كلية المنصور الجامعة

Al-Mansour University College

قسم الهندسة المدنية
المرحلة الثانية

Civil Eng. Dept
2nd. Stage

Mechanics of materials

2023– 2022

Sheet.1

مقاومة

- *Direct Stresses*
- *Thermal Stresses*

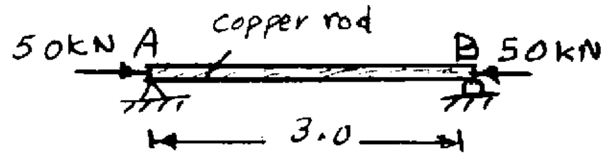
Dr. Maloof Mahmood

Ex. 3

The copper rod AB shown below is initially of length 3.0 m and of X-sectional area $A = 30\ cm^2$. Find the change in the length of the rod if an axial force 50 kN is applied at the ends and the surrounding temp. rises from 5°C to 50°C neglecting the weight of the rod.

Given: $E_{cu} = 110 \times 10^3\ N/mm^2$, $\alpha_{cu} = 17 \times 10^{-6}/°C$.

$\Delta = \dfrac{PL}{AE}$, $A = 3000\ mm^2$



Shortening of rod:

$$\Delta = \frac{(50 \times 10^3) \times (3.0 \times 10^3)}{3000 \times 110 \times 10^3} = 0.45\ mm$$

Extension due to rise in temp. $= \Delta = \alpha L \delta T$

$\delta T = 50 - 5 = 45°C$

$\Delta = (17 \times 10^{-6}) \times (3.0 \times 10^3) \times (45) = 2.295\ mm$

$\therefore$ Net increase in length of rod $= 2.295 - 0.45$

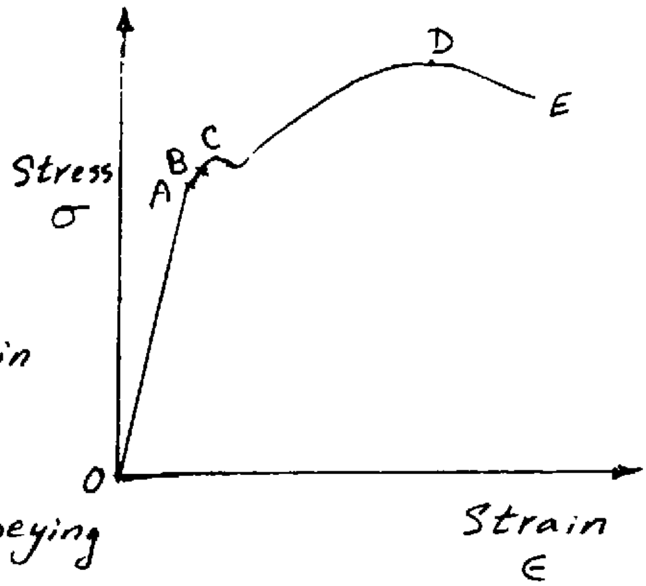
$\cong \underline{1.85}\ mm$

Mechanics of Materials

References

1. "Strength of Materials"
   by: G. H. Ryder.
2. "Strength of Materials"
   by: William A. Nash.
3. "Strength of Materials"
   by: J. P. Den Hartog.
4. "Mechanics of Materials"
   by: R. C. Hibbeler.
5. "Strength of Materials"
   by: E. C. POPOV.

Stress – Strain Curve for Mild Steel



Consider the Standard tensile test of mild Steel bar of uniform cross-section (usually Circular).
The load is gradually increased from zero and hence the strain is proportional to the load (hence to the stress). $\overline{\overline{OA}}$ is a straight line relation obeying Hooke's law.

point A: is called Limit of proportionality.

point B: is that of Elastic Limit beyond which plastic deformation will occur, i.e., strain will not be recovered when load is removed.

point C: is called the yield point, the metal shows appreciable Strain even without increase in load.

The Stress–Strain curve continues to rise up to 'D'. The strain from C to D is mainly plastic and is in the range 100 times the strain from O to C.

The ultimate maximum tensile stress = Load at D divided by original cross-sectional area.
Necking in the bar will start beyond point D until the bar finally breaks.

Factor of Safety (F.S.):

It's the ratio between the ultimate stress and the working stress

allowable stress

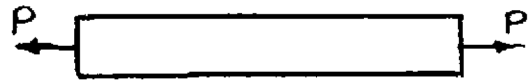
$$F.S. = \frac{\text{ultimate stress}}{\text{working stress}}$$

Direct Stress

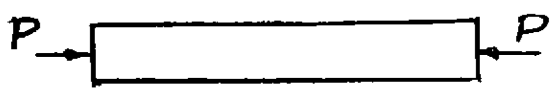
Tension & Compression

Axially Loaded Bar

Assume a straight metal bar of uniform cross-section loaded at its ends by a pair of oppositely directed collinear forces along the longitudinal axis of the bar.



Bar in Tension



Bar in Compression

If the forces are directed away from the bar, it is said to be in tension. If the forces are directed toward the bar, it is said to be in a state of compression.

Normal Stress

The intensity of normal force per unit area is termed the normal stress (σ). It is expressed in units of force per unit area : lb/in^2 (PSi) or Newton/mm^2 (N/mm^2) (Mpa)

$$\sigma = \frac{P}{A}$$

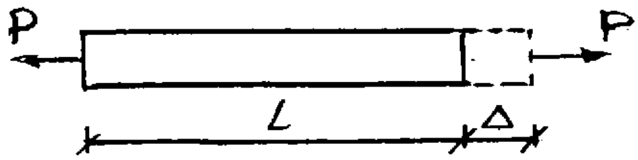
σ = stress

A = cross-sectional area of bar

P = External normal force

Normal Strain

It is the ratio of total elongation of a member under tension divided by its original length.



$$\epsilon = \frac{\Delta}{L}$$

Note:

Strain is a ratio or change per unit length, hence it is dimensionless.

Normally, tensile strains are considered positive (+) while compressive strains (decrease in length) are negative (−).

Hooke's Law

This states that strain is proportional to the stress producing it.

$$\sigma = E\epsilon$$

where, σ = applied stress ,

E = Young's modulus

Modulus of Elasticity (Young's Modulus)

Within the limits for which Hooke's law is obeyed, the ratio of the direct stress to the strain produced is called "Young's Modulus" or "the Modulus of Elasticity"

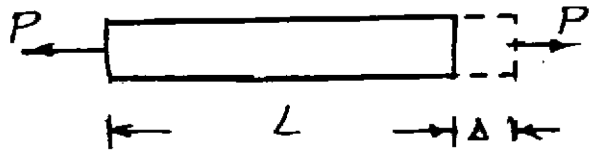
$$E = \frac{\sigma}{\epsilon}$$

Note:

E is constant for a given material and it is the same in tension or compression.

Ex. 1

Determine the total elongation of an initially straight bar of length (L), cross-sectional area (A) and Modulus of Elasticity (E), if a tensile load (P) acts on the ends of the bar.



Stress $\sigma = \frac{P}{A}$

Strain $\epsilon = \frac{\Delta}{L}$

$$E = \frac{\sigma}{\epsilon} = \frac{\frac{P}{A}}{\frac{\Delta}{L}} = \frac{PL}{A\Delta}$$

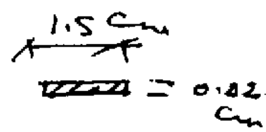
$\therefore \Delta = \frac{PL}{AE}$, units in inches, ft, mm etc.

Ex. 2

A steel surveyor's tape 50 m long, has a cross-sectional area of (1.5 x 0.02) cm . Determine the elongation when the whole tape is stretched and held tight by a force of 6 kgm.

Given $E_s = 200{,}000\ N/mm^2$

1 KN = 100 kgm



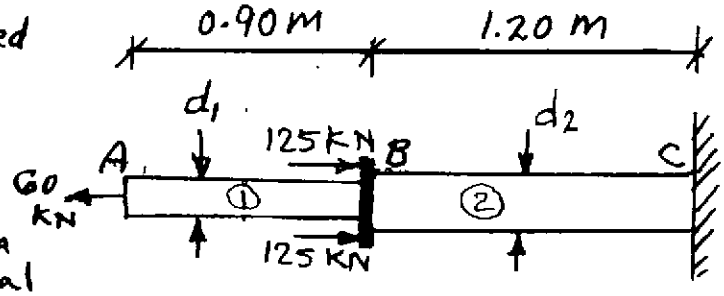
1 kgm $\cong$ 10 N , $P = 6$ kgm $= 60$ N

$A = (1.5 \times 10) \times (0.02 \times 10) = 3\ mm^2$

$$\Delta = \frac{PL}{AE} = \frac{60 \times (50 \times 1000)}{3 \times (200{,}000)} = 5\ mm$$ the elongation of the tape

Ex. 5

Two solid steel cylindrical rods AB and BC are welded together at (B) and loaded as shown. If $d_1 = 30$ mm & $d_2 = 50$ mm & $E_s = 200 \times 10^3$ MPa



(1) Find the average normal stress in the mid section of rod AB & rod BC

(2) Find the total change in length of the 2-rods due to the applied external loads.

---

(1) Rod AB:

Force $P_1 = 60$ KN (tension)

$$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2$$

$$\text{Normal stress} = \sigma_{AB} = \frac{P_1}{A_1} = \frac{60 \times 10^3}{706.86} = 84.9 \text{ MPa } (N/mm^2)$$ (tensile stress)

Rod BC

Force $P_2 = 60 - 2 \times 125 = -190$ KN (compression)

$$\text{Area } A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(50)^2 = 1963 \text{ mm}^2$$

$$\sigma_{BC} = \frac{P_2}{A_2} = \frac{-190 \times 10^3}{1963} = -96.8 \text{ MPa}$$ (Comp. Stress)

(2) $\Delta = \dfrac{PL}{AE}$

$$\Delta_{tot} = \frac{P_1 L_1}{A_1 E} + \frac{P_2 L_2}{A_2 E}$$

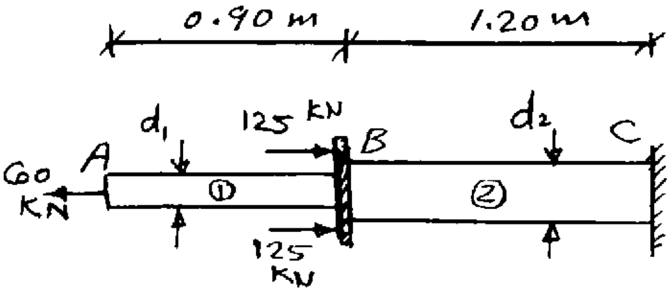
$$= \frac{\sigma_{AB} L_1}{E} + \frac{\sigma_{BC} L_2}{E}$$

$$= \frac{84.9 \times 0.9 \times 10^3}{200 \times 10^3} - \frac{96.8 \times 1.2 \times 10^3}{200 \times 10^3}$$

$$= \frac{1}{200}(76.4 - 116.2) \cong -0.20 \text{ mm}$$ total reduction in length of rods AB & BC.

Ex. 6

Two solid cylindrical rods AB & BC are welded together at (B) and loaded as shown. If the average normal stress must not exceed 150 MPa in either rod, determine the smallest allowable values of the diameters d_1 & d_2.



Rod AB

Force $P_1 = 60$ KN , stress $\sigma_{AB} = 150$ MPa (N/mm²)

Area $A_1 = \frac{\pi}{4} d_1^2$

$$\sigma_{AB} = \frac{P_1}{A_1} \Rightarrow A_1 = \frac{P_1}{\sigma_{AB}}$$

$$\frac{\pi}{4} d_1^2 = \frac{60 \times 10^3}{150} = 400$$

$$d_1^2 = \frac{1600}{\pi}$$

$$\underline{d_1 = 22.6 \text{ mm}}$$

Rod BC

Force $P_2 = 60 - 2 \times 125 = -190$ KN (-ve → Comp.)

$\sigma_{BC} = -150$ MPa (N/mm²) , $A_2 = \frac{\pi}{4} d_2^2$

$$\sigma_{BC} = \frac{P_2}{A} \Rightarrow A_2 = \frac{P_2}{\sigma_{BC}}$$

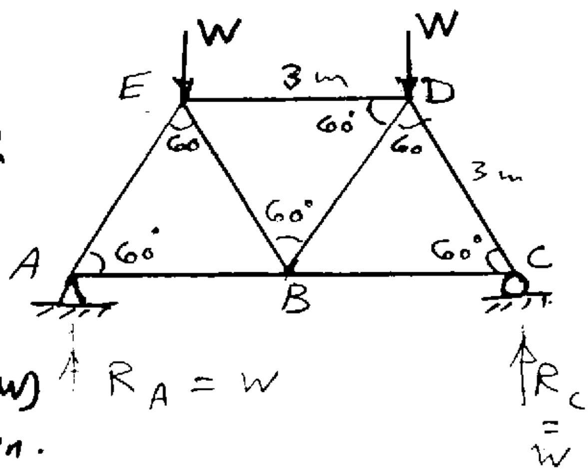
$$\frac{\pi}{4} d_2^2 = \frac{(190 \times 10^3)}{(-150)}$$

$$d_2^2 = \frac{4(-190 \times 10^3)}{\pi(-150)} = 1613 \text{ mm}^2$$

$$d_2 = \sqrt{1613} = \underline{40.2} \text{ mm}$$

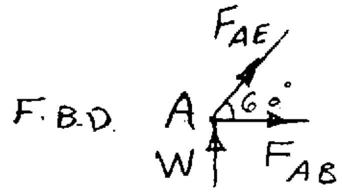
Ex. 10

A steel truss is made of equal members' length (3m) and equal areas (2000 mm²). The allowable axial stress in tension is (200 MPa) and in compression is (150 MPa). Find the safe allowable load (W) placed at E and D as shown.



$\Sigma F_y = 0$ for the whole truss;
From symmetry, $R_A = R_C = W$

Joint A : F.B.D. :



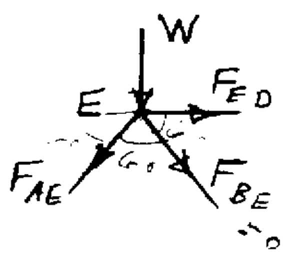
$$\Sigma F_y = 0 \Rightarrow F_{AE} \sin 60 + W = 0 \qquad \left(\sin 60 = \tfrac{\sqrt{3}}{2}\right)$$

$$\therefore F_{AE} = \frac{-W}{\sin 60} = \frac{-2W}{\sqrt{3}} \quad \text{(comp.)}$$

$$\Sigma F_x = 0 : \quad F_{AB} + F_{AE} \cos 60 = 0$$

$$F_{AB} = -\frac{-2W}{\sqrt{3}} \times \frac{1}{2} = +\frac{W}{\sqrt{3}} \quad \text{(tension)}$$

Joint E :



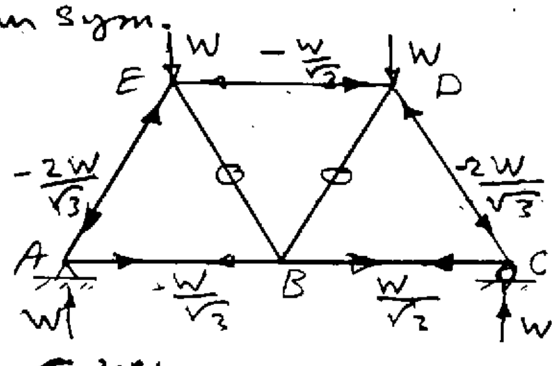
$$\Sigma F_y = 0 \Rightarrow W + F_{AE} \sin 60 + F_{BE} \sin 60 = 0$$

$$W + \frac{-2W}{\sqrt{3}} \times \frac{\sqrt{3}}{2} + F_{BE} \times \frac{\sqrt{3}}{2} = 0$$

$$\therefore \boxed{F_{BE} = 0} \ \& \ \boxed{F_{BD} = 0} \text{ from sym.}$$

$$\Sigma F_x = 0 : \quad F_{ED} - F_{AE} \cos 60 = 0$$

$$F_{ED} = F_{AE} \times \frac{1}{2} = \frac{-2W}{\sqrt{3}} \times \frac{1}{2} = \frac{-W}{\sqrt{3}} \quad \text{comp.}$$



Max tens. $= \frac{W}{\sqrt{3}}$

Max comp. $= \frac{-2W}{\sqrt{3}}$

$$\frac{W_1}{\sqrt{3}} = 200\,(2000) \Rightarrow W_1 = 400\sqrt{3} \text{ KN}$$

$$\frac{2W_2}{\sqrt{3}} = 150\,(2000) \Rightarrow W_2 = \underline{150\sqrt{3}}$$

$$\therefore \boxed{W_{all} = 150\sqrt{3} \cong 260 \text{ KN}}$$ ← the allowable Load (W) on the truss

Thermal Stresses

A change in temperature can cause a body to change its dimensions. If the temperature increases, the body will expand and vice versa. If the material is isotropic and homogeneous, it has been found from experiment that the displacement of a member having a length L can be calculated using the formula:

$$\Delta = \alpha L \delta T$$

where, Δ = the change in the length of the member.
α = the linear coefficient of thermal expansion.
δT = the change in temperature of the member.

Ex.1

A straight aluminum wire 30 m long is subjected to a tensile stress of 75 N/mm² (MPa).

1) Determine the total elongation of the wire.
2) What temperature change would produce this same elongation? and what is the thermal strain produced? Given: $E = 70 \times 10^3$ N/mm², $\alpha = 39 \times 10^{-6}/°C$.

1) Total elongation $\Delta = \frac{PL}{AE} = \frac{\sigma L}{E} = \frac{75 \times (30 \times 10^3)}{70 \times 10^3} = 32$ mm

2) $\Delta = \alpha L \delta T$

$32 = 39 \times 10^{-6} \times (30 \times 10^3) \times \delta T$

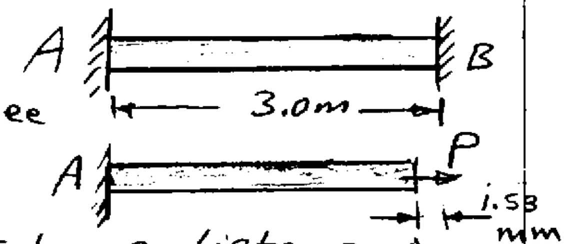
$\delta T = \frac{32 \times 10^3}{39 \times 30} = 27.4°C$ the required temp. change.

Thermal Strain produced $= \frac{\Delta}{L} = \frac{32}{30 \times 10^3} = 1.07 \times 10^{-3}$ mm/mm

Ex. 2

The copper rod shown below is rigidly attached to the walls at the ends A & B. Rod diameter $D = 6\,cm$.
At $35^\circ C$ the rod is stress free. Determine the stress in the rod if the temperature is dropped to $5^\circ C$.
For copper, $E_c = 110 \times 10^3\ N/mm^2$, $\alpha = 17 \times 10^{-6}/^\circ C$.

—·—



Assume that the rod is cut free from the wall at the end B. Hence the rod will contract by a distance Δ.

$\Delta = \alpha L \delta T$, $\delta T = 35 - 5 = 30^\circ c$.

$\Delta = (17 \times 10^{-6}) \times (3.0 \times 10^3) \times (30) = 1.53\ mm$

X-sectional area of rod $= A = \dfrac{\pi D^2}{4} = \dfrac{\pi (60)^2}{4}$

$\therefore A = 2827\ mm^2$

Let $P =$ the axial force at B required to stretch the rod by 1.53 mm.

Note: We know that the end 'B' does not move at all when the temp. drops.

$$\Delta = \frac{PL}{AE} \Rightarrow 1.53 = \frac{P \times (3.0 \times 10^3)}{2827 \times 110 \times 10^3}$$

$$\therefore P = \frac{1.53 \times 2827 \times 110}{3} \times 10^{-3} = \underline{\underline{158.6\ kN}}$$

Axial stress developed $= \sigma = \dfrac{P}{A}$

$$\sigma = \frac{158.6 \times 10^3}{2827} = 56.1\ N/mm^2$$

the stress developed in the rod due to drop in temp. of $30^\circ c$.

كلية المنصور الجامعة

Al-Mansour University College

قسم الهندسة المدنية
المرحلة الثانية

Civil Eng. Dept
2nd. Stage

Mechanics of materials

2022 – 2023

Sheet.2

مقاومة

- Direct Stresses
(Solved Problems)

Dr. Maloof Mahmood

(Q.3)

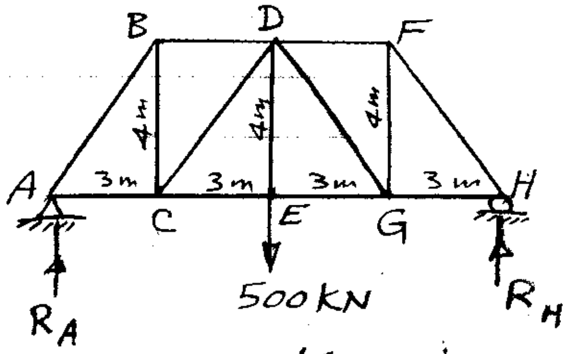
The truss shown below supports a single load of (500kN) at joint (E). All members have a yield stress point of ($420\ N/mm^2$). Assume a factor of safety of (2.4) for (tension) and (3.2) for compression with ($E = 200\times10^3\ N/mm^2$)

(a) Determine the required x-sectional area of (AB, AC & DE)

(b) Find the change in length of bars (AB) and (DE).

From Symmetry: $R_A = R_H = 250\ kN$

(a) Joint A:



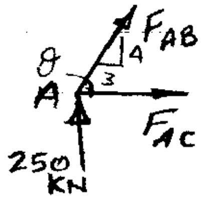
$\Sigma F_y = 0 \Rightarrow F_{AB}\sin\theta + 250 = 0$

$F_{AB}\times\frac{4}{5} = -250 \Rightarrow F_{AB} = \frac{-5\times250}{4} = -312.5\ kN$ (comp.)

$\Sigma F_x = 0$:

$F_{AC} + (-312.5)\cos\theta = 0$

$F_{AC} = +312.5\times\frac{3}{5} = +187.5\ kN$ (tens.)



Joint E:

$\Sigma F_y = 0 \Rightarrow F_{DE} = 500\ kN$ (tens.)

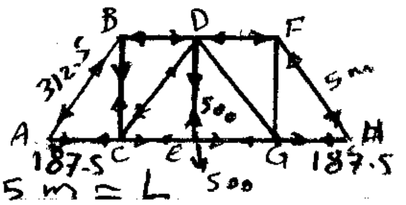
For steel, $\sigma_y = 420\ N/mm^2 = \sigma_u$, Working stress $(\sigma_{work}) = \frac{\sigma_u}{F.S.}$

$(\sigma_{work})_{tens} = \frac{420}{2.4} = 175\ N/mm^2$

$(\sigma_{work})_{comp} = \frac{420}{3.2} = 131.25\ N/mm^2$

X-Sec. Area $= A_{AC} = \frac{P}{\sigma_w} = \frac{187.5\times10^3}{175} = 1071\ mm^2$ (AC)

$A_{DE} = \frac{500\times1000}{175} = 2857\ mm^2$ (DE)



$A_{AB} = \frac{312.5}{131.25}\times1000 = 2381\ mm^2$ (AB)

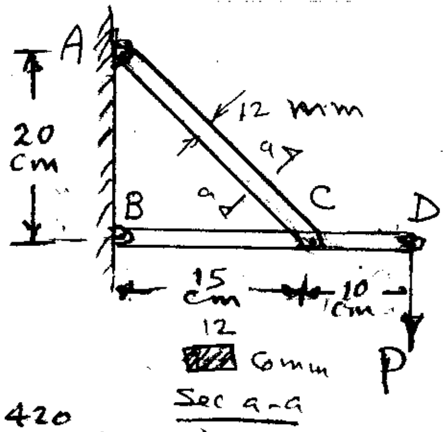
(b) $\Delta = \frac{PL}{AE}$, Length of AB $= \sqrt{3^2+4^2} = 5\ m = L$

contraction of AB $= \Delta_{comp} = \frac{312.5\times10^3\times5\times10^3}{2381\times200\times10^3} = 3.28\ mm$ contraction

Elongation of DE $= \frac{500\times10^3\times4\times10^3}{2857\times200\times10^3} = 3.5\ mm$ elongation

Q.1)

The link AC has a uniform rectangular X-section of (6x12)mm and is made of steel with Ultimate normal stress ($\sigma_u = 420 N/mm^2$). It is connected to a support at (A) and to a member (BCD) by a single shear pin of diameter (10mm). The member BCD is connected to a hinge support at (B). The pin at (C) has an ultimate Shearing stress of (180 N/mm²).
Using a factor of safety of (3.25), determine the allowable value of the load (P) applied at (D). (see fig. below)



Consider the F.B.D of member (BCD)

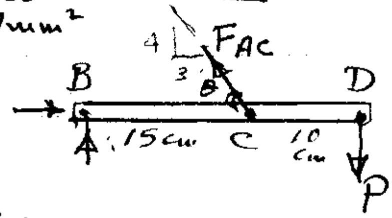
Take moments about B: $\boxed{\Sigma M_B = 0}$

$P \times 25 - (F_{AC} \sin\theta) \times 15 = 0$

$25P - \frac{4}{5} \times 15 F_{AC} = 0 \Rightarrow F_{AC} = \frac{25}{12} P$

$F_{AC} = 2.083\, P$

$\therefore P = \frac{F_{AC}}{2.083} = 0.48 F_{AC}$ —— (1)



Based on Strength of link (AC), $\sigma_u = 420\ N/mm^2$

$A_{net} = 6(12-10) = 12 mm^2$

$(F_{AC})_u = \sigma_u \times A_{net}$

$= 420 \times 12 = 5040\ N$

Eq.(1)

$P_u = 0.48 \times \underset{F_{AC}}{5040} = 2419\ N = 2.419\ kN$

Based on strength of pin at (C) ($\tau_u = 180 N/mm^2$)

$(A)_{pin} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(100) = 78.5\ mm^2$

Ult. shear stress of pin $\tau_u = 180\ N/mm^2$

$(F_{AC})_u = \tau_u (A)_{pin} = \frac{180 \times 78.5}{10^3} = 14.1\ kN$

$P_u = 0.48 F_{AC} = 0.48 \times 14.1 = 6.786\ kN$

Choose the smallest $P_u \Rightarrow P_u = 2.419\ kN$ Control

$\therefore$ Allowable value for $P = \frac{P_u}{F.S.} = \frac{2.419}{3.25} = 0.744\ kN$

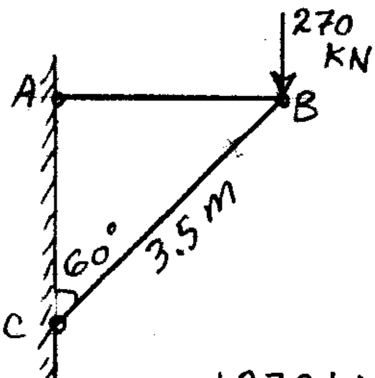
Q.2)

In the figure shown below, the two steel bars AB and BC are pinned at each end and support the load (270KN) at B. The bars' material has a maximum yield stress of (420 N/mm²). A factor of safety of (2.0) for tensile members and (3.5) for compressive members are to be used.

(a) Determine the required cross-sectional areas of the 2-bars.

(b) Calculate the change in the length of each bar.

$E_s = 200 \times 10^3\ N/mm^2$



Length $AB = 3.5 \sin 60 = \underline{3.03}$ m

Consider F.B.D. of point B :

(a) Joint B :

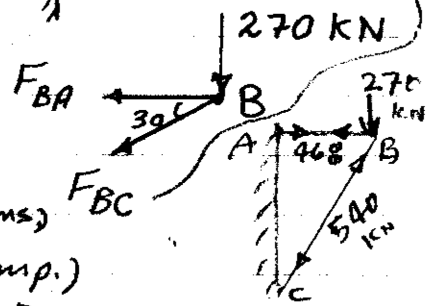
$\Sigma F_y = 0$: $270 + F_{BC} \sin 30 = 0$

$270 + \frac{1}{2} F_{BC} = 0$

$F_{BC} = \underline{-540}$ KN (comp.)

$\Sigma F_x = 0 \Rightarrow F_{BA} + F_{BC} \cos 30 = 0$

$F_{BA} = -(-540 \times \frac{\sqrt{3}}{2}) = +468$ KN (tens.)



Allowable stress $\sigma_t = \frac{420}{2} = 210\ N/mm^2$ for (tens.)

$\sigma_c = \frac{420}{3.5} = 120\ N/mm^2$ for (comp.)

$\sigma_t = \frac{P}{A} = \frac{468 \times 10^3}{A_{AB}} = 210\ N/mm^2 \Rightarrow A_{AB} = \frac{468 \times 10^3}{210} \approx \underline{2229}\ mm^2$

$\sigma_c = \frac{540 \times 10^3}{A_{BC}} = 120 \Rightarrow A_{BC} = \frac{540 \times 10^3}{120} = \underline{4500}\ mm^2$

(b)

$\Delta_{AB} = \frac{PL}{AE} = \frac{468 \times 10^3 \times 3.03 \times 10^3}{2229 \times 200 \times 10^3} = 3.18$ mm (Increase)

$\Delta_{BC} = \frac{-540 \times 10^3 \times 3.5 \times 10^3}{4500 \times 200 \times 10^3} = 2.1$ mm (decrease in length)