كلية المنصور الجامعة

Al-Mansour University College

قسم الهندسة المدنية
المرحلة الثالثة

Civil Eng. Dept
3rd. Stage

Theory of Structures

2023 - 2022

Sheet-1

انشاءات

Stability
& Determinacy of Structures

Dr. Maloof Mahmood

Theory of Structures

References

1. " Structural Analysis "
   By: R.C. Hibbeler, (Seventh Edition)

2. "Elementary Theory of Structures"
   By: Yuan-Yu Hsieh, (Second Edition)

3. " Structural Analysis"
   By: Jack C. McCormac

4. "Analysis of Statically Indeterminate Structures"
   By: Clifford S. Williams

5. " Statically Indeterminate Structural Analysis"
   By: R.L. Sanks

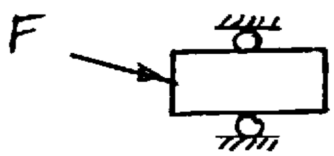
Stability and Determinacy of Structures

Stability and Determinacy of a Rigid Body

Consider a rigid body supported by a number of supports. Two reactions are not sufficient to ensure the stability of a rigid body when they are collinear, parallel or concurrent. This case is called statically unstable because of insufficient number of support elements.

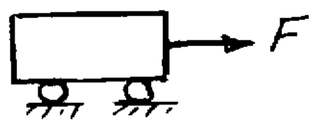
Examples:

1. Collinear Reactions:
   The reactions, in this case, cannot resist external load that has a component normal to the line of reactions.

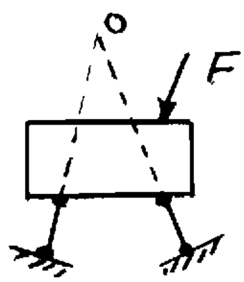


2. Parallel Reactions:

   The reactions, in this case, cannot prevent the body from lateral sliding.



3. Concurrent Reactions;
   The reactions, in this case, cannot resist the moment about concurrent Point (o).



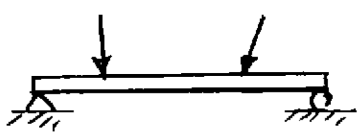
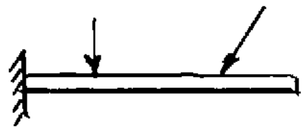
Remarks:

In cases (1) and (2) above, equation of equilibrium $\sum Fx = 0$ is not satisfied, while for case (3) the condition $\sum Mo = 0$ is not satisfied.

Elements of Reactions and Stability

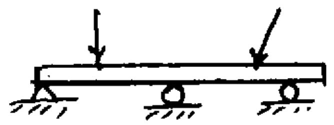
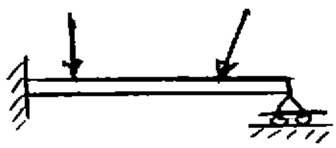
1. At least 3- elements of reaction are necessary for stable equilibrium of a body.

   Examples:

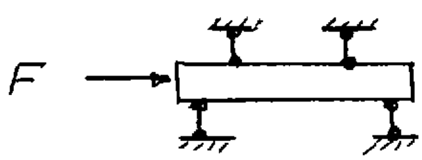



The satisfaction of the 3-equilibrium equations: $\sum Fx = 0$, $\sum Fy = 0$ and $\sum Mo = 0$ for loads and reactions acting on the body guarantees that the body will not move horizontally, vertically or rotate. The system is said to be "Statically Stable and Determinate".

2. If there are more than 3-elements of reactions, as in the cases below, the body is said to be more stable. Since the number of unknown elements of reaction is more than the number of equations for static equilibrium, the system is said to be " Statically Indeterminate".

3. When the lines of reaction are all parallel as in the figure below, the body is said to be unstable as it is subject to lateral sliding. The inadequate arrangement of supports will cause "External Geometric Instability".



Theory of Structures

Stability & Determinacy of Structures

(Solved Problems)

BEAMS

Discuss the stability and determinacy of the beams shown below:

1)

$r = 3+1+1+3 = 8, \quad c = 0, \quad c + 3 = 3$

$8 > 3 \;, \; r > c + 3$

Hence, the beam is stable and statically indeterminate to the 5th degree

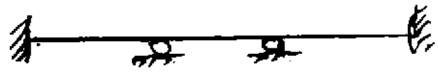


Fig.(1)

2)

$r = 1 + 3 = 4, \quad c = 2, \quad c + 3 = 5$

$4 < 5, \quad r < c + 3$

Hence , the beam is unstable.

Fig.(2)

3)

$r = 2+1+1+2 = 6, \quad c = 1+2 = 3, \quad c + 3 = 6$

$6 = 6, \quad r = c + 3.$

$\therefore$ The beam is stable and statically determinate.

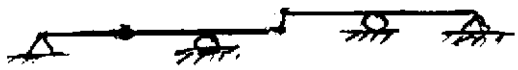


Fig.(3)

4)

$r = 6 \;, \; c = 1, \; c + 3 = 4$

$6 > 4, \quad r > c + 3$

$\therefore$ The beam is stable and statically indeterminate to the 2nd degree

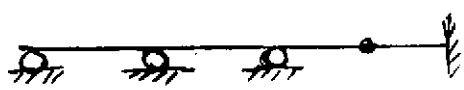


Fig.(4)

TRUSSES

Discuss the stability and determinacy of the trusses shown below:

1)

$b = 27,\ r = 3,\ j = 15,\ \ b + r = 30,\ \ 2j = 30$

i.e., $b + r = 2j$

Hence the truss is stable and determinate.

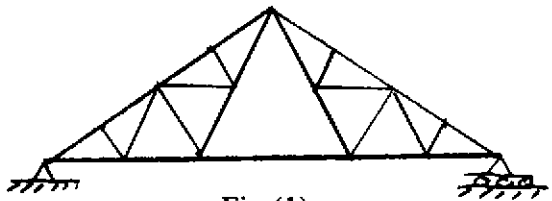


Fig.(1)

2)

$b = 15,\ r = 4,\ j = 8,\ b + r = 19,\ 2j = 16$

$b + r > 2j\ ,\ 19 > 16$

Hence the truss is indeterminate to the 3rd degree.

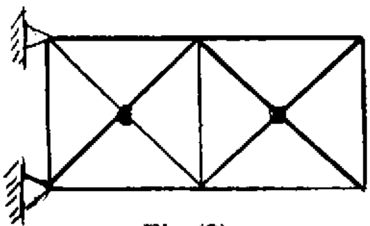


Fig.(2)

3)

$b = 14,\ r = 4,\ j = 9,\ b + r = 18 = 2j$

The truss is unstable internally.

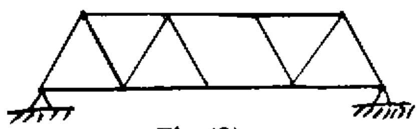


Fig.(3)

4)

$b = 7,\ r = 3,\ j = 5,\ \ b + r = 10 = 2j$

The truss is stable and determinate.

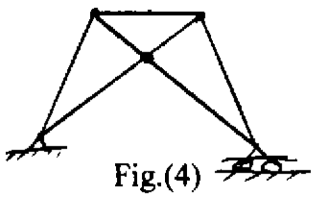


Fig.(4)

5)

$b = 22,\ r = 4,\ j = 13,\ b + r = 26 = 2j$

The truss is stable and determinate.

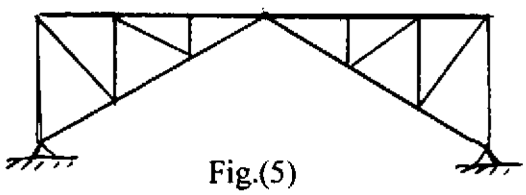


Fig.(5)

6)

$b = 9,\ r = 3,\ \ j = 6,\ b + r = 12 = 2j$

The truss is statically stable and determinate.

Fig.(6)

RIGID FRAMES

Discuss the stability and determinacy of the frames shown below:

1)

$b = 8,\ r = 9,\ c = 0,\ j = 8,\ 3b + r = 33,\ 3j + c = 24$

$3b + r > 3j + c,\ 33 > 24$

Frame is stable and indeterminate to 9^{th} degree.

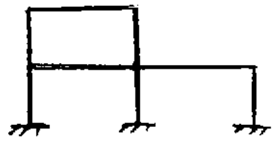


Fig.(1)

2)

$b = 9,\ c = 0,\ j = 10,\ r = 8,\ 3b + r = 35,\ 3j + c = 30$

$3b + r > 3j + c,\ 35 > 30$

Frame is stable and indeterminate to 5^{th} degree.

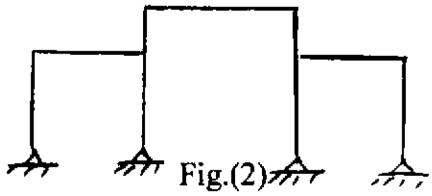


Fig.(2)

3)

$b = 6,\ c = 3\text{-}1 = 2,\ j = 6,\ r = 6,\ 3b + r = 24,\ 3j + c = 20$

$3b + r > 3j + c,\ 24 > 20$

Frame is stable and indeterminate to 4^{th} degree.

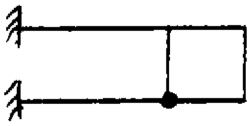


Fig.(3)

4)

$b = 2,\ r = 3,\ c = 0,\ j = 3,\ 3b + r = 9,\ 3j + c = 9$

$3b + r = 3j + c,\ 9 = 9$

The frame is unstable externally, since the roller support may slide downward.

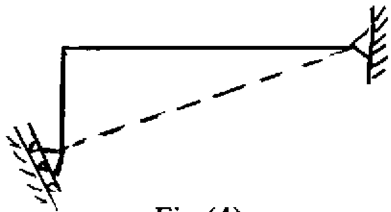


Fig.(4)

5)

Since the frame can be cut to form 10 stable and determinate parts by 34 cuts in the beams, the frame is statically indeterminate to 3 x 34 = 102 degrees

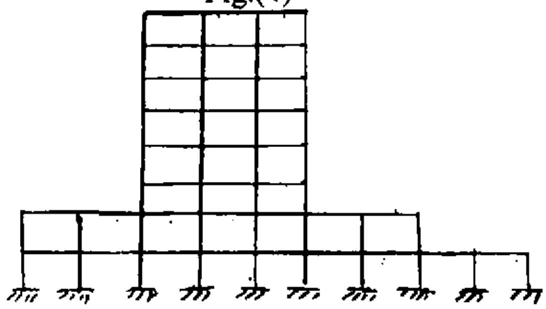


Fig.(5)

6)

$r = 5,\ b = 9,\ j = 8,\ c = 0,$

$3b + r = 32,\ 3j + c = 24,\ 3b + r > 3j + c,\ 32 > 24$

The frame is statically indeterminate to 8^{th} degree.

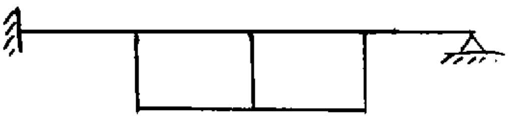


Fig.(6)

7)

$r = 6,\quad b = 4,\quad j = 5,\quad c = 0$

$3b + r = 12 + 6 = 18,\quad 3j + c = 15$

$3b + r > 3j + c,\ 18 > 15$

The frame is indeterminate to 3rd degree.

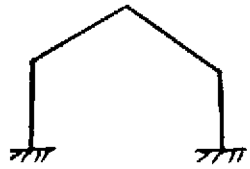


Fig.(7)

8)

$r = 6,\ b = 4,\ j = 5,\ c = 2$

$3b + r = 18,\quad 3j + c = 17,\quad 18 > 17$

The frame is indeterminate to 1°.

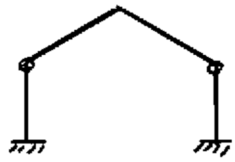


Fig.(8)

9)

$r = 4,\ b = 5,\ j = 6,\ c = 3$

$3b + r = 19,\quad 3j + c = 21,\quad 19 < 21$

The frame is unstable

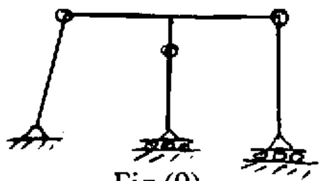


Fig.(9)

10)

$r = 11, b = 6, j = 7, c = 0$

$3b + r = 18 + 11 = 29,\quad 3j + c = 21$

$3b + r > 3j + c, 29 > 21$

The frame is indeterminate to 8°.

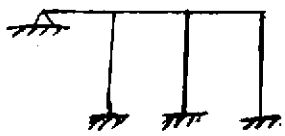


Fig.(10)

11)

$r = 4, b = 2,\quad j = 3,\quad c = 1$

$3b + r = 10,\quad 3j + c = 10$

$3b + r = 3j + c,\quad 10 = 10$

The frame is stable and statically determinate.

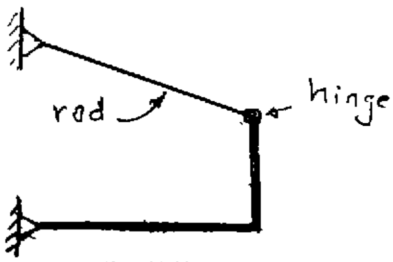


Fig.(11)

12)

$r = 9, b = 7, j = 8, c = 0$

$3b + r = 30,\quad 3j + c = 24$

$3b + r > 3j + c,\quad 30 > 24$

The frame is indeterminate to 6°.

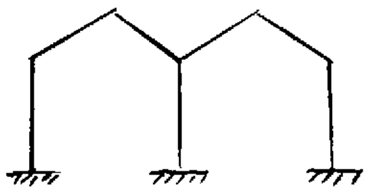


Fig.(12)

13)

Cut beams as shown, since all supports are fixed.

No. of beams cut = 8

The frame is indeterminate to (3x8) = 24°

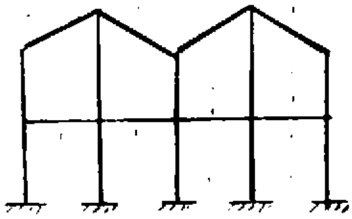


Fig.(13)

14)

Cut all beams of frame, since all supports are fixed.

No. of beams cut = 21

The frame is indeterminate to 3x21 = 63°

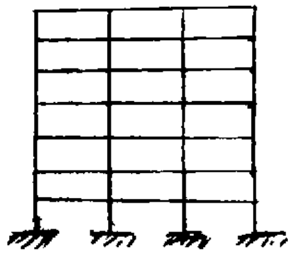


Fig.(14)

15)

$r = 3, b = 9, j = 8, c = 0$

$3b + r = 30, \quad 3j + c = 24, \quad 30 > 24$

The frame is indeterminate to 6°

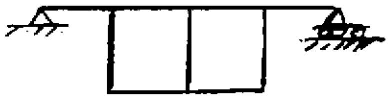


Fig.(15)

16)

$r = 6, b = 5, j = 5, c = 0$

$3b + r = 21, \quad 3j + c = 15, \quad 21 > 15$

The frame is indeterminate to 6°

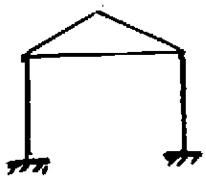


Fig.(16)

17)

$r = 8, b = 8, j = 8, c = 0$

$3b + r = 32, \quad 3j + c = 24, \quad 32 > 24$

The frame is indeterminate to 8°

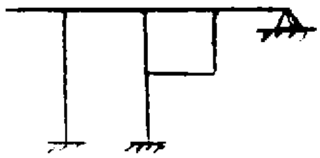


Fig.(17)

18)

$r = 10, b = 14, j = 12, c = 0$

$3b + r = 42 + 10 = 52, 3j + c = 36, 52 > 36$

The frame is indeterminate to 16°

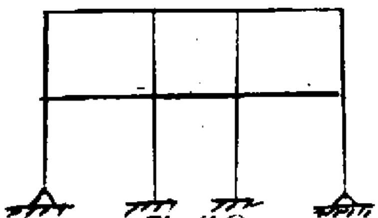


Fig.(18)

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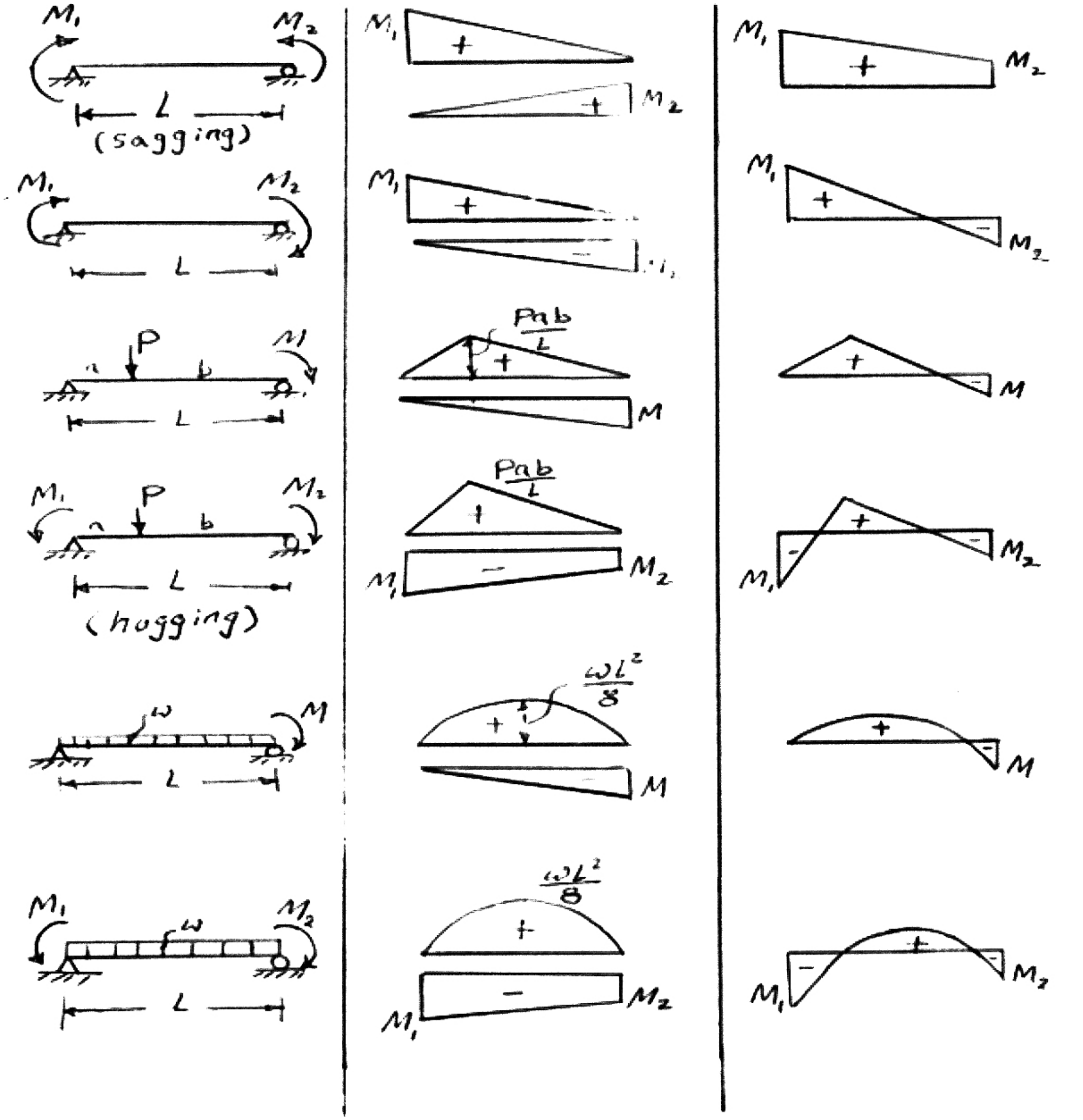
Sheet-2

انشاءات

Statically Determinate Beams
(S.F.D. & B.M.D.)

Dr. Maloof Mahmood

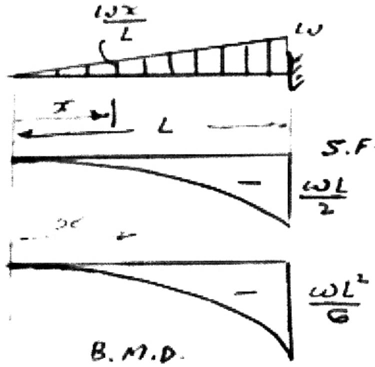
Case	Separate Moment Diagram	Combined Moment Diagram



Examples

Draw S.F.D. & B.M.D. for the following Beams:

Ex (1) Cantilever with triangular loading.

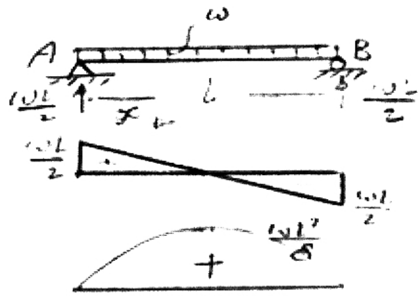


At any section, distance x from free end:

$V_x = -\frac{1}{2}\left(\frac{\omega x}{L}\right) \cdot x = -\frac{\omega x^2}{2L}$ — 2nd degree parabola

$M_x = -\frac{1}{2}\left(\frac{\omega x}{L}\right) \cdot x \cdot \frac{x}{3} = -\frac{\omega x^3}{6L}$ — curved eq.

Ex (2) U.d.L.



$V_x = \frac{\omega L}{2} - \omega x$

$M_x = \frac{\omega L}{2}x - \frac{\omega x^2}{2}$

$0 \le x \le \frac{L}{2} \Rightarrow \hat{M}_x = \frac{\omega L^2}{4} - \frac{\omega L^2}{8} = \frac{\omega L^2}{8}$

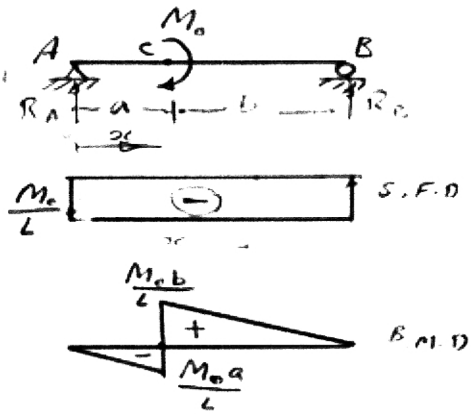
OR

Area of $\Delta = \frac{1}{2} \cdot \frac{\omega L}{2} \cdot \frac{L}{2} = \frac{\omega L^2}{8}$

OR $\frac{dM_x}{dx} = \frac{\omega L}{2} - \frac{\omega x}{2} = 0 \Rightarrow x = \frac{L}{2}$

$\therefore \hat{M}\Big]_{x=\frac{L}{2}} = \frac{\omega L^2}{8}$

Ex 3) $\Sigma M_B = 0 \Rightarrow R_A \cdot L + M_o = 0 \Rightarrow R_A = \frac{-M_o}{L}$



i.e., $R_A = \frac{-M_o}{L}\downarrow$, $R_B = \frac{+M_o}{L}\uparrow$

S.F.D. →

B.M.D.

$M_x = -\frac{M_o}{L}x$ — linear, $0 \le x \le a$

$M_x\Big]_{x=0} = 0$, $M_x\Big]_{x=a} = -\frac{M_o}{L}a$

For $a \le x \le L$

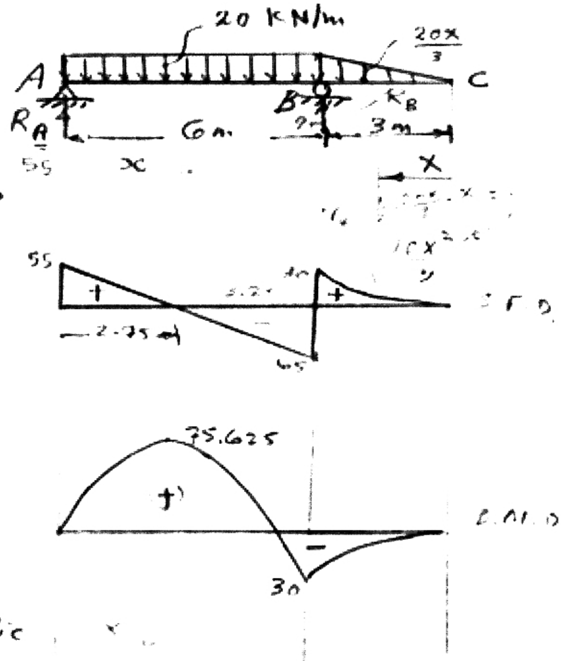
$M_x = -\frac{M_o}{L}x + M_o$

$M_x\Big]_{x=a} = -\frac{M_o}{L}a + M_o = M_o\left(-\frac{a}{L} + 1\right) = \frac{M_o b}{L}$

$M_x\Big]_{x=L} = -M_o + M_o = 0$

Ex. 4

Draw the S.F.D. & B.M.D. for the loaded beam shown.



—— . ——

$\sum M_B = 0$

$R_A \cdot 6 + (\frac{1}{2} \times 20 \times 3)\frac{3}{3} - (20 \times 6)3 = 0$

$\therefore R_A = 55$ KN , $R_B = 95$ KN

S.F.D.

$0 \le x \le 6$

$V_x = 55 - 20x$ (linear eq.)

$V_x\big]_{x=0} = 55$ KN , $V_x\big]_{x=6} = -65$ KN

At $V_x = 0 \Rightarrow 55 - 20x = 0$

$\therefore x = \frac{55}{20} = 2.75$ m

Taking origin at end C :

$V_X = \frac{1}{2} \cdot \frac{20}{3} X \cdot X = \frac{10X^2}{3}$ ← Parabolic

B.M.D.

$\hat{M} = \frac{1}{2} \times 55 \times 2.75 = 75.625$ KN-m

$M_B = 75.625 - \frac{1}{2} \times 3.25 \times 65 = -30$ KN-m

OR Moment of cantilever about B :

$M_B = (\frac{1}{2} \times 20 \times 3) \cdot \frac{3}{3} = 30$ KN-m

OR : for $0 \le x \le 6$

$M_x = 55x - \frac{20x^2}{2}$

$M_x\big]_{x=6} = 55 \times 6 - 10 \times 36$

$= 330 - 360 = -30$ KN-m

$A_1 = \frac{1}{3} bd$

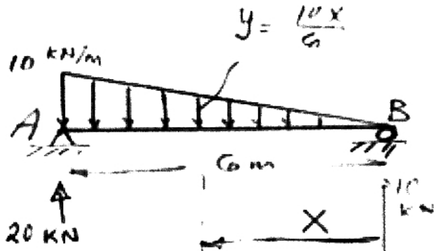
$A_2 = \frac{2}{3} bd$

$\bar{x}_1 = \frac{3}{4} b$

$\bar{x}_2 = \frac{3}{8} b$

Ex. 5

For the beam with triangular loading shown. Draw S.F.D. & B.M.D.

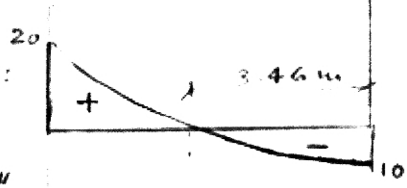


Take origin at B as shown.

1^{st} calculate the reactions at ends:

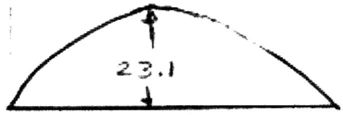
$\Sigma M_B = 0 \Rightarrow R_A \cdot 6 - (\frac{1}{2} \times 10 \times 6) \cdot \frac{2}{3} \times 6 = 0$

$\therefore \quad R_A = 20 \text{ kN} \quad , \quad R_B = \frac{1}{2} \times 10 \times 6 - 20 = \underline{\underline{10 \text{ kN}}}$



S.F.D. :

origin at B :



B.M.D.

$V_x = -10 + (\frac{1}{2} \times \frac{10}{6} x) \cdot x$

$\quad = \frac{10x^2}{12} - 10 \quad$ parabolic

$V_x\Big]_{x=0} = -10^{kN}, \quad V_x\Big]_{x=6} = +20 \text{ kN}$

Slope of S.F. $= \frac{dV_x}{dx} = \frac{10x}{6}$

$V_x = \frac{10x^2}{12} - 10 = 0$

$x^2 = 12 \Rightarrow x = 2\sqrt{3} = \underline{3.46 m}$ pt. of zero shear = max. moment

B.M.D :-

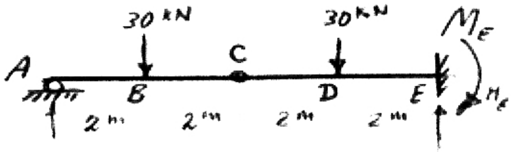
$M_x = -10x + (\frac{1}{2} \times \frac{10x^2}{6}) \cdot \frac{x}{3}$

$M\Big]_{x=3.46} = -10 \times 3.46 + \frac{10}{36}(3.46)^3 = -34.6 + 11.5$

$= \underline{\underline{-23.1}} \text{ kN-m}$

Ex. 6

Analyze the compound beam shown and draw the S.F.D and B.M.D



$r = 1+3 = 4$, $c = 1$

$\Sigma F_x = 0$, $H_E = 0$

$\therefore$ $r = 4 = c + 3 = 4 \Rightarrow$ Beam is stable & det.

To find the reactions, make a cut at C

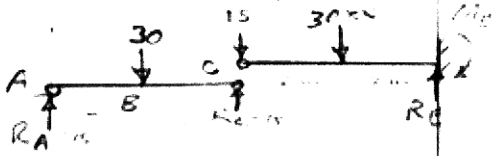
$\therefore$ From left part, $R_A = R_c = 15$ KN

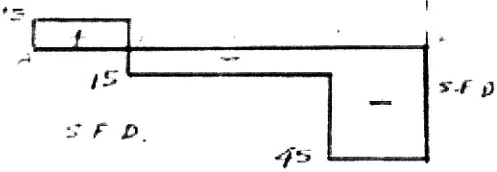
From right part, $R_E = 15 + 30 = 45$ KN

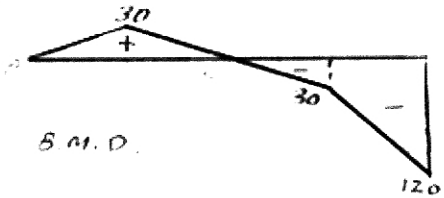
or from entire beam: $\Sigma F_y = 0$: $R_E = 60 - 15 = 45$ KN

From right portion, Take moments about E:

$M_E = 15 \times 4 + 30 \times 2 = 120$ KN-m







S.F.D

Draw S.F.D. as shown.

B.M.D.

Note: Since, there is a hinge at C, M_c should $= 0$

Draw B.M.D. from areas of S.F.D.

$M_B = 15 \times 2 = 30$ KN-m

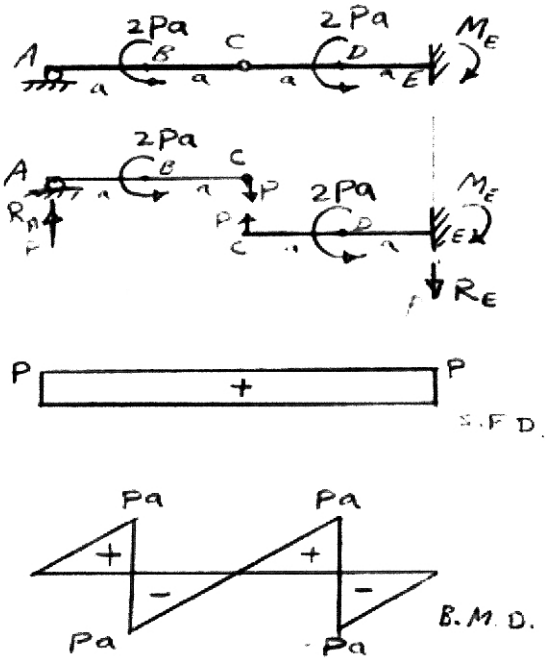
$M_c = 30 - 15 \times 2 = 0$

$M_D = 30 - 15 \times 4 = -30$ KN-m

$M_E = -30 - 45 \times 2 = -120$ KN-m.

Ex. 7

Draw S.F.D. & B.M.D. of the beam shown.



Make a cut at C:

$\Sigma M_C = 0 \Rightarrow R_A \cdot 2a - 2Pa = 0$

$\therefore R_A = P\uparrow$, for beam AC:

$\Sigma F_y = 0 \Rightarrow R_c = -P\downarrow$, $R_c' = P\uparrow$

Beam CE:

$\Sigma M_E = 0 \Rightarrow P \cdot 2a - 2Pa + M_E = 0$

$\therefore M_E = 0$

$\Sigma F_y = 0 \Rightarrow R_E = -P\downarrow$

S.F.D. As shown

B.M.D

$M_B = Pa$

$M_B' = Pa - 2Pa = -Pa$

$M_C = P \cdot 2a - 2Pa = 0$

$M_D = P \cdot 3a - 2Pa = Pa$

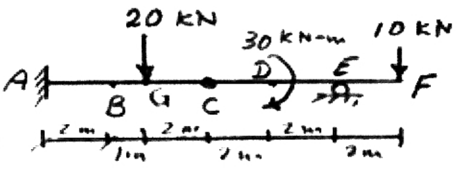
$M_D' = P \cdot 3a - 2Pa - 2Pa = -Pa$

$M_E = P \cdot 4a - 2Pa - 2Pa = 0$

Ex. 9

Analyze the compound beam shown below and draw S.F.D & B.M.D. Find also the shear and moment at point 'B' which is 2m from the support 'A'.

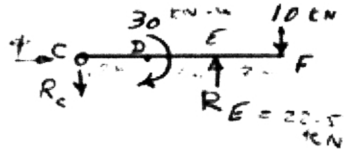
Check stability of beam:



$r = 3+1 = 4$, $c = 1 \Rightarrow c+3 = 4$

$\therefore r = c+3 \Rightarrow$ the beam is stable & det.

Make a cut at C:



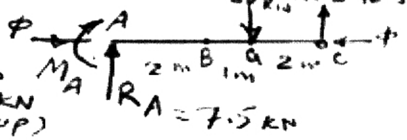
For the right portion of beam:

$\Sigma M_c = 0 \Rightarrow 10(6) + 30 - R_E(4) = 0$

$\therefore R_E = \frac{90}{4} = 22.5 \uparrow$ KN (up)

$\Sigma F_y = 0 \Rightarrow R_c = 10 - 22.5 = -12.5$ KN $\downarrow$ (down)

For left portion of beam:



$\Sigma F_y = 0 \Rightarrow R_A = 20 - 12.5 = 7.5 \uparrow$ KN (up)

$\Sigma M_c = 0$:

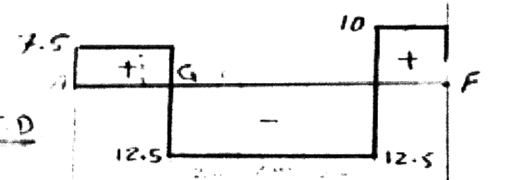
$M_A + 7.5(5) - 20(2) = 0$

$\therefore M_A = +2.5$ KN-m

S.F.D.

Draw S.F.D.

S.F.D

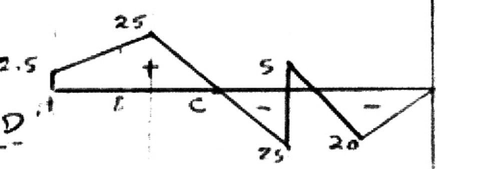


Draw B.M.D. M_G =

S.F. at B = + 7.5 KN

$M_B = 2.5 + 7.5(2) = \underline{17.5}$ KN-m

B.M.D



Al-Mansour University College
قسم الهندسة المدنية
Civil Eng. Dept.
المرحلة الثالثة
3rd. Stage

Theory of Structures

2022 - 2023

Sheet-3

انشاءات

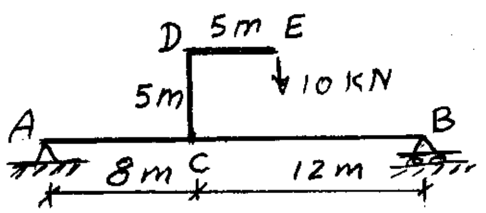
Rigid Frames
(Solved Problems)

Dr. Maloof Mahmood

Problem Sheet (2) – Structures

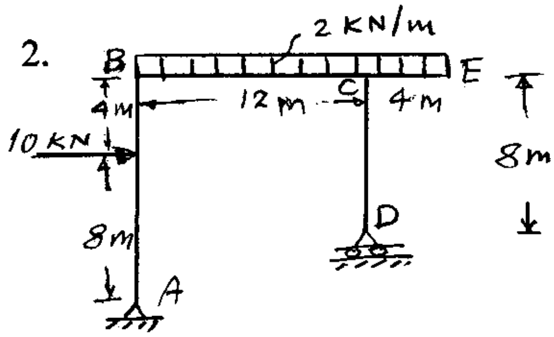
Analyze each of the frames shown below and draw A.F.D., S.F.D. & B.M.D.

1.



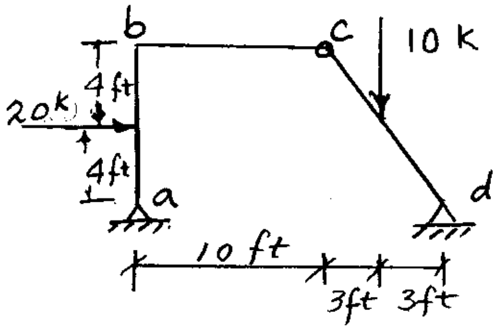
Ans. $(M_c)_{max}$ = 78 kN-m

2.



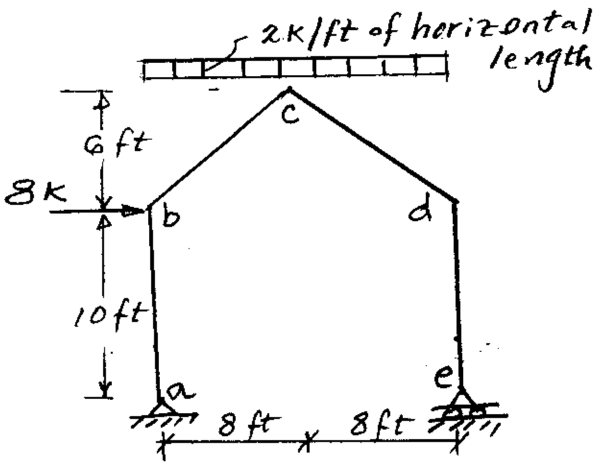
Ans. M_B = 80 kN-m

3.



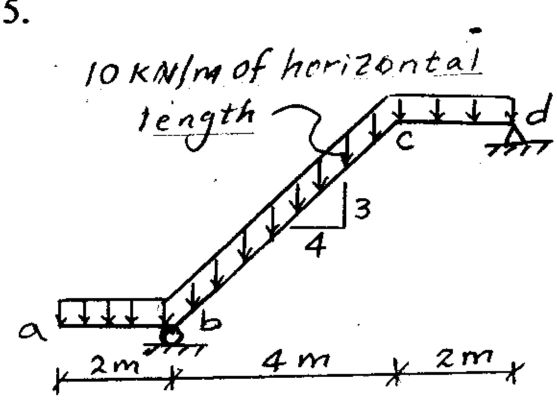
Ans. M_b = 31.25 kip-ft

4.



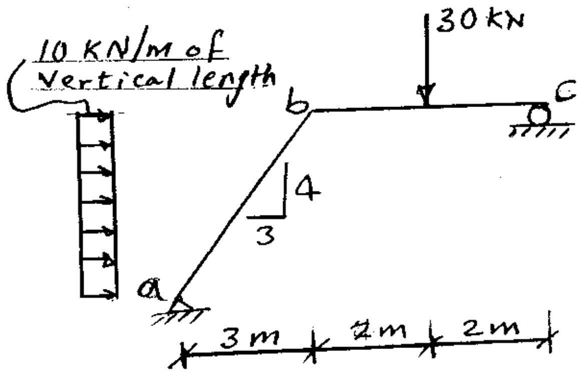
Ans. M_b = 80 kip-ft, M_c = 104 kip-ft

5.



Ans. M_c = 33.3 kN-m

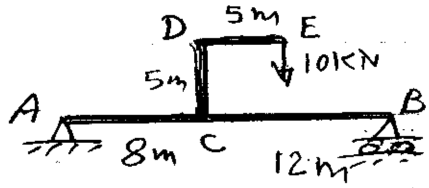
6.



Ans. M_b = 71.4 kN-m

Prob.(1)

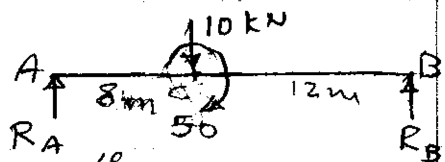
Analyze the frame shown and draw A.F.D., S.F.D. and B.M.D.

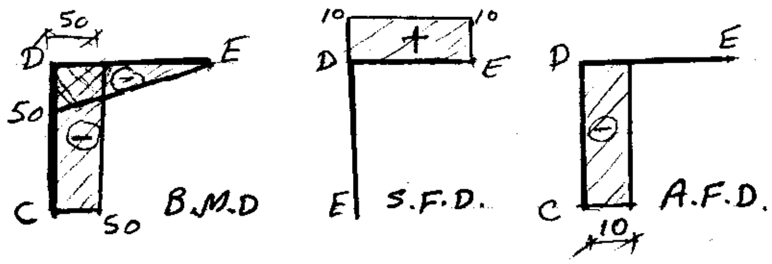


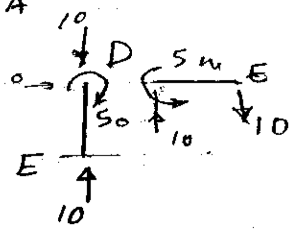
$\Sigma M_B = 0$

$R_A(20) + 50 - 10(12) = 0$

$R_A = \frac{70}{20} = 3.5\ KN$, $\Sigma F_x = 0 \Rightarrow R_B = 6.5\ KN$







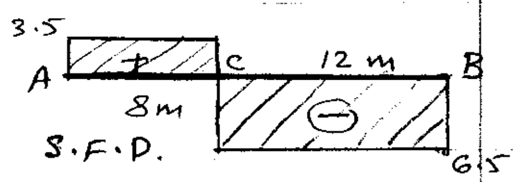
B.M.D.

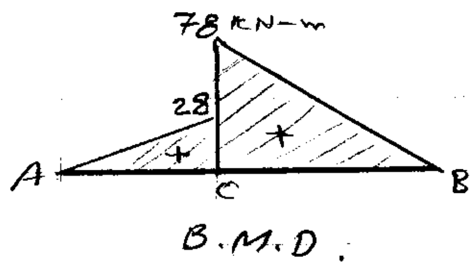
$AC \Rightarrow M_c = 3.5 \times 8 = 28\ KN\text{-}m$

$M_c = 28 + 50 = 78\ KN\text{-}m$

$M_B = 78 - 6.5(12) =$

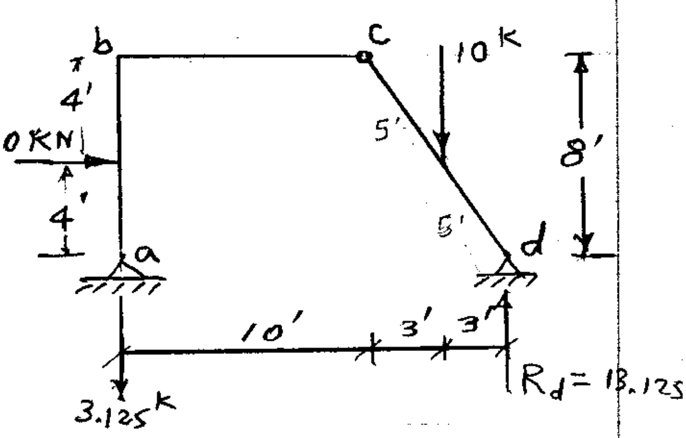
$= 78 - 78 = 0$





Prob.(3)

Analyze the frame shown and draw A.F.D., S.F.D.&B.M.D.

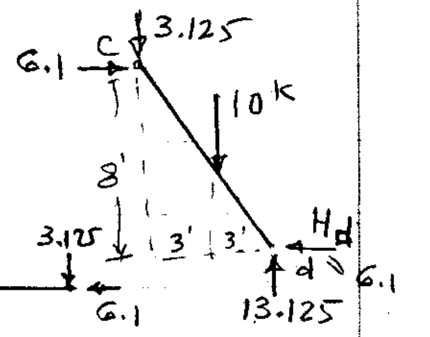


$\overset{+\circlearrowleft}{\sum} M_d = 0$ for the whole frame:

$R_a(16) + \overset{80}{20(4)} - \overset{30}{10(3)} = 0$

$\boxed{R_a} = -\frac{50}{16} = \boxed{-3.125\ k\downarrow}$

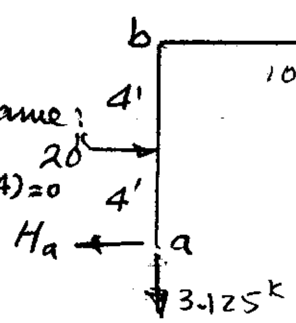
$\sum F_y = 0 \Rightarrow \boxed{R_D} = 3.125 + 10 = \boxed{13.125^k \uparrow}$



Make a cut at c ; right portion:

$\overset{+\circlearrowleft}{\sum} M_c = 0 \Rightarrow 10(3) + H_d(8) - \underset{=78.75}{13.125(6)} = 0$

$\therefore \boxed{H_d} = \frac{48.75}{8} \cong \boxed{\overleftarrow{6.1}^{k}}$



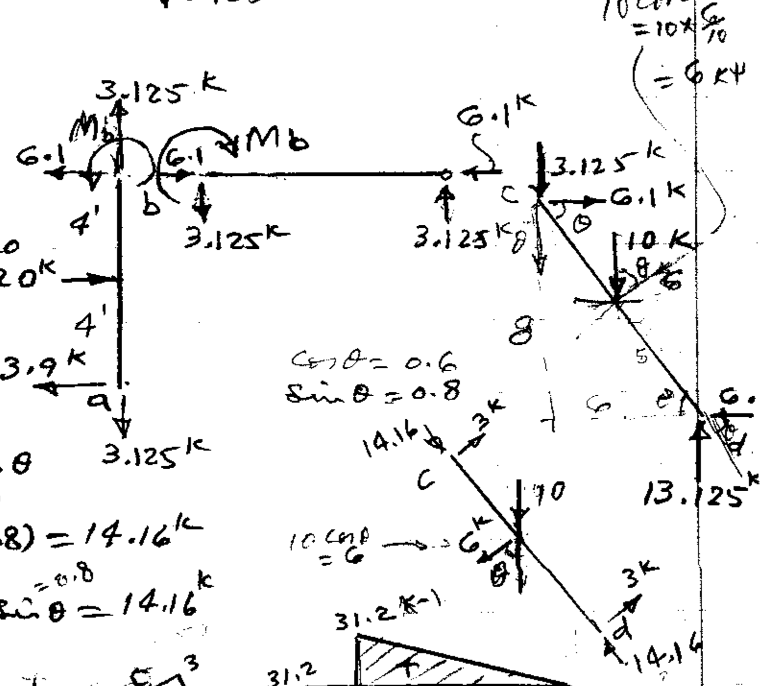
Take Left portion of frame:

$\overset{+\circlearrowleft}{\sum} M_c = 0 \Rightarrow -3.125(10) + H_a(8) - 20(4) = 0$

$\therefore \boxed{H_a} = \frac{111.25}{8} \cong 13.9^k$

Check $\sum F_x = 0 \Rightarrow H_a + H_d = 20$

$13.9 + 6.1 = 20^k \Rightarrow OK$



Take a cut at (b):

$\overset{+\circlearrowleft}{\sum} M_b = 0 \Rightarrow \overset{111.2}{13.9(8)} - \overset{80}{20(4)} - M_b = 0$

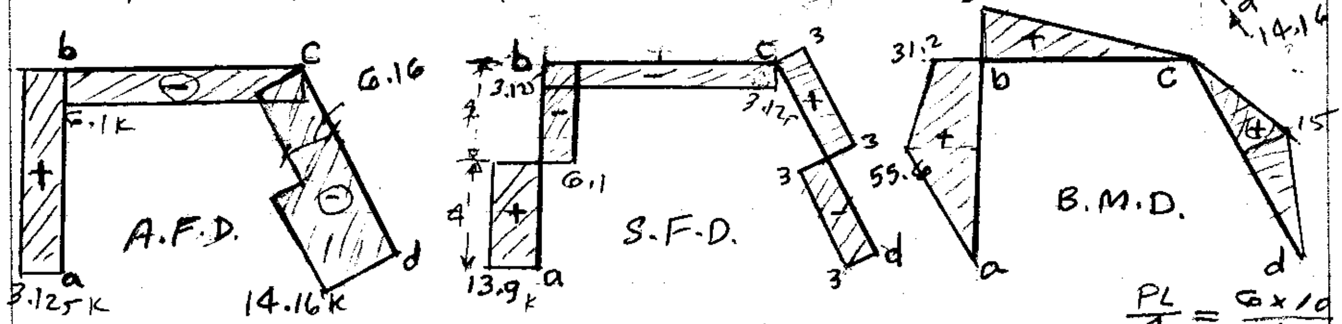
$\therefore M_b = 31.2^k$

Axial force at c (bar cd):

$= 6.1\cos\theta + 3.125\sin\theta + 10\sin\theta$

$= \underset{=3.66}{6.1 \times (0.6)} + \underset{2.5}{3.125(0.8)} + \underset{8.0}{10(0.8)} = 14.16^k$

Axial force at (d) $= 6.1\overset{=0.6}{\cos\theta} + 13.125\overset{=0.8}{\sin\theta} = 14.16^k$



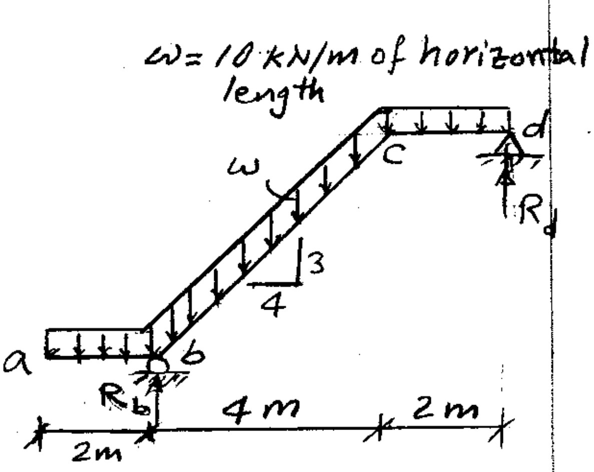
$\frac{PL}{4} = \frac{6 \times 10}{4} = 15$ kN-L

Axial force at 'c' = 6.1 cos θ 3.125 sin θ

$= 6.1 \times \frac{6}{10} + 3.125 \times \frac{8}{10} = 3.66 + 2.50 = 6.16$ k

Prob. 5

Analyze the frame shown and draw the A.F.D., S.F.D. & B.M.D.



For the whole frame:

$\overset{+}{\curvearrowleft}\sum M_d = 0 \Rightarrow R_b(6) - 10(8)(4) = 0$

$\therefore \boxed{R_b = 53.33 \text{ KN}}$

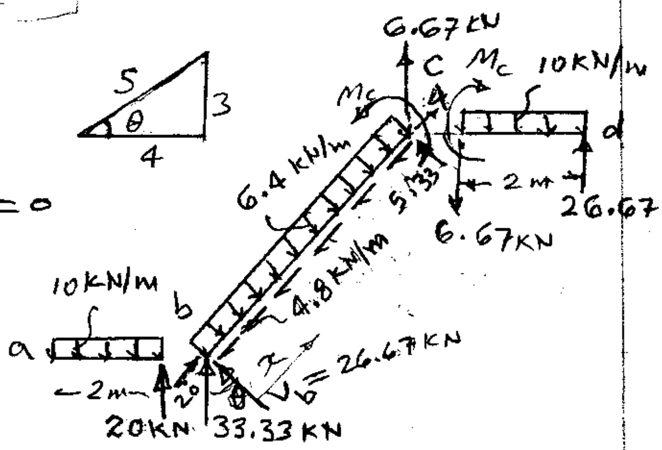
$\sum F_y = 0 \Rightarrow \boxed{R_d} = 80 - 53.33 = \boxed{26.67 \text{ KN}}$

Take a section at (c) and at (b):

For the portion (cd):

$\overset{+}{\curvearrowleft}\sum M_c = 0 \Rightarrow 10(2)(1) + M_c - 26.67(2) = 0$

$M_c = \underline{33.3}$ KN-m



For beam (bc):

$(10\times4)(\frac{4}{5}) = 32$ KN

$\frac{32}{5} = 6.4$ KN/m $\perp$ (bc)

$(10\times4)(\frac{3}{5}) = 24$ KN parallel to bc

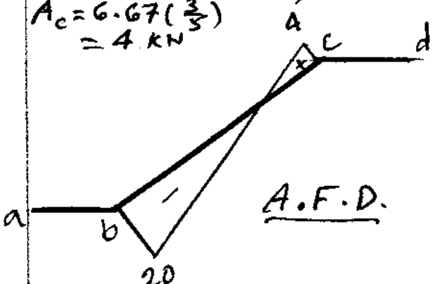
$\frac{24}{5} = 4.8$ KN/m along bc

$V_b = 33.33\cos\theta = 33.33(0.8) = \underline{26.66}$ KN

$V_c = 6.4\times5 - 26.67 = \underline{5.33}$ KN

Axial force at (b) $= 33.33(\frac{3}{5}) = 20$ KN

$A_c = 6.67(\frac{3}{5}) = 4$ KN

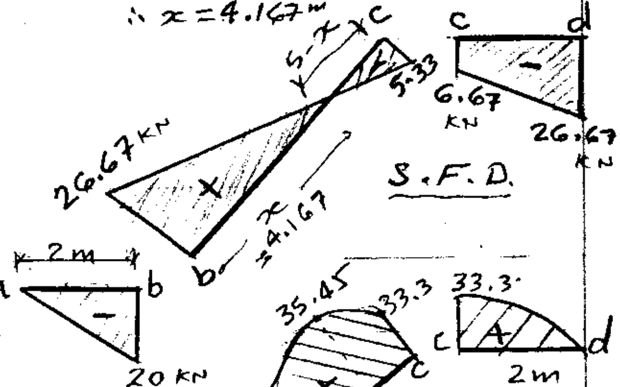


A.F.D.

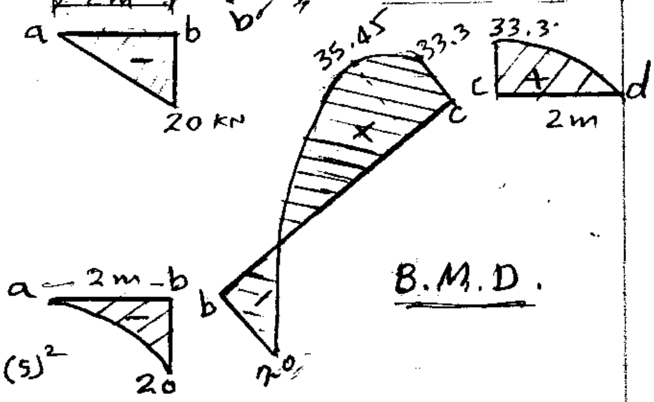
$\frac{26.67}{5.33} = \frac{x}{5-x}$ $(=5)$

$x = 25 - 5x$

$\therefore x = 4.167$ m



S.F.D.



B.M.D.

B.M. Beam bc:

$M_x = -20 + 26.66x - \frac{6.4x^2}{2}$ (3.2)

$M_x\Big]_{x=0} = -20$ KN-m, $M_x\Big]_{x=5} = -20 + 26.66(5) - 3.2(5)^2$

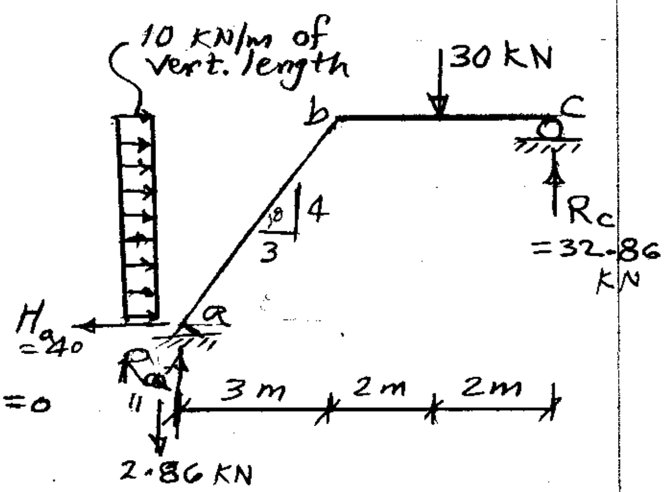
$= -20 + 133.3 - 80 = 33.3$ KN-m

$M_x\Big]_{x=4.167} = \hat{M} = -20 + 26.66(4.167) - 3.2(4.167)^2 = \underline{35.45}$ KN-m $\quad (111, \ 55.55)$

7

Prob. 6

Analyze the frame shown and draw A.F.D., S.F.D. & B.M.D.



For the whole frame,

$\Sigma F_x = 0 \Rightarrow H_a = 10 \times 4 = 40$ KN

$\overset{+}{\curvearrowleft}\Sigma M_a = 0 \Rightarrow -R_c(7) + 30(5) + 40(2) = 0$

$R_c = \frac{230}{7} = 32.86$ KN $\uparrow$

$\Sigma F_y = 0 \Rightarrow R_a = 32.86 - 30 = -2.86$ KN $\downarrow$

Take a section at (b):

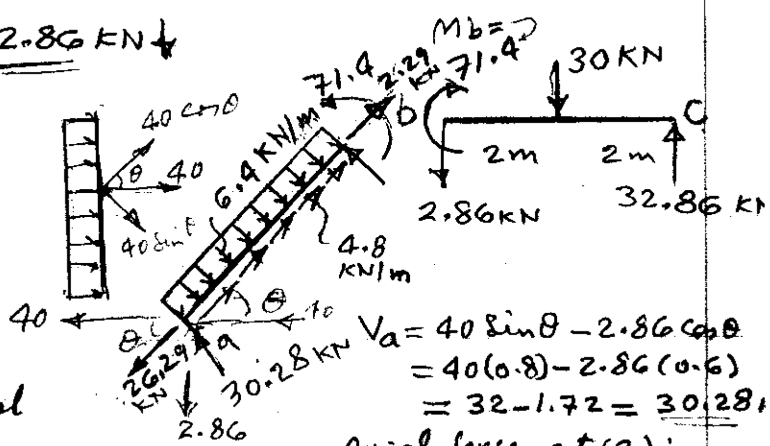
For beam (ab)

$40 \sin\theta = 40(\frac{4}{5}) = 32$ KN

$\frac{32}{5} = 6.4$ KN/m $\perp^r$ ab

$40 \cos\theta = 40(\frac{3}{5}) = 24$ KN parallel to (ab)

$\frac{24}{5} = 4.8$ KN/m along ab



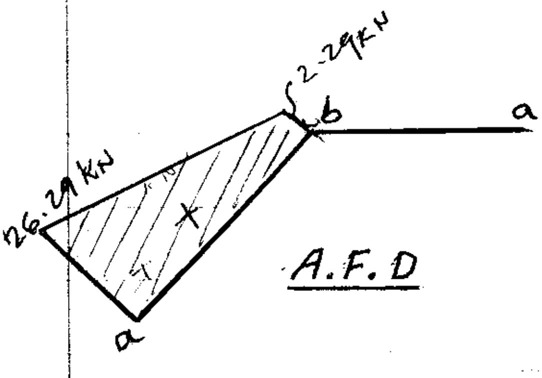
$V_a = 40\sin\theta - 2.86\cos\theta$
$= 40(0.8) - 2.86(0.6)$
$= 32 - 1.72 = 30.28$

Axial force at (a):
$= 40(0.6) + 2.86(0.8)$
$= 24 + 2.29 = 26.29$ KN

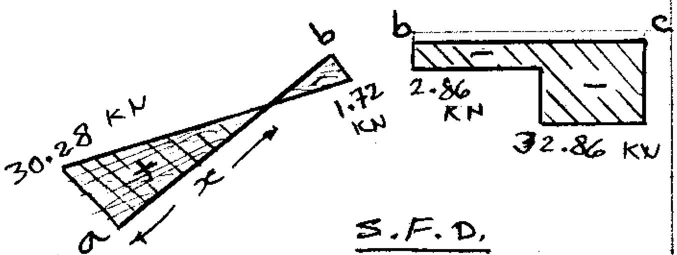
Axial force at (b):
$= 26.29 - 4.8(5) = 2.29$ KN

For beam (bc): $\overset{+}{\curvearrowleft}\Sigma M_b = 0 \Rightarrow M_b + \underset{=60}{30(2)} - \underset{=131.4}{32.86(4)} = 0$

$\therefore M_b = 71.4$ KN-m



A.F.D



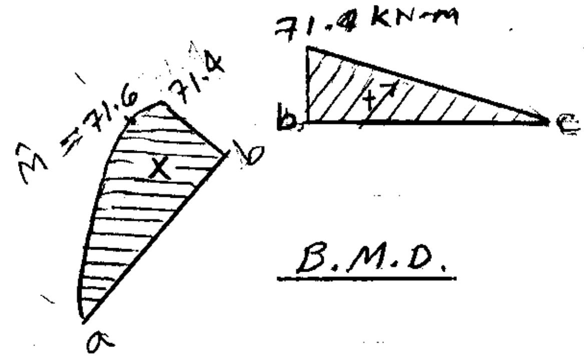
S.F.D.

$\frac{30.28}{1.72} = \frac{x}{5-x}$ (=17.6)

$x = 88 - 17.6x$

$x = \frac{88}{18.6} = 4.73$ m

$\hat{M} = 30.28 \times (4.73) = 71.6$ KN-m



B.M.D.