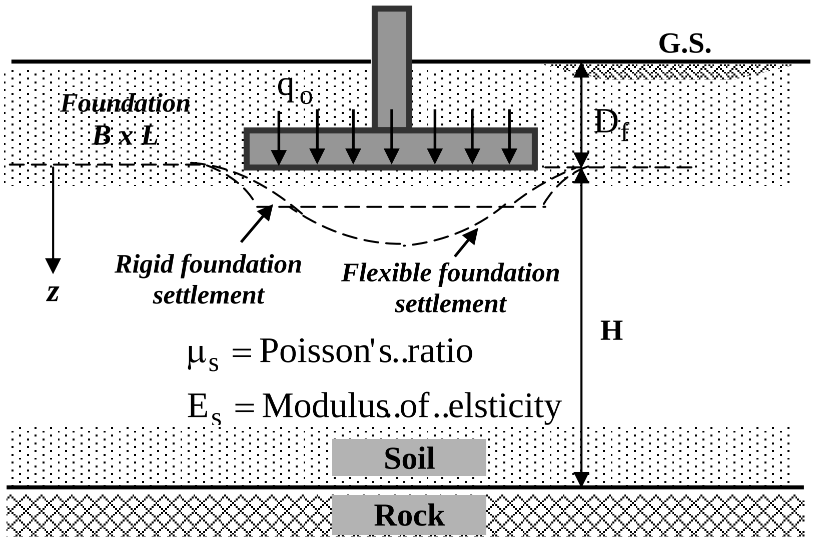


Fig.(5.2): Elastic settlement of flexible and rigid foundations.

where,

S_i = immediate or elastic,

q_o = net applied pressure on the foundation,

B' = **B/2** *for center* of foundation, and

= **B** *for corners* of foundation,

μ_s = Poisson's ratio of soil, ***(see Table (5.7) for typical values).***

E_s = weighted average modulus of elasticity of the soil over a depth of H. For a multi-layered soil stratum it is computed as:

$$E_{s(avg.)} = \frac{\sum Es_{(i)} . H_i}{\sum H_i}$$

in which, H_i and E_i are the thickness and modulus of elasticity of layer **i**, and $\sum H_i$ = **H** (the depth of hard stratum) or **5B** whichever is smaller,

(see Table (5.8) for typical values of E_s***).***

I_s = Shape factor (Steinbrenner, 1934) computed by:

$$I_s = I_1 + \frac{1-2\mu_s}{1-\mu_s} I_2$$

Table (5.9a): Values of I_1 to compute Steinbrenner's influence factor

$$I_s = I_1 + \frac{1-2\mu_s}{1-\mu_s} I_2 \ .$$

H/B'	L/B										
	1.0	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**	**1.8**	**1.9**	**2**
0.2	0.009	0.008	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007
0.4	0.033	0.032	0.031	0.030	0.029	0.028	0.028	0.027	0.027	0.027	0.027
0.6	0.066	0.064	0.063	0.061	0.060	0.059	0.058	0.057	0.056	0.056	0.055
0.8	0.104	0.102	0.100	0.098	0.096	0.095	0.093	0.092	0.091	0.090	0.089
1.0	0.142	0.140	0.138	0.136	0.134	0.132	0.130	0.129	0.127	0.126	0.125
1.5	0.224	0.224	0.224	0.223	0.222	0.220	0.219	0.217	0.216	0.214	0.213
2	0.285	0.288	0.290	0.292	0.292	0.292	0.292	0.292	0.291	0.290	0.289
3	0.363	0.372	0.378	0.384	0.389	0.393	0.396	0.398	0.400	0.401	0.402
4	0.408	0.421	0.431	0.440	0.448	0.455	0.460	0.465	0.469	0.473	0.476
5	0.437	0.452	0.465	0.477	0.487	0.496	0.503	0.510	0.516	0.522	0.526
6	0.457	0.473	0.488	0.501	0.513	0.524	0.533	0.542	0.549	0.556	0.562
7	0.471	0.489	0.506	0.520	0.533	0.545	0.556	0.566	0.575	0.583	0.590
8	0.482	0.502	0.519	0.534	0.549	0.561	0.573	0.584	0.594	0.602	0.611
9	0.491	0.511	0.529	0.545	0.560	0.574	0.587	0.598	0.609	0.618	0.627
10	0.498	0.519	0.537	0.554	0.570	0.584	0.597	0.610	0.621	0.631	0.641
20	0.529	0.553	0.575	0.595	0.614	0.631	0.647	0.662	0.677	0.690	0.702
500	0.560	0.586	0.612	0.635	0.656	0.677	0.696	0.714	0.731	0.748	0.763

H/B'	L/B										
	2.5	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**25**	**50**	**100**
0.2	0.007	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006
0.4	0.026	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024
0.6	0.053	0.051	0.050	0.050	0.050	0.049	0.049	0.049	0.049	0.049	0.049
0.8	0.086	0.082	0.081	0.080	0.080	0.080	0.093	0.092	0.091	0.090	0.089
1.0	0.121	0.115	0.113	0.112	0.112	0.112	0.111	0.111	0.110	0.110	0.110
1.5	0.207	0.197	0.194	0.192	0.191	0.190	0.190	0.189	0.188	0.188	0.188
2	0.284	0.271	0.267	0.264	0.262	0.261	0.260	0.259	0.257	0.256	0.256
3	0.402	0.392	0.386	0.382	0.378	0.376	0.374	0.373	0.378	0.367	0.367
4	0.484	0.484	0.479	0.474	0.470	0.440	0.464	0.462	0.453	0.451	0.451
5	0.543	0.554	0.552	0.548	0.543	0.540	0.536	0.534	0.522	0.522	0.519
6	0.585	0.609	0.610	0.608	0.604	0.601	0.598	0.595	0.579	0.576	0.575
7	0.618	0.653	0.658	0.658	0.656	0.653	0.650	0.647	0.628	0.624	0.623
8	0.643	0.688	0.697	0.700	0.700	0.698	0.695	0.692	0.672	0.666	0.665
9	0.663	0.716	0.730	0.736	0.737	0.736	0.735	0.732	0.710	0.704	0.702
10	0.679	0.740	0.758	0.766	0.770	0.770	0.770	0.768	0.745	0.738	0.735
20	0.756	0.856	0.896	0.925	0.945	0.959	0.969	0.977	0.982	0.965	0.957
500	0.832	0.977	1.046	1.102	1.150	1.191	1.227	1.259	1.532	1.721	1.879

B' = **B/2** *for center of foundation, and* = **B** *for corners of foundation,*
H = *depth of hard stratum (rock) under the footing.*

Table (5.9b): Values of I_2 to compute Steinbrenner's influence factor

$$I_s = I_1 + \frac{1-2\mu_s}{1-\mu_s} I_2 .$$

H/B'	L/B										
	1.0	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**	**1.8**	**1.9**	**2**
0.2	0.041	0.042	0.042	0.042	0.042	0.042	0.043	0.043	0.043	0.043	0.043
0.4	0.066	0.068	0.069	0.070	0.070	0.071	0.071	0.072	0.072	0.073	0.073
0.6	0.079	0.081	0.083	0.085	0.087	0.088	0.089	0.090	0.091	0.091	0.092
0.8	0.083	0.087	0.090	0.093	0.095	0.097	0.098	0.100	0.101	0.102	0.103
1.0	0.083	0.088	0.091	0.095	0.098	0.100	0.102	0.104	0.106	0.108	0.109
1.5	0.075	0.080	0.084	0.089	0.093	0.096	0.099	0.102	0.105	0.108	0.110
2	0.064	0.069	0.074	0.078	0.083	0.086	0.090	0.094	0.097	0.100	0.102
3	0.048	0.052	0.056	0.060	0.064	0.068	0.071	0.075	0.078	0.081	0.084
4	0.037	0.041	0.044	0.048	0.051	0.054	0.057	0.060	0.063	0.066	0.069
5	0.031	0.034	0.036	0.039	0.042	0.045	0.048	0.050	0.053	0.055	0.058
6	0.026	0.028	0.031	0.033	0.036	0.038	0.040	0.043	0.045	0.047	0.050
7	0.022	0.024	0.027	0.029	0.031	0.033	0.035	0.037	0.039	0.041	0.043
8	0.020	0.022	0.023	0.025	0.027	0.029	0.031	0.033	0.035	0.036	0.038
9	0.017	0.019	0.021	0.023	0.024	0.026	0.028	0.029	0.031	0.033	0.034
10	0.016	0.017	0.019	0.020	0.022	0.023	0.025	0.027	0.028	0.030	0.031
20	0.008	0.099	0.010	0.010	0.011	0.012	0.013	0.013	0.014	0.015	0.016
500	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001

H/B'	L/B										
	2.5	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**25**	**50**	**100**
0.2	0.043	0.044	0.044	0.044	0.044	0.044	0.044	0.044	0.044	0.044	0.044
0.4	0.074	0.075	0.075	0.075	0.076	0.076	0.076	0.076	0.076	0.076	0.076
0.6	0.094	0.097	0.097	0.098	0.098	0.098	0.098	0.098	0.098	0.098	0.098
0.8	0.107	0.111	0.112	0.113	0.113	0.113	0.113	0.114	0.114	0.114	0.114
1.0	0.114	0.120	0.122	0.123	0.123	0.124	0.124	0.124	0.125	0.125	0.125
1.5	0.118	0.130	0.134	0.136	0.137	0.138	0.138	0.139	0.140	0.140	0.140
2	0.114	0.131	0.136	0.139	0.141	0.143	0.144	0.145	0.147	0.147	0.148
3	0.097	0.122	0.131	0.137	0.141	0.144	0.145	0.147	0.152	0.153	0.154
4	0.082	0.110	0.121	0.129	0.135	0.139	0.142	0.145	0.154	0.155	0.156
5	0.070	0.098	0.111	0.120	0.128	0.133	0.137	0.140	0.154	0.156	0.157
6	0.060	0.087	0.101	0.111	0.120	0.126	0.131	0.135	0.153	0.157	0.157
7	0.053	0.078	0.092	0.103	0.112	0.119	0.125	0.129	0.152	0.157	0.158
8	0.047	0.071	0.084	0.095	0.104	0.112	0.118	0.124	0.151	0.156	0.158
9	0.042	0.064	0.077	0.088	0.097	0.105	0.112	0.118	0.149	0.156	0.158
10	0.038	0.059	0.071	0.082	0.091	0.099	0.106	0.112	0.147	0.156	0.158
20	0.020	0.031	0.039	0.046	0.053	0.059	0.065	0.071	0.124	0.148	0.156
500	0.001	0.001	0.002	0.002	0.002	0.003	0.003	0.003	0.008	0.016	0.031

B' = **B/2** *for center of foundation, and* = **B** *for corners of foundation,*
H = *depth of hard stratum (rock) under the footing.*

5.5.2 Schmertmann's Method (1978): Use of Strain Influence Factor

This method is based on the Dutch cone penetration resistance q_c using the strain influence factor diagram. It is proposed for two cases, square foundation (L/B = 1) where axisymmetric stress and strain conditions occur and strip foundation (L/B = 10) where plane strain conditions exist.

For square foundation: $$S_i = \frac{C_1 C_2}{2.5} \Delta p \sum_0^{2B} \frac{I_z \Delta z}{q_c} \quad \text{...} \mathbf{(5.6a)}$$

For strip foundation: $$S_i = \frac{C_1 C_2}{3.5} \Delta p \sum_0^{4B} \frac{I_z \Delta z}{q_c} \quad \text{...} \mathbf{(5.6b)}$$

where, P = gross applied pressure,

P'_o = effective stress at the foundation level,

ΔP = net applied pressure = $P - P'_o$ (in kN/m^2),

q_c = cone end resistance, kN/m^2, for each soil layer,

Δz = thickness for each soil layer, (in meters),

C_1 = correction for depth of foundation $= 1 - 0.5\dfrac{P'_o}{\Delta p} \geq 0.5$

C_2 = correction for creep or time related settlement $= 1 + 0.2\log_{10}\dfrac{t}{0.1}$

t = time in (years) after construction,

I_z = average strain influence factor for each soil layer obtained as the value at the mid−point of each soil layer from a diagram drawn alongside the q_c − depth graph with a depth of 2B for square foundation and 4B for strip foundation as shown in **Fig.(5.3)**, and

$I_{z\,max} = 0.5 + 0.1\sqrt{\dfrac{\Delta p}{\sigma v'}}$ is the maximum value of I_z, where σ'_v = vertical effective stress at a depth of B/2 for a square foundation and B for strip foundation.

Notes:

- *Values of Δz, average q_c and average I_z for each soil layer are required for the summation term.*
- *Settlements for shapes intermediate between square and strip can be obtained by interpolation.*

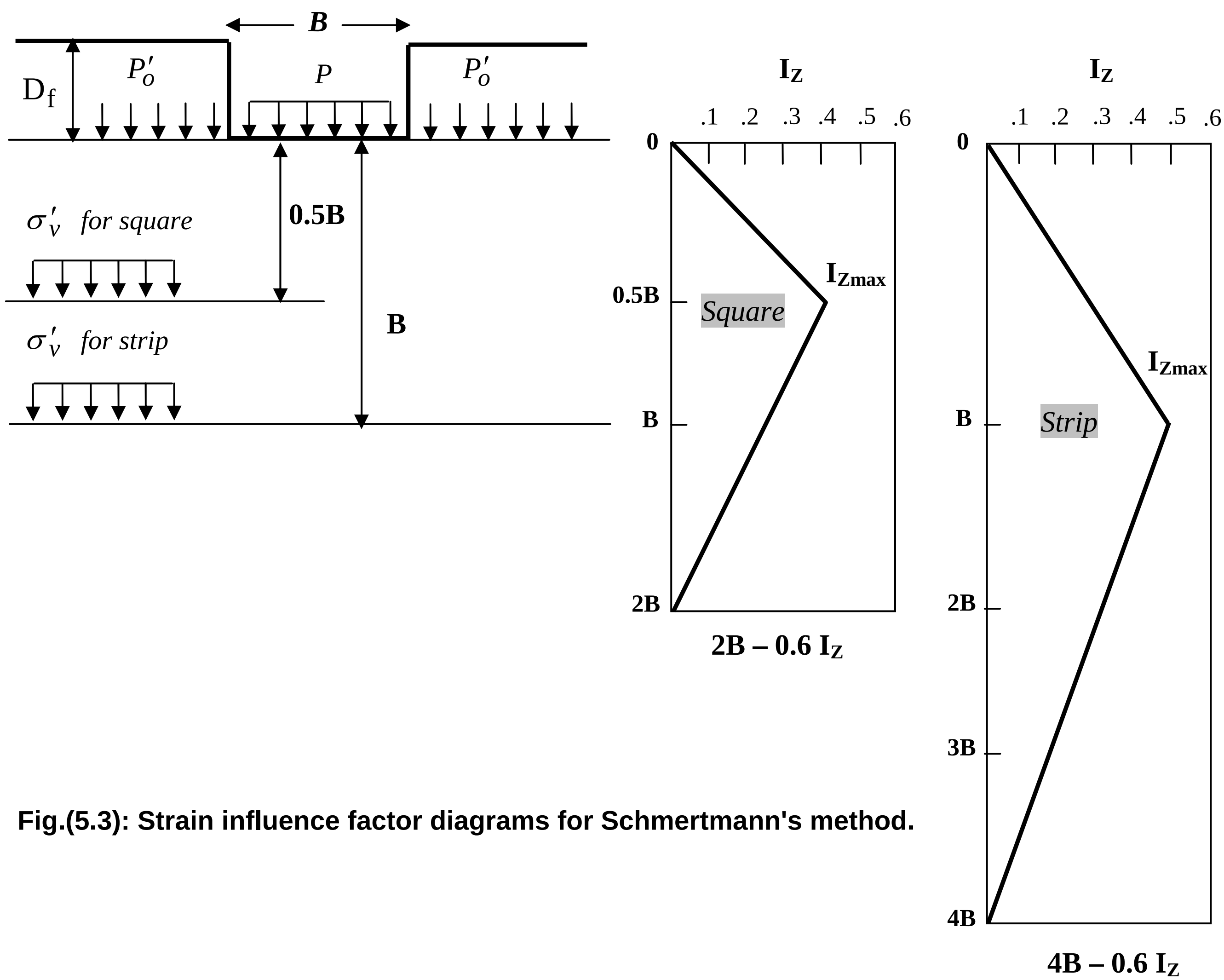


Fig.(5.3): Strain influence factor diagrams for Schmertmann's method.

5.5.3 Bjerrum's Method for Average Settlement of Layered Clay Soil

$$S_{i(average)flexible} = \mu_o . \mu_1 \frac{q.B}{E_u} \quad \text{……………………………………..(5.7)}$$

where, μ_o and μ_1 are factors for depth of embedment and thickness of soil layer beneath the foundation, respectively; obtained from **Fig.(5.4)**. Remember that the principle of layering could be applied with this method such that the overlapping is equal to the number of layers – 1.

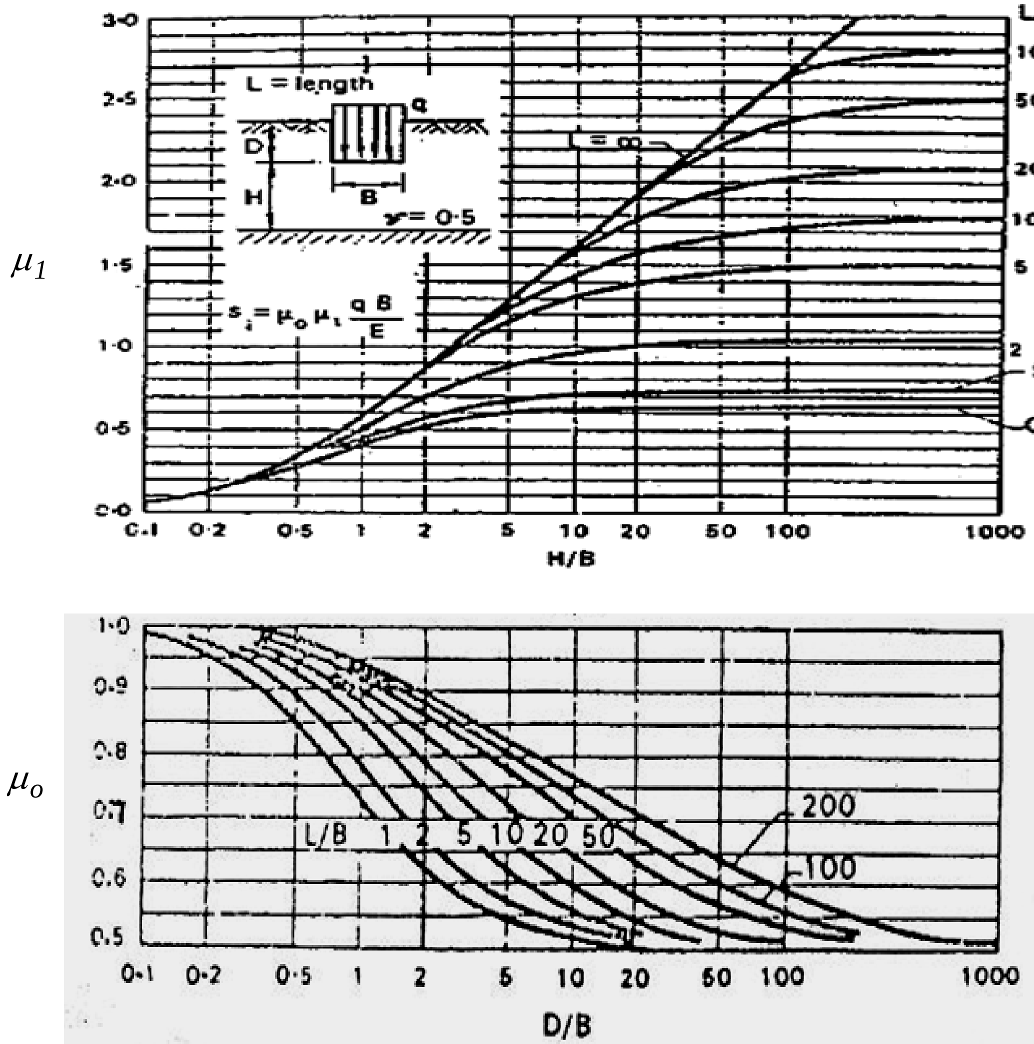
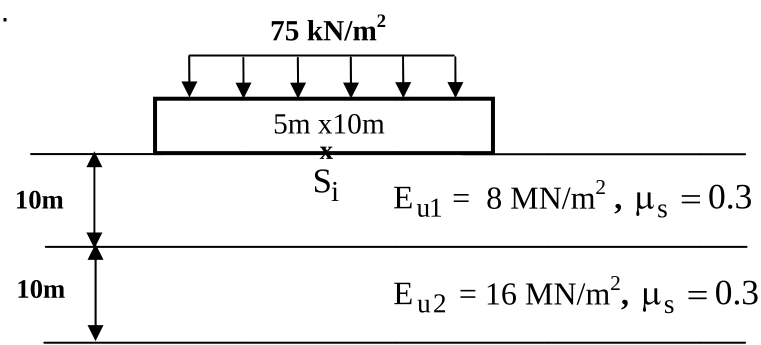


Fig.(5.4): Coefficients of vertical displacement for foundations on saturated clays
(after Janbu et al., 1956).

Problem (5.1): A (5m x 10m) rectangular flexible foundation is placed on two layers of clay, both 10m thick as shown in the figure below. The modulus of elasticity of the upper layer is 8 MN/m^2 and that of the lower layer is 16 MN/m^2. Determine the immediate settlement at the center of the foundation using:

(1) Elastic Theory Method.
(2) Bjerrum Method.



Solution:

(1) (Elastic Theory Method):

$$E_{avg.} = \frac{8(10)+16(10)}{20} = 12.MN/m^2 = 12000.kN/m^2$$

$$S_{i(flexible)} = q_o.B'\frac{1-\mu_s^{\ 2}}{E_s}I_s.I_D.C_N \quad \text{.......................................(5.3)}$$

For H/B' =20/2.5 = 8, L/B = 10/5 = 2**:** $I_1 = 0.611$ and $I_2 = 0.038$; *from **Table (4.9)***

$$I_s = I_1 + \frac{1-2\mu_s}{1-\mu_s}I_2 = 0.611 + \frac{1-2(0.3)}{1-0.3}0.038 = 0.633$$

I_D = 1 *(for* $D_f = 0$)**;** and C_N = 4 (*for center)*.

$$S_{i(flexible)_{(surface,center)}} = (75)(2.5)\frac{1-(0.3)^2}{12000}(0.633)(1)(4) = \underline{\textbf{36 mm.}}$$

(2) (Bjerrum Method):

- **Settlement of 1st. layer** (average settlement)**:**

From **Fig.(5.4):** for D_f/B = 0 and L/B = 2; μ_o = 1.00

For H/B = 10/5 = 2 and L/B = 2; μ_1 = 0.70

$$S_{i(average)flexible} = \mu_o.\mu_1\frac{q.B}{E_u} \quad \text{.......................................(5.7)}$$

$$S_{1(average)flexible} = (1.00)(0.70)\frac{(75)(5)(1000)}{(8x1000)} = 32.81 \text{ mm}$$

- **Settlement of 2nd. layer** (average settlement)**:**

From **Fig.(5.4):** for D_f/B = 0 and L/B = 2; μ_o = 1.00

For H/B = 20/5 = 4 and L/B = 2; μ_1 = 0.85

$$S_{2(average)flexible} = (1.00)(0.85)\frac{(75)(5)(1000)}{(16x1000)} = 19.92 \text{ mm}$$

- **The interaction between the 1st. and 2nd. Layers:**

$$S_{3(average)flexible} = (1.00)(0.70)\frac{(75)(5)(1000)}{(16x1000)} = 16.41 \text{ mm}$$

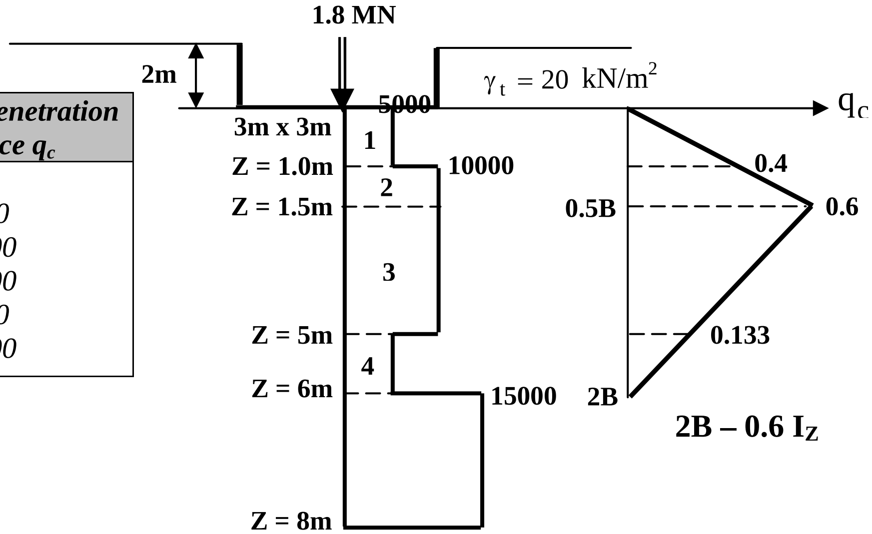
The immediate settlement at foundation center = $S_1 + S_2 - S_3$

= 32.81 + 19.92 – 16.41 = **36.32 mm**

Problem (5.2): (Schmertmann's method–settlement on sand)

A (3m x 3m) square footing rested at a depth of (2m) below the ground surface. Estimate the immediate settlement of the footing under the load and soil conditions shown in the figure below after (0.1 year) from construction.

Depth from base (m)	*Static cone penetration resistance q_c*
0-1.0	*5000*
1.0-1.5	*10000*
1.5-5.0	*10000*
5.0-6.0	*5000*
6.0-8.0	*15000*



Solution:

For square foundation:

$$S_i = \frac{C_1 C_2}{2.5}\Delta p \sum_0^{2B} \frac{I_z \Delta z}{q_c} \quad \textbf{(5.6a)}$$

- C_1 = correction for depth of foundation $= 1 - 0.5\dfrac{P'_o}{\Delta p} \geq 0.5$

  P'_o = effective stress at the foundation level = $D_f . \gamma = 2(20) = 40$ kN/m^2

  ΔP = net increase in stress at footing level = $P - P'_o = \dfrac{1.8x10^3}{3x3} - 40 = 160$ kN/m^2

  $C_1 = 1 - 0.5\dfrac{40}{160} = 0.875 > 0.5$ (O.K.)

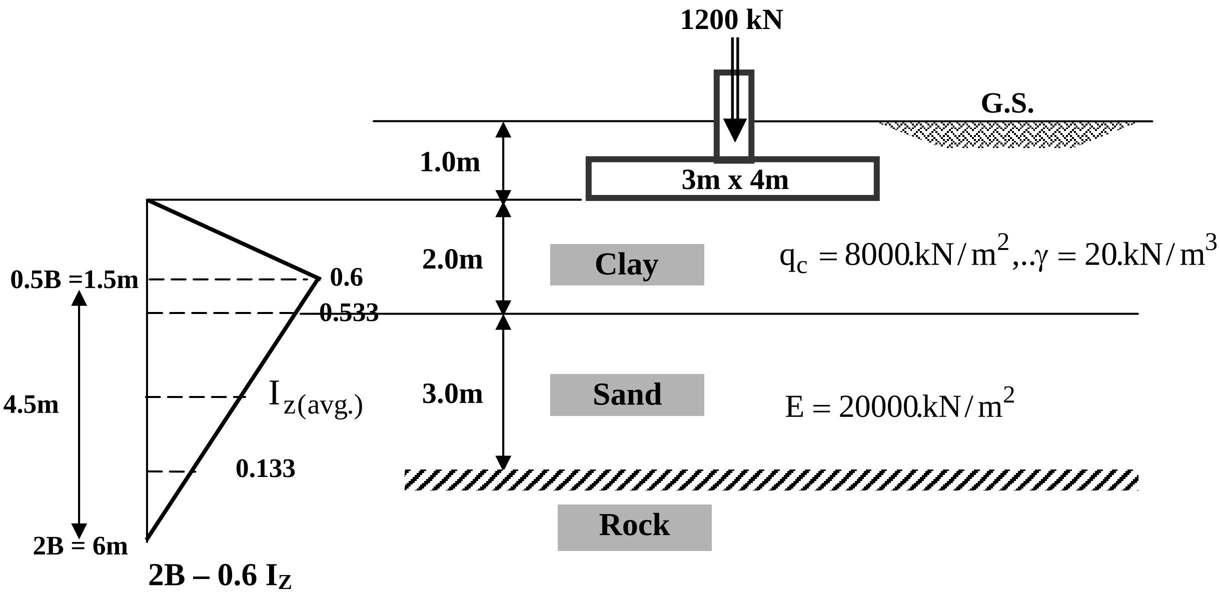
- C_2 = Time correction factor $= 1 + 0.2\log_{10}\dfrac{t}{0.1} = 1 + 0.2\log_{10}\dfrac{0.1}{0.1} = 1.0$

No.	Δ_Z (m)	q_c	I_Z (average)	$\dfrac{\Delta_Z . I_Z}{q_c}$
1	1.0	5000	(0 + 0.4)/2= 0.2	0.000040
2	0.5	10000	0.5	0.000025
3	3.5	10000	0.366	0.000128
4	1.0	5000	0.066	0.0000132
				$\sum 20.62x10^{-5}$

$$S_i = \frac{(0.875)(1.0)}{2.5}(160)(20.62x10^{-5}) = 0.01155 \text{ m} = \mathbf{11.55\ mm}$$

Problem (5.3): (Total immediate settlement)

Determine the total immediate settlement of the rectangular footing shown in figure below after 2 months.



Solution:

Since the soil profile is made up of two different soils, then the total immediate settlement will be:

$$S_{i(Total)} = S_{i(clay)} + S_{i(sand)}$$

- **Immediate Settlement of clay by Bjerrum's method:**

$$S_{i(average)flexible} = \mu_o \cdot \mu_1 \frac{q.B}{E_u} \quad \text{..........(5.7)}$$

From **Fig.(5.4):** for D_f /B = 1/3 = 0.33 and L/B = 4/3 = 1.33; μ_o = 0.93

for H/B = 2/3 = 0.66 and L/B = 1.33; μ_1 = 0.38

$$S_{1(average)flexible} = (0.93)(0.38)\frac{(1200/3x4)(3)(1000)}{(2x8x1000)} = \underline{\mathbf{6.6\ mm}}$$

- **Immediate Settlement of sand by Schmertmann's method:**

For square foundation:

$$S_i = \frac{C_1 C_2}{2.5} \Delta p \sum_0^{2B} \frac{I_z \Delta z}{q_c} \quad \text{..........(5.6a)}$$

$$C_1 = 1 - 0.5\frac{P'_o}{\Delta p} \geq 0.5$$

At foundation level:

$$P'_o = D_f \cdot \gamma = 1(20) = 20 \text{ kN/m}^2, \quad \Delta P = P/A - P'_o = \frac{1200}{3x4} - 20 = 80 \text{ kN/m}^2.$$

On sand surface:

$P'_o = D_f.\gamma = 3(20) = 60\ kN/m^2$, $\quad \Delta P = \dfrac{(80)(3)(4)}{(3+2)(4+2)} = 32\ kN/m^2$ *(2:1 method)*

$C_1 = 1 - 0.5\dfrac{60}{32} = 0.06 < 0.5 \quad \therefore \quad$ Use $C_1 = 0.5$

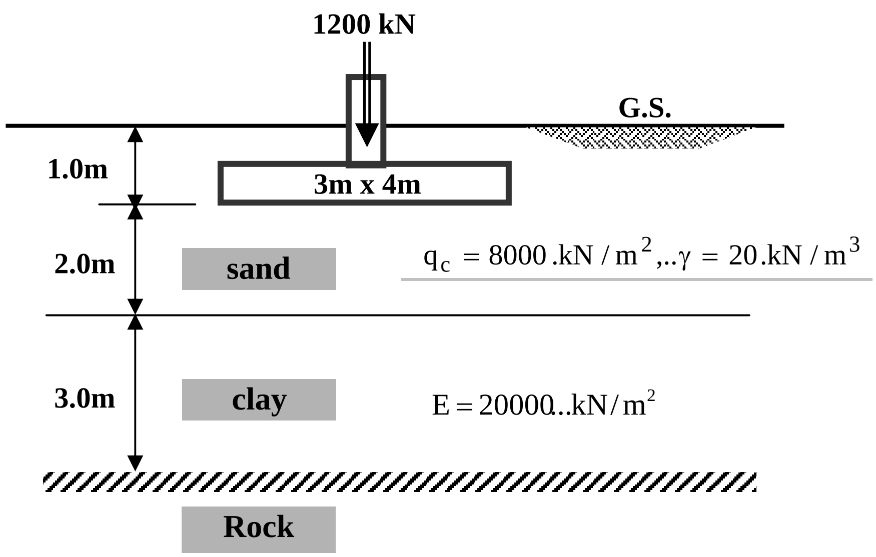
$C_2 = 1 + 0.2\log_{10}\dfrac{t}{0.1} = 1 + 0.2\log_{10}\dfrac{2/12}{0.1} = 1.04$

$I_{z\,(avg.)} = \dfrac{0.533 + 0.133}{2} = 0.333$, $\quad \dfrac{I_z.\Delta z}{E} = \dfrac{(0.333)(3)}{20000} = 4.9.x.10^{-5}$

$S_{i\,(sand)} = (0.5)(1.04)(32)(4.9.x.10^{-5}) =$ **<u>0.815 mm</u>**

$\therefore \quad S_{i\,(Total)} = 6.6 + 0.815 =$ **<u>7.415 mm</u>**

<u>Home work:</u> ***Redo problem (5.3) but with sand instead of clay as shown in the figure below.***
*(Ans.: $S_{i\,(Total)}$ = **5.75 mm**).*



5.6 PRIMARY CONSOLIDATION SETTLEMENT

5.6.1 Compression Index C_c Method:

This method is adopted for normally and lightly overconsolidated clays. The compression index C_c is the gradient of $e - log\ P$ plot for normally consolidated clay. While for overconsolidated clay, C_c is also the slope of the $e - log\ P$ but beyond the preconsolidation pressure P'_c. C_c values obtained from oedometer tests are likely to be underestimated due to

1. For thick clay layer, better results in settlement calculation can be obtained by dividing a given clay layer into (n) sub-layers.
2. Calculate the effective stress $\sigma'_{o(i)}$ at the middle of each clay sub-layer.
3. Calculate the increase of stress at the middle of each sub-layer $\Delta\sigma_{(i)}$ due to the applied load.
5. Calculate $\Delta e_{(i)}$ for each sub-layer from **Eqs.(5.8a to 5.8e)** mentioned before in method (A) −step 3, whichever is applicable.
5. Calculate the total consolidation settlement of the entire clay layer from:

$$S_c = \sum_{i=1}^{n} \Delta S_c = \sum_{i=1}^{n} \frac{\Delta e_i}{1+e_o} \Delta H_i \quad \text{where} \quad e_o = \omega_o . G_s \quad \text{.....................(5.9)}$$

Layer	Values at mid-point of each sub-layer						
	$\sigma'_{o(i)}$	$\Delta\sigma_{(i)}$	$\Delta e_{(i)}$	ω_o	e_o	ΔH_i	$\frac{\Delta e_{(i)}}{1+e_o}\Delta H_i$
1							
2							
3							

$S_c = \sum$ ▬

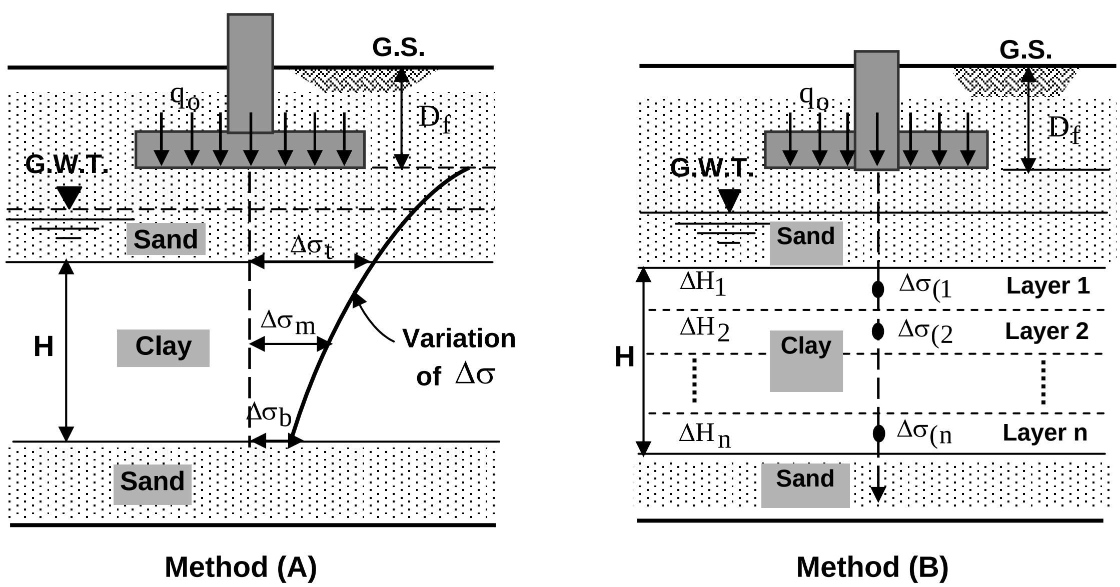


Fig.(5.5): Calculation of consolidation settlement Methods.

5.7 SKEMPTON - BJERRUM MODIFICATION FOR 3-DIMENTIONAL CONSOLIDATION

In one-dimensional consolidation tests, there is no lateral yield of the soil specimen and the ratio of minor to major principal effective stresses, K_o, remains constant. In that case, the increase of pore water pressure due to an increase of vertical stress is equal in magnitude, (i.e., $\Delta u = \Delta\sigma$) where Δu is the increase in pore water pressure and $\Delta\sigma$ is the increase of vertical stress. While for actual simulation of field condition, in 3-dimensions, any point in a clay layer due to a given load suffers from lateral yield and therefore, K_o does not remain constant.

$$\therefore \qquad S_c = \rho \; S_{c\,(Oed)} \qquad \textbf{(5.11)}$$

where, ρ = correction factor depends on pore-pressure parameter (A); obtained from **Fig.(4.6)**.

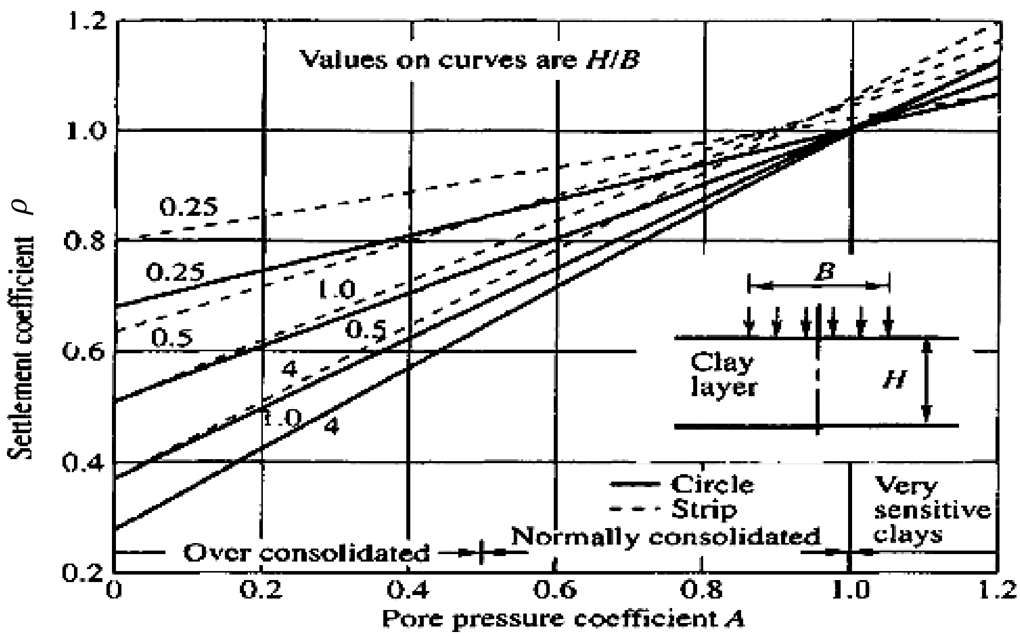
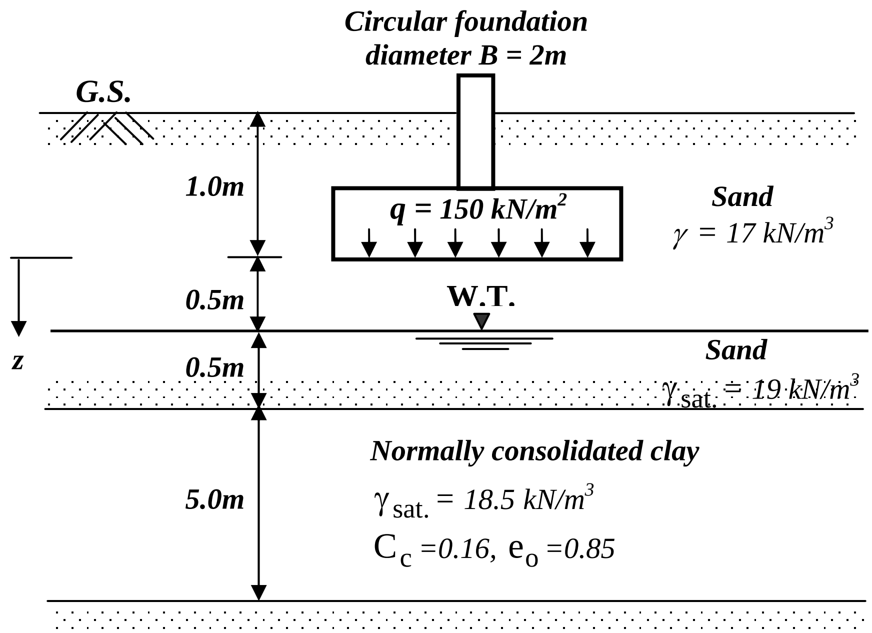


Fig.(5.6): Settlement correction factor versus pore-pressure coefficient for circular and strip footings *(after Skempton and Bjerrum, 1957).*

Problem (5.4): *(consolidation settlement$-C_c$ method)*

A circular foundation 2m in diameter is shown in the figure below. A normally consolidated clay layer 5m thick is located below the foundation. Determine the consolidation settlement of the clay.



Solution:

(1) As one layer of clay of 5m thick:

At the center of clay: $\sigma'_o = 1.5(17) + 0.5(19\text{-}9.81) + 2.5(18.5\text{-}9.81) = 51.82\ \text{kN/m}^2$

For circular loaded area, the increase of stress below the center is given by:

$$\Delta\sigma = q\left\{1 - \frac{1}{[(b/z)^2 + 1]^{3/2}}\right\}$$ where: b = the radius of the circular foundation,

At mid-depth of the clay layer: z = 3.5m; $\Delta\sigma = 150\left\{1 - \frac{1}{[(1/3.5)^2 + 1]^{3/2}}\right\} = 16.66\ \text{kN/m}^2$

$$\Delta e = C_c \log_{10}\frac{\sigma'_o + \Delta\sigma}{\sigma'_o} = 0.16.\log_{10}\frac{51.82 + 16.66}{51.82} = 0.0194$$

$$S_c = \frac{\Delta e}{1 + e_o} H_t = \frac{0.0194}{1 + 0.85}(5)(1000) = \underline{\mathbf{52.4\ mm}}$$

(2) Divide the clay layer into (5) sub-layers each of 1m thick:

- ***Calculation of effective stress at the middle of each sub-layer*** $\sigma'_{o(i)}$ **:**

For 1st. Layer: $\sigma'_{o(1)} = 1.5(17) + 0.5(19\text{-}9.81) + 0.5(18.5\text{-}9.81) = 35.44\ \text{kN/m}^2$

For 2nd. Layer: $\sigma'_{o(2)} = 35.44 + 1.0(18.5\text{-}9.81) = 35.44 + 8.69 = 43.13\ \text{kN/m}^2$

For 3rd. Layer: $\sigma'_{o(3)} = 43.13 + 8.69 = 51.81\ \text{kN/m}^2$

For 4th. Layer: $\sigma'_{o(4)} = 51.81 + 8.69 = 60.51\ \text{kN/m}^2$

For 5th. Layer: $\sigma'_{o(5)} = 60.51 + 8.69 = 69.20$ kN/m²

- ***Calculation of increase of stress below the center of each sub-layer*** $\Delta\sigma_{(i)}$ **:**

For 1st. Layer: $\Delta\sigma_{(1)} = 150\left\{1 - \frac{1}{[(1/1.5)^2+1]^{3/2}}\right\} = 63.59$ kN/m²

For 2nd. Layer: $\Delta\sigma_{(2)} = 150\left\{1 - \frac{1}{[(1/2.5)^2+1]^{3/2}}\right\} = 29.93$ kN/m²

For 3rd. Layer: $\Delta\sigma_{(3)} = 150\left\{1 - \frac{1}{[(1/3.5)^2+1]^{3/2}}\right\} = 16.66$ kN/m²

For 4th. Layer: $\Delta\sigma_{(4)} = 150\left\{1 - \frac{1}{[(1/4.5)^2+1]^{3/2}}\right\} = 10.46$ kN/m²

For 5th. Layer: $\Delta\sigma_{(5)} = 150\left\{1 - \frac{1}{[(1/5.5)^2+1]^{3/2}}\right\} = 7.14$ kN/m²

Layer no.	ΔH_i **m**	$\sigma'_{o(i)}$ **kN/m²**	$\Delta\sigma_{(i)}$ **kN/m²**	$\Delta e^*_{(i)}$	$\frac{\Delta e_{(i)}}{1+e_o}\Delta H_i$ **m**
1	1	35.44	63.59	0.0727	0.0393
2	1	43.13	29.93	0.0366	0.0198
3	1	51.82	16.66	0.0194	0.0105
4	1	60.51	10.46	0.0111	0.0060
5	1	69.20	7.14	0.00682	0.0037
					$\sum$ = 0.0793

$\Delta e^*_{(i)} = C_c \log_{10}\frac{\sigma'_{o(i)} + \Delta\sigma_{(i)}}{\sigma'_{o(i)}}$; $C_c = 0.16$, $e_o = 0.85$, $S_c = 0.0793$ m = **79.3 mm.**

(3) Weighted average pressure increase (Simpson's rule):

At the center of clay: $\sigma'_o = 1.5(17) + 0.5(19-9.81) + 2.5(18.5-9.81) = 51.82$ kN/m²

At z = 1.0m from the base of foundation: $\Delta\sigma = 150\left\{1 - \frac{1}{[(1/1)^2+1]^{3/2}}\right\} = 75$ kN/m² where, m_v = volume.change.coefficient $= \dfrac{a_v}{1+e_o}$, $a_v = \dfrac{\Delta e}{\Delta p} =$ compressibility coefficient and k = permeability of soil.

(2) Second, the time factor (T_v) is calculated from:

$$T_v = \frac{C_v.t}{(H_d)^2} \quad \text{...}\textbf{(5.14)}$$

where, H_d (drainage path) = H *(for one-way drainage)* and
= H/2 *(for two-way drainage).*

(3) Third, with (T_v) value obtained from Eq. (5.14), the degree of consolidation $U\%$ at any time (t) is calculated from **Fig.(5.8)** depending on the distribution of the excess pore water pressure; or one of the following equations:

$$T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2 \quad \text{for } U \le 60\% \text{ ....}\textbf{(5.15a)}$$

$$T_v = 1.781 - 0.933.\log_{10}(100 - U\%) \quad \text{for } U > 60\% \text{ ....}\textbf{(5.15b)}$$

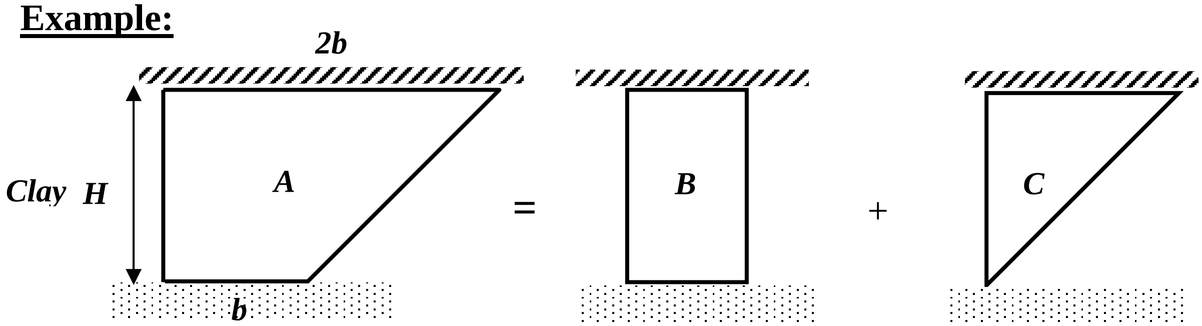
(4) From the degree of consolidation $U\%$ at any time (t), the settlement at any time is calculated from the following relation if the total settlement is known:

$$U_t = \frac{S_t}{S_\infty} = \frac{\text{Settlement.at.any.time(t)}}{\text{Total.settlement}} \text{ ....}\textbf{(5.16)}$$

where, $S_\infty = S_T = S_i + S_c + S_{sc}$.

Note: $U\%$ for any layer depends on pore water pressure distribution using Figs.(5.20a and 5.20b) to find U_t at any time. But, for other shapes use division to suit with figures above as shown in the following example.

Example:



Curve (1+3) *Curve (1)* *Curve (3)*

$$U_A = \frac{U_B.A_B + U_c.A_C}{\sum A}$$

At z = 3.5m from the base of foundation: $\Delta\sigma = 150\left\{1 - \dfrac{1}{[(1/3.5)^2 + 1]^{3/2}}\right\} = 16.66 \text{ kN/m}^2$

At z = 6.0m from the base of foundation: $\Delta\sigma = 150\left\{1 - \dfrac{1}{[(1/6)^2 + 1]^{3/2}}\right\} = 6.04 \text{ kN/m}^2$

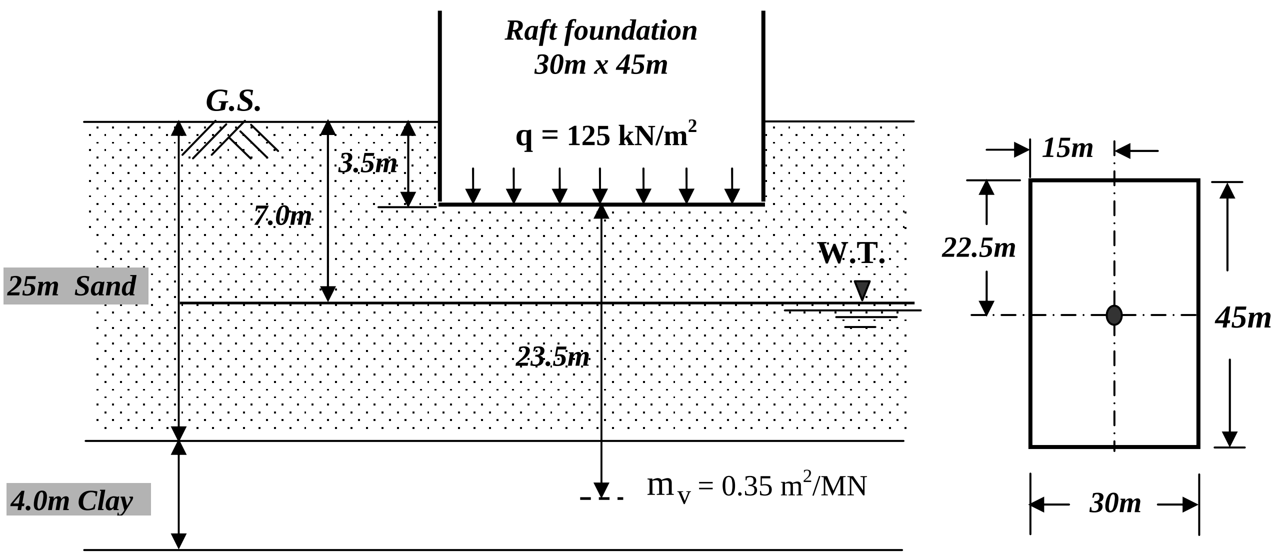
$$\therefore \quad \Delta\sigma_{avg.} = \frac{1}{6}(\Delta\sigma_t + 4\Delta\sigma_m + \Delta\sigma_b) = \frac{1}{6}\left[75 + 4(16.66) + 6.04\right] = 24.61 \text{ kN/m}^2$$

$$\Delta e = C_c \log_{10} \frac{\sigma'_o + \Delta\sigma}{\sigma'_o} = 0.16.\log_{10} \frac{51.82 + 24.61}{51.82} = 0.027$$

$$S_c = \frac{\Delta e}{1 + e_o} H_t = \frac{0.027}{1 + 0.85}(5)(1000) = \underline{\mathbf{72.9\ mm}}$$

Problem (5.5): *(consolidation settlement – m_v method)*

A building is supported on a raft of (30m x 45m), the net pressure being 125 kN/m^2 as shown in the figure below. Determine the settlement under the center of the raft due to consolidation of the clay.



Solution:

From Ch.(4) the vertical stress below the corner of flexible rectangular or square loaded area

$$\Delta\sigma_z = I.q_o$$

At mid-depth of the layer , z = 23.5m below the center of the raft:

m/z = 22.5/23.5 = 0.96 and n/z = 15/23.5 = 0.64 therefore; I = 0.140

$\Delta\sigma_z$ = (4)(0.140)(125) = 70 kN/m^2

$$S_c = m_v.H_t.\Delta\sigma \text{ ....(5.10)}$$

S_c = (0.35)(70)(4)(1000) = **98 mm.**

5.8 SECONDARY CONSOLIDATION SETTLEMENT

It occurs after the primary consolidation settlement has finished when all pore water pressures have dissipated (see **Fig.(5.7)**). Secondary consolidation can be ignored for hard or overconsolidated soils. But, it is highly increased for organic soil such as peat. This can explained due to the redistribution of forces between particles after large structural rearrangements that occurred during the normal consolidation stage of the soil.

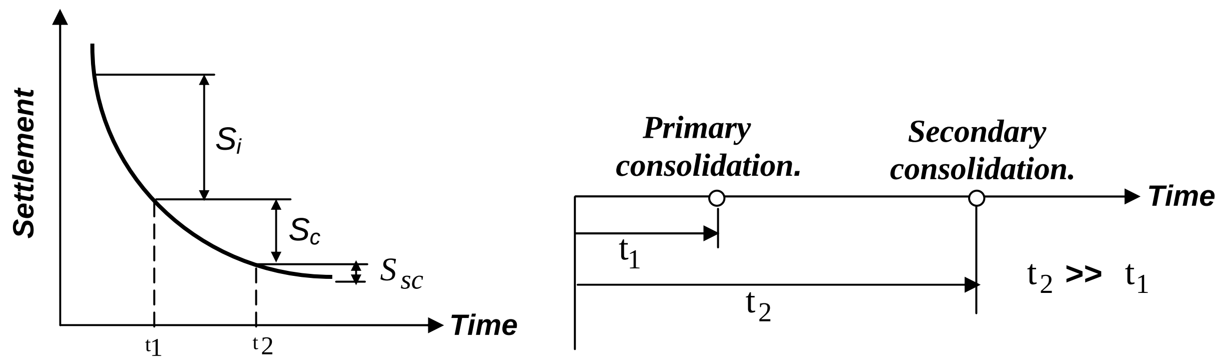


Fig. (5.7): Definition of secondary compression.

$$S_{cs} = C_\alpha.H.\log_{10}\frac{t_2}{t_1} \text{ ....(5.12)}$$

where, S_{cs} = secondary consolidation settlement.

C_α = coefficient of secondary consolidation; obtained from table below.

H = thickness of clay layer.

t_1 = time of primary consolidation settlement, and

t_2 = time of secondary consolidation settlement.

To determine t_1: from $T_v = \frac{C_v.t}{H^2}$ take $T_v = 1.0$ and $t = t_1$; then $1.0 = \frac{C_v.t_1}{H^2}$ or $t_1 = \frac{H^2}{C_v}$

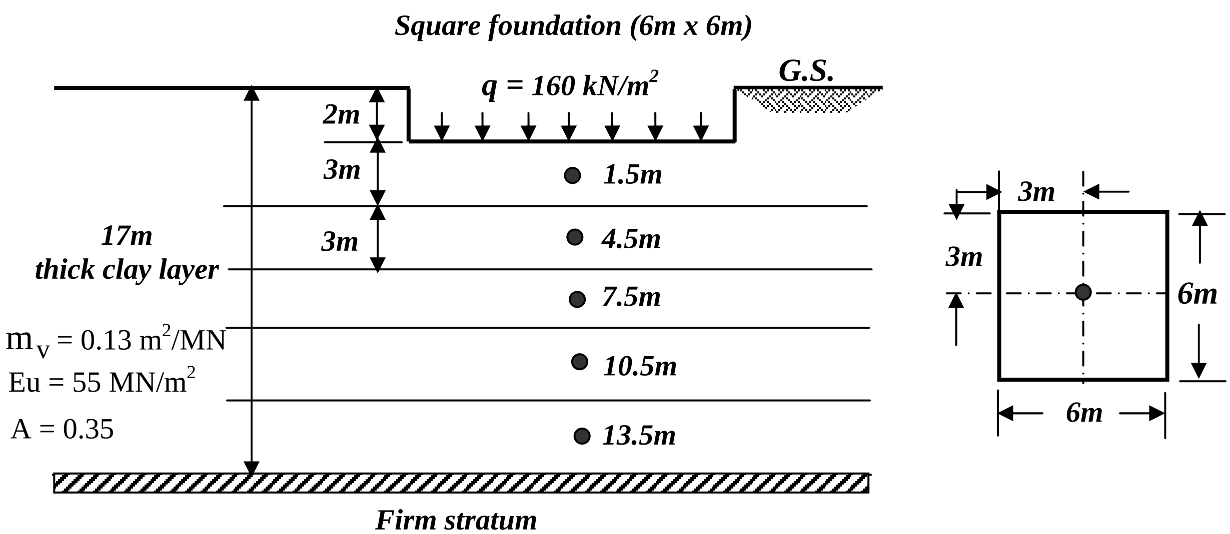
Table (5.7): Values of C_α for some typical soils.

Type of clay	C_α
Normally consolidated clay	*0.005-0.02*
Plastic or organic soil	*≥ 0.03*

Hard clay or overconsolidated clay with O.C.R > 2	*0.001 - 0.0001*

Problem (5.6): *(Total settlement)*

As shown in the figure below, a footing 6m square, carrying a net pressure of 160 kN/m^2 is located at a depth of 2m in a deposit of stiff clay 17m thick; a firm stratum lies immediately below the clay. Form Oedometer tests on specimens of the clay, the value of m_v was found to be 0.13 m^2/MN and from Triaxial tests the value of A was found to be 0.35. The undrained Young's modulus for the clay is estimated to be 55 MN/m^2. Determine the total settlement under the center of the footing.



Solution:

(1) Immediate settlement (Using Bjerrum method):

From **Fig.(5.4):** for H/B = 15/6 = 2.5, L/B = 1 and D_f /B = 2/6 = 0.33

$\mu_o = 0.91$ and $\mu_1 = 0.60$

$$S_{i(average)flexible} = \mu_o . \mu_1 \frac{q.B}{E_u} \quad \text{...........(5.7)}$$

$$S_{i(average)flexible} = (0.91)(0.60)\frac{(160)(6)(1000)}{(55x1000)} = 9.5 \text{ mm}$$

(2) Consolidation settlement (m_v - method):

From Ch.(4), the vertical stress below the corner of flexible rectangular or square loaded area

$$\Delta\sigma_z = I.q_o$$

At mid-depth of each 3 m depth as shown in the table below:

Layer no.	z **(m)**	$m/z,\ n/z$	I **From Ch.(4) Fig.(4.15)**	$\Delta\sigma'$ **(kN/m²)**	$S_{c(oed)} = m_v.H_t.\Delta\sigma'$ **(mm)**
1	1.5	2.00	0.233	149	58.1
2	5.5	0.67	0.121	78	30.4
3	7.5	0.40	0.060	38	15.8
4	10.5	0.285	0.033	21	8.2
5	13.5	0.222	0.021	13	5.1
					$\sum$ = 116.6

For 1st. Layer: m/z = 3/1.5 = 2.00 and n/z = 3/1.5 = 2.00 therefore; I = 0.233

$\Delta\sigma'_z$ = (4)(160)(I)..........................(kN/m^2)

$$S_{c(oed)} = m_v.H_t.\Delta P \quad \textbf{..(5.10)}$$

$S_{c(oed)}$ = (0.13)($\Delta\sigma'_z$)(3)(1000) .... (mm).

(3) Correction for A pore water pressure:

From **Fig.(5.6):** for H/B = 15/6.77 = 2.2 (equivalent diameter = 6.77 m) and A = 0.35; ρ_{circle}=0.55 then, $S_{c(oed)}$ = (0.55)(116.6) = 64 mm.

∴ Total settlement = $S_T = S_i + S_c$ = 9.5 + 64 = **73.5 mm**

5.9 DEGREE OR RATE OF SETTLEMENT

It is the ratio of consolidation at time (t) to that of 100% consolidation when the pore water pressure was diminishes. It is calculated as follows:

(1) First, from Oedometer tests, the coefficient of consolidation (C_v) is calculated as:

$$C_v = \frac{k}{m_v.\gamma_w} \quad \textbf{...(5.13)}$$

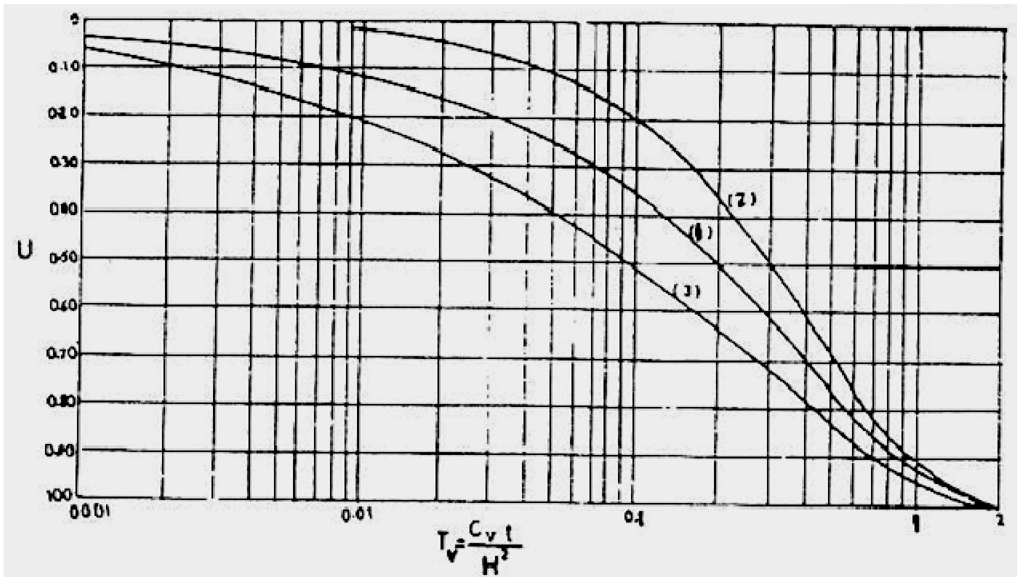
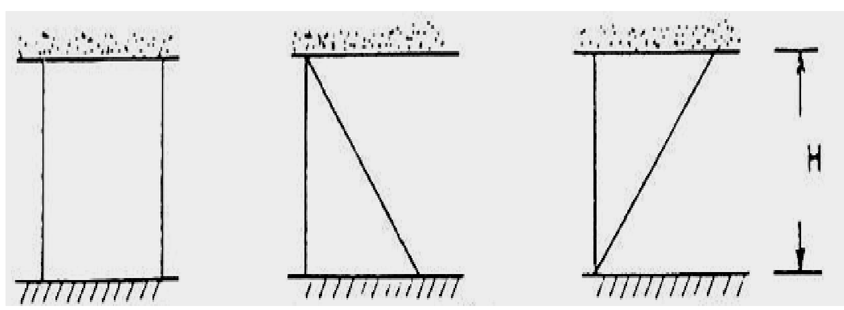


Fig.(5.8): Variation of average degree of consolidation and time factor
(for EPWP conditions given in Figs. a, and b) .

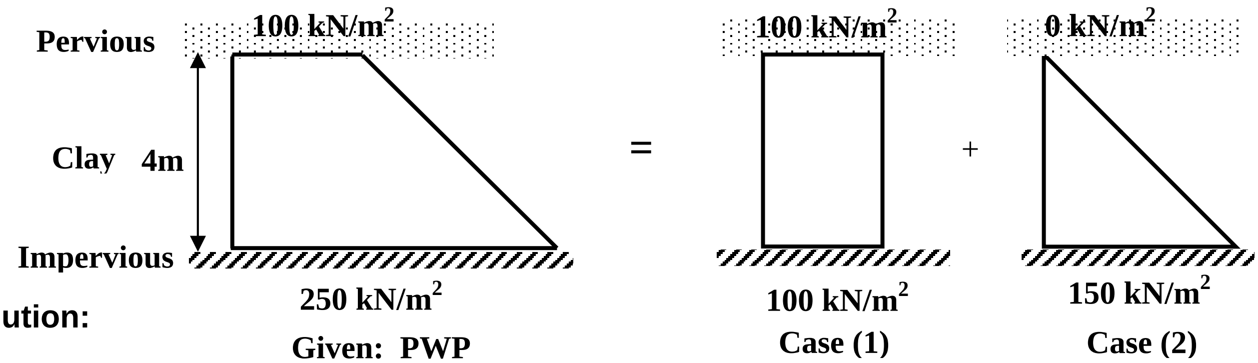
(a) 1-D drainage



Curve (1) ***Curve (2)*** ***Curve (3)***

(b) 2-D drainage

Curve (1) ***Curve (2)*** ***Curve (3)***



Solution:

Given: PWP

$C_v = 0.4\ m^3/year$

(1) $$T_v = \frac{C_v.t}{(H_d)^2} = \frac{(0.4)(20)}{(4)^2} = 0.50$$

From **Fig.(5.8)** for $T_v = 0.50$; $U_1 =$ 76% **(curve 1)** and

for $T_v = 0.50$; $U_2 =$ 69% **(curve 2)**

$$U_{avg.} = \frac{U_1.A_1 + U_2.A_2}{\sum A} = \frac{0.76(4)(100) + 0.69(4)(150)/2}{(\frac{100+250}{2})(4)} = 0.73 = 73\%$$

(2) To calculate the time required for any degree % of consolidation, take several times $t_{i\,(year)}$ and find the corresponding $U_{i\,(avg.)}$ as follows:

t_i *(year)*	T_v	U_1	U_2	$U_{avg.} = \frac{U_1.A_1 + U_2.A_2}{\sum A}$ *(%)*
10	0.25	0.55	0.45	50.7 < 62
12	0.30	0.62	0.50	56.8 < 62
15	0.375	0.67	0.57	62.7 $\approx$ 62
18	0.45	0.72	0.64	68.5 > 62

Sample of Calculation:

- For t = 10 (years): $$T_v = \frac{C_v.t}{(H_d)^2} = \frac{(0.4)(10)}{(4)^2} = 0.25$$

From **Fig. (5.8)** for $T_v = 0.25;\ U_1 =$ 55% **(curve 1)** and

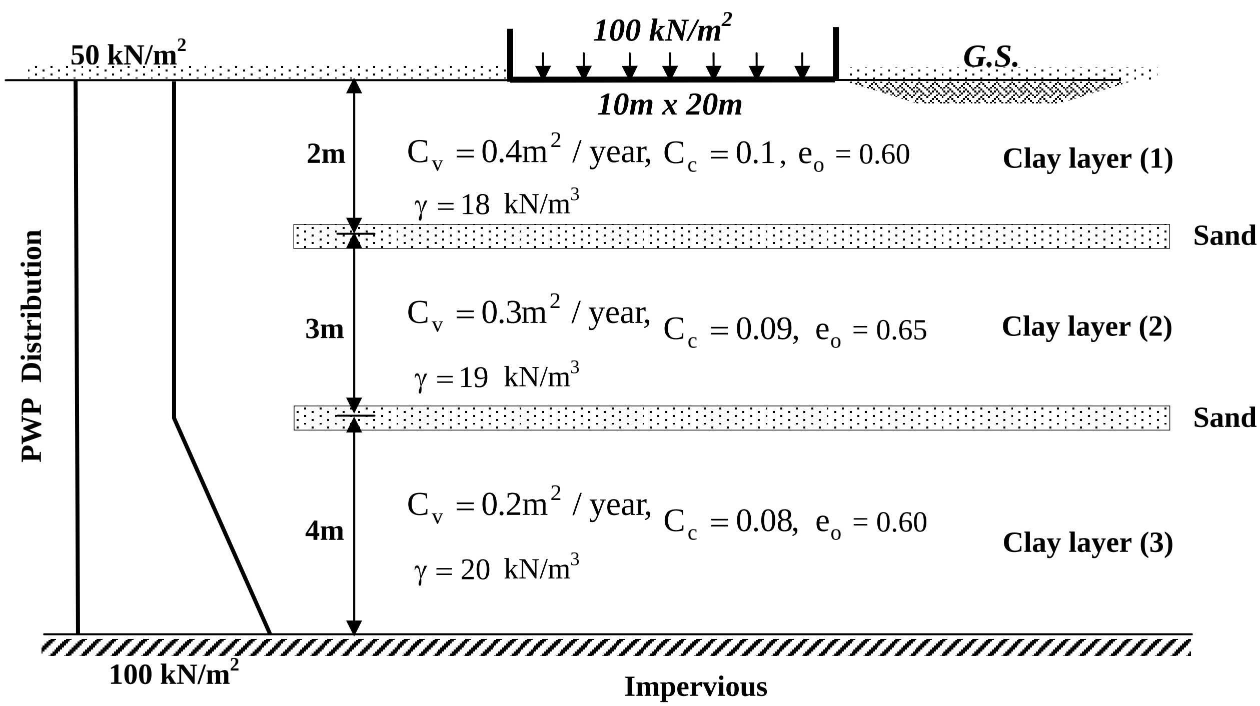
for $T_v = 0.25;\ U_2 =$ 45% **(curve 2)**

$$U_{avg.} = \frac{U_1.A_1 + U_2.A_2}{\sum A} = \frac{0.55(4)(100) + 0.45(4)(150)/2}{(\frac{100+250}{2})(4)} = 0.507 = 50.7\%$$

- After drawing $U_{i(avg.)}$ versus $t_{i(year)}$ as obtained from table above; it can be seen that ***the required time for 62 % consolidation = 15 (years).***

Problem (5.9): *(consolidation for layered soils)*

A raft foundation is placed at surface of a normally consolidated clay layers with internal sand drain layers as shown in the figure below. Determine the % degree of consolidation after 10 years if the PWP distribution is as given in the same figure.



Solution:

(1) Calculate (T_v) for each clay layer; 1, 2, 3:

$$T_{v1} = \left[\frac{C_{v1}.t}{(H_1)^2}\right] = \frac{0.4(10)}{(2/2)^2} = 4,$$

$$T_{v2} = \left[\frac{C_{v2}.t}{(H_2)^2}\right] = \frac{0.3(10)}{(3/2)^2} = 1.333, \text{ and}$$

$$U_3 = \frac{U_1.A_1 + U_2.A_2}{\sum A} = \frac{0.38(4)(50) + 0.22(4)(50)/2}{(\frac{50+100}{2})(4)} = 0.326 = 33\%$$

The average degree of consolidation after (10) years for all layers is calculated from:

$$U_{i(avg.)} = U_{(t)} = \frac{1}{S_c}(S_{c1}U_1 + S_{c2}U_2 + S_{c3}U_3 + ...)$$

$$U_{i(avg.)} = \frac{1}{16.86}\left[(9.55)(1.00) + (4.85)(0.95) + (2.46)(0.33)\right] = 0.89 = \mathbf{89\ \%}$$