Al-Mansour University College

قسم الهندسة المدنية
المرحلة الثالثة

Civil Eng. Dept
3rd. Stage

Reinforced Concrete Design

2022 – 2023

Lec.1

خرسانة مسلحة

Refernces & Syllabus

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Lecture # 01: References & Syllabus

REFERENCES:

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2- Nawy, E. G., "***Reinforced Concrete: A Fundamental Approach***", 5th Edition, 2005, pp. 848.

3- Nilson, A. H., et. al., "***Design of Concrete Structures***", 13th Edition, 2004, pp. 783.

4- Nilson, A. H. and Winter, G., "***Design of Concrete Structures***", 11th Edition, 1991, pp. 922.

5- Ghoneim, M. and El-Mihilmy, M., "***Design of Reinforced Concrete Structures***", 1st Edition, 2008, Vol. 1,2 & 3.

6- Reynolds, C. E. and Steedman, J. C., "***Reinforced Concrete Designer's Handbook***", 10th Edition, 1999, pp. 449.

7- Ray, S. S., "***Reinforced Concrete: Analysis & Design***", 1st Edition, 1995, pp. 556.

8- Merritt, F. S. and Ricketts, J. T., "***Building Design & Construction Handbook***", 6th Edition, 2001, pp. 1722.

Chapter 6: Torsional Design of Beams

Chapter 07: Serviceability (Crack-Width and Deflection)

Chapter 08: Bond Stresses, Splices & Development Lengths

Chapter 09: One-Way & Continuous Slabs Design

Chapter 10: Two-Way Slabs Design

The percent elongation at fracture, which varies with the grade, bar diameter and manufacturing source, ranges from 4.5 to 12% over an 8" (203 mm) gage length. For most steels, the behavior is assumed to be elastoplastic, and Young's modulus is taken as 29 x 10^6 psi (200 x 10^6 MPa). Table 1, presents the reinforcement-grade strength.

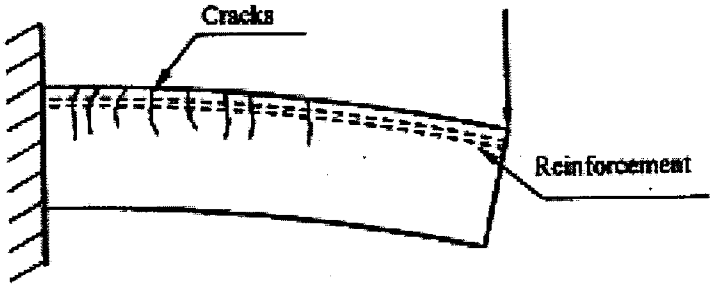
Table 1: Reinforcement strength and grades.

1982 Standard Type	Minimum Yield Point or Yield Strength, f_y (psi)	Ultimate Strength, f_u (psi)
Billet steel (A615)		
Grade 40	40,000	70,000
Grade 60	60,000	90,000
Axle steel (A617)		
Grade 40	40,000	70,000
Grade 60	60,000	90,000
Low-alloy steel (A706): Grade 60	60,000	80,000
Deformed wire		
Reinforced	75,000	85,000
Fabric	70,000	80,000
Smooth wire		
Reinforced	70,000	80,000
Fabric	65,000, 56,000	75,000, 70,000

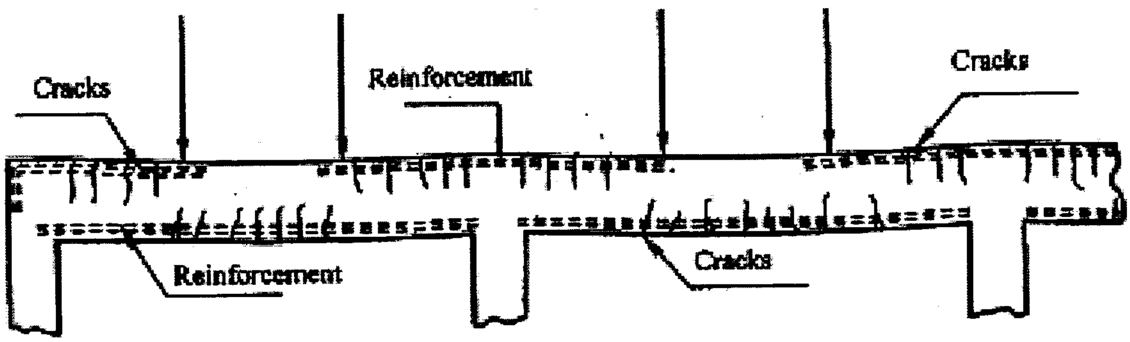
Welded wire fabric is increasingly used for slabs because of the ease of placing the fabric sheets, control of reinforcement spacing and better bond. The fabric reinforcement is made of smooth or deformed wires that run in perpendicular directions and are welded together at intersections.

Chapter 11: Short Columns

Chapter 12: Long Columns



B-Cantilever beam



C-Continuous beam

Fig. 1: Reinforcement placement in different types of beams.

1.2 Concrete

Concrete is a stonelike material obtained by permitting a carefully proportioned mixture of cement, sand and gravel or other aggregate, and water to harden in forms of the shape and dimensions of the desired structure. The bulk of the material consists of fine and course aggregate. Cement and water interact chemically to bind the aggregate particles into a solid mass.

Additional water, over and above that needed for this chemical reactions, in necessary to give the mixture the workability that enables it to fill the forms and surround the embedded reinforcing steel prior to hardening. Concrete in a wide range of strength properties can be obtained by appropriate adjustment of the proportions of the constituent materials.

1.3 Mechanical Properties of Concrete

1.3.1 Compressive Strength:

Many factors affect the concrete compressive strength such as the water/cement ratio, the type of cement, aggregate properties, age of concrete, and time of curing. The most important factor of all is the water/cement ratio. The lower w/c ratio with good workability leads to higher concrete compressive strength. Increasing the w/c ratio from 0.45 to 0.65 can decrease the compressive strength by 30-40%

The compressive strength of concrete is usually determined by loading a 150 mm cube up to failure in uniaxial compression after 28 days of casting and is referred to as f'c. It should be mentioned that in other countries such as the U.S.A & Canada, the compressive strength is measured by compression tests on 150 mm x 300 mm cylinder tested after 28 days of moist curing.

Since concrete is used mostly in compression, its compressive stress-strain curve is of a prime interest. Fig. 2 shows a typical set of such curves obtained from uniaxial compression test of cylinder. They consist of an initial relatively straight elastic portion in which stresses and strains are closely proportional, then begin to curve to reach a maximum value at a strain of 0.002 to 0.003. These is a descending branch after the peak stress is reached.

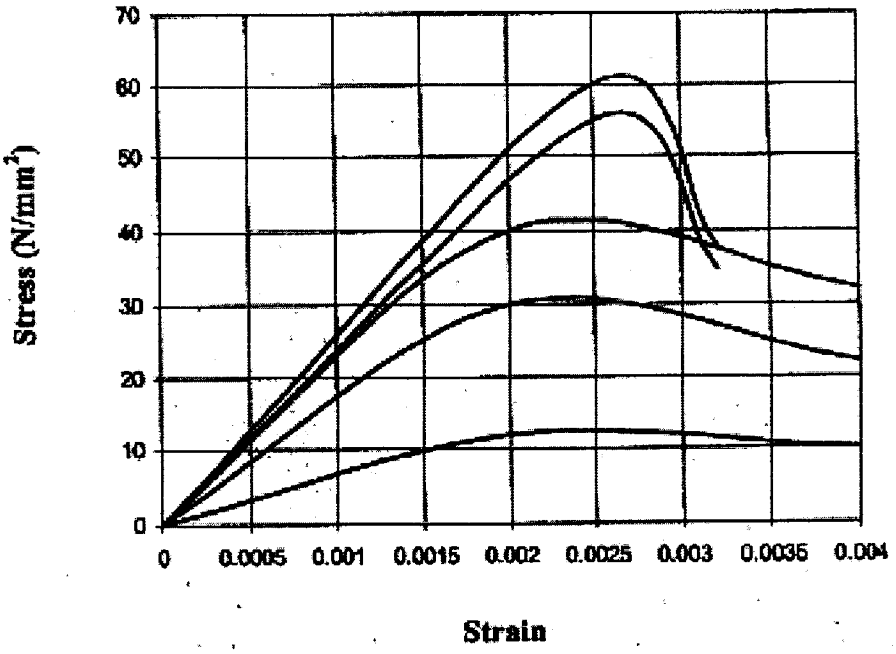


Fig. 2: Typical concrete stress-strain curves in compression

1.3.2 Tensile Strength:

Experimental tests indicate that tensile strength of concrete is highly variable and ranges from 8-12% of its compressive strength. The actual value depends on the type of test and crack propagation pattern at failure. Tensile strength is usually determined by the bending test or by the split cylinder test (as shown in the Figs. 3 &4). The ACI-code states that the value of concrete tensile strength can be taken from experimental tests as follows:

60% from the concrete tensile strength determined from bending test.

85% from the concrete tensile strength determined from split cylinder test.

Where λ is a modification factor reflecting the reduced mechanical properties of lightweight concrete.

1.3.3 Modulus of elasticity

It is clear from the stress-strain curve of the concrete shown in Fig. 5 that the relation between the stress and the strain is not linear. Thus, the modulus of elasticity changes from point to another. Furthermore, its value varies with different concrete strengths, concrete age, type of loading, and characteristics of cement and aggregate. The initial tangent is sometimes used to estimate the concrete modulus of elasticity, in which, the slope of the stress-strain curve of concrete at the origin is evaluated. The ACI-code (8.5.1) gives the following formula for estimating the concrete modulus of elasticity:

$E_c = w_c^{1.5}\ 0.043\ \sqrt{f'c}$ (in MPa) for values of w_c between 1440 and 2560 kg/m^3.

$E_c = 4700\ \sqrt{f'c}$ (in MPa) for normal weight concrete.

Where, w_c is the concrete unit weight in kg/m^3 and $f'c$ is the is the concrete compressive strength in MPa.

The magnitude of the modulus of elasticity is required when calculating deflection, evaluating bracing condition and cracking of a structure.

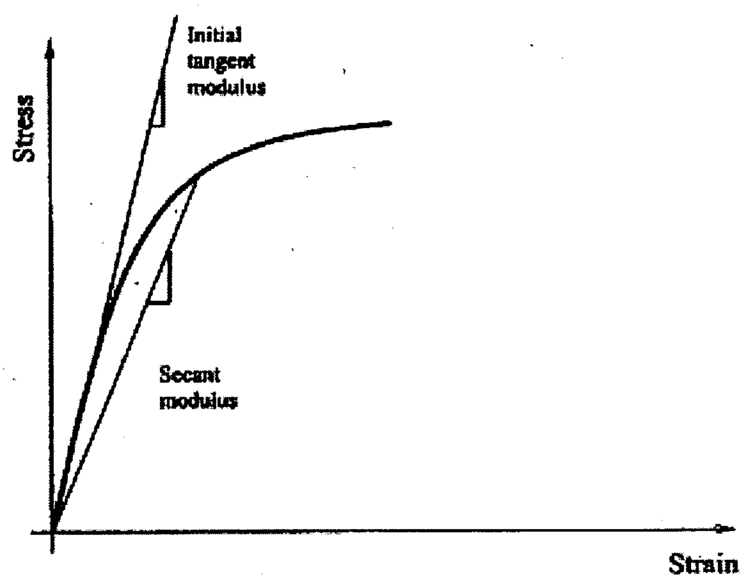


Fig. 5: Initial Tangent Modulus of Concrete

1.3.4 Strength of Concrete under Biaxial Loading

Portions of many concrete members may be subjected to stresses in two perpendicular directions (biaxial state). The strength of the concrete is affected greatly by the applied stress in the perpendicular direction as shown in Fig. 6. All stresses are normalized in terms of the uniaxial compressive strength (f'c). The curve has three regions: biaxial compression-compression, biaxial tension-tension, biaxial tension-compression.

In compression-compression zone, it can be seen that the compressive strength of the concrete can be increased by 20-25% when applying compressive stress in the perpendicular direction. In the tension-tension zone, it is clear that the tensile strength of the concrete is not affected by the presence of tension stresses in the normal direction.

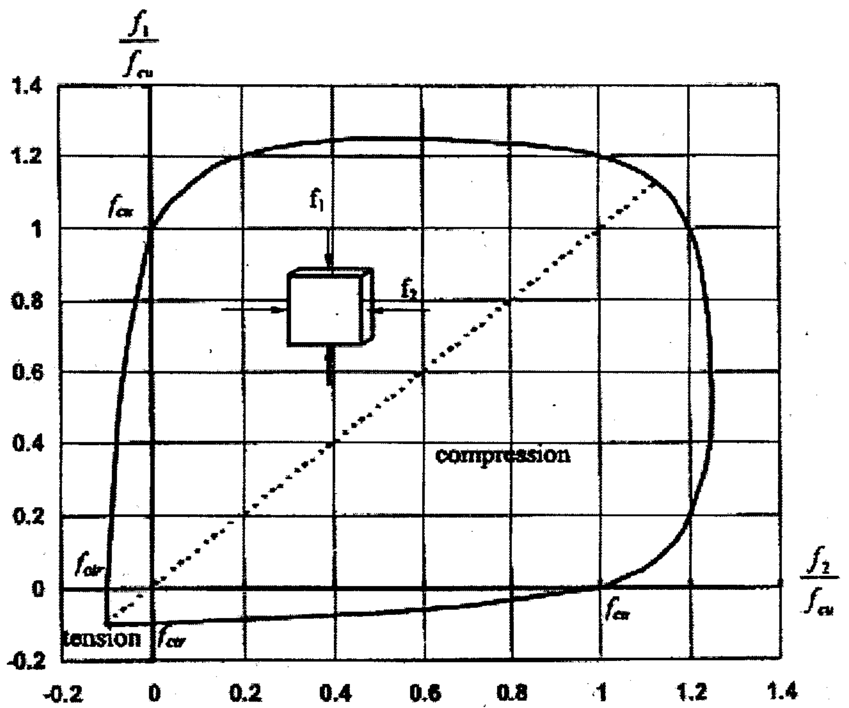


Fig. 6: Strength of concrete under biaxial stress.

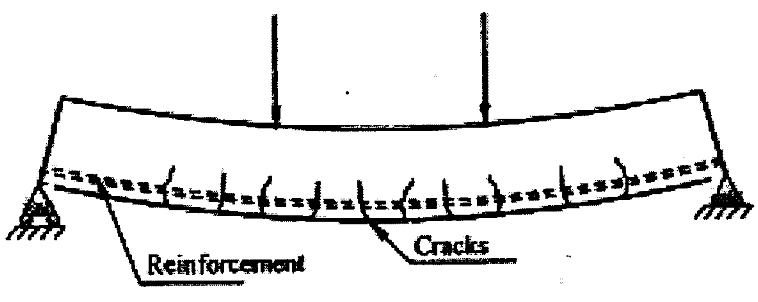
Lecture # 02: Properties of Concrete and Reinforcing Steel

1.1 Reinforced Concrete

Concrete is stong in compression but weak in tension. Therfore, reinforcement is needed to resist the tensile stresses resulting from the induced loads. Additional reinforcement is occasionally used to reinfoce the compression zone of concrete beam sections. Such steel is necessary for heavy loads in order to reduce long-term deflections.

Concrete and steel work together in reinforced concrete structures. The advantages of each material seem to compensate for the disadvantages of the other. The tensile strength of steel is approximately equal to 100-140 times the tensile strength of the usual concrete mix. Also, the two materials bond together very well with no slippage and thus act together as one unit in reisiting the applied loads.

The disadvantages of steel is corrosion, but the concrete surrounding the reinforcement provides as excellent protection. The strength of the exposed steel subjected to fire is close to zero, but again the enclosure of the reinforcement in the concrete produced very satisfactory fire protection. Finally, concrete and steel work very well together in temperature changes because their coefficient of thermal expansion are almost the same. The coefficient of thermal expansion for steel is 6.5 x 10^{-6}, while that for concrete is about 5.5 x 10^{-6}. Fig. 1 shows typical reinforcement arrangements for different types of beams.



A- Simple beam

1.3.5 Shrinkage

As the concrete dries, it shrinks in volume due to the excess water used in concrete mixing. The shortening of the concrete per unit length due to moisture loss is called shrinkage strain. The magnitude of the shrinkage strain is a function of the initial water content, the composition of the concrete and the relative humidity of the surroundings.

Shrinkage is also a function of member size and shape. Drying shrinkage occurs as the moisture diffuses out of the concrete. As a result, the exterior shrinks more rapidly than the interior. This leads to tensile stresses in the outer skin of the concrete member and compressive stress in its interior. The rate of the shrinkage increases as the exposed area to the volume increases.

Although shrinkage continues for many years as shown in Fig. 7, approximately 90% of the ultimate shrinkage occurs during the first year.

Fig. 7: Variation of shrinkage with time for a typical concrete mix.

1.3.6 Creep

When a reinforced concrete members is loaded, an initial deformation occurs as shown in Fig. 8. Experimental studies show that this initial deformation increases with time under constant loading.

The total deformation is usually divided in to two parts:

1- Initial deformation.
2- Time-dependent deformation "Creep".

As shown in the figure, more than 75% of the creep deformation occurs during the first year and 95% in the first five years. If the load is removed, immediate recovery occurs, followed by a time dependent recovery (creep recovery). The member will never recover all the developed deformation and there will be a non-recoverable deformation called permanent deformation.

The creep deformations are within a range of 1-3 times the instantaneous elastic deformations. Creep causes an increase in the deflection with time that may lead to undesirable deformation of the member. Thus, the deflection must be ensure that the deformations are within the allowable limits of the code.

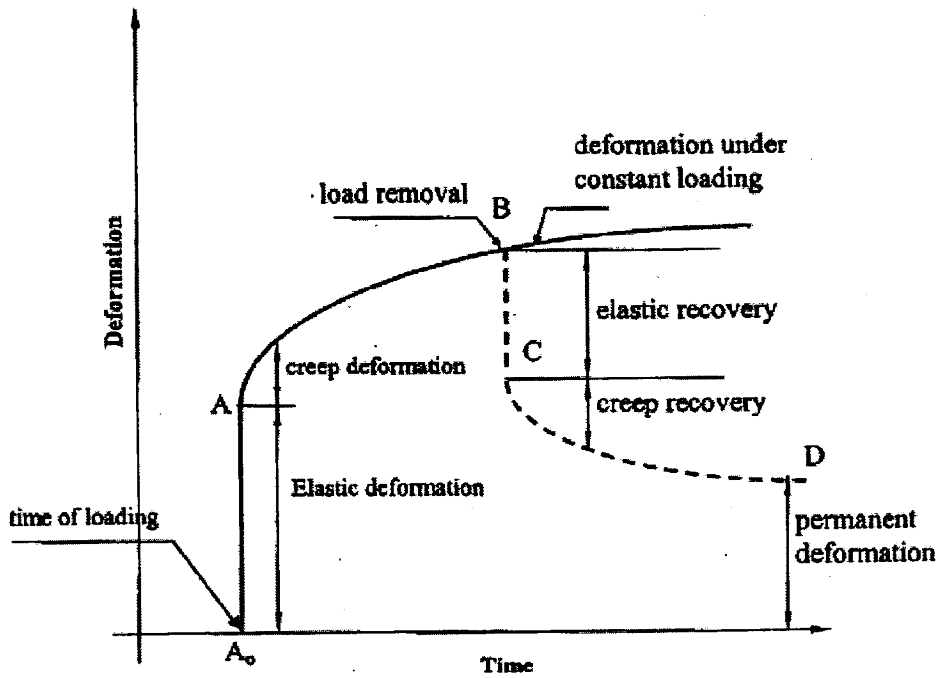


Fig. 8: Elastic and creep deformations of concrete.

2.2 Loads

One of the basic duties of the civil engineer is to estimate the loads applied to the structure. There are many types of loads that the structure can subject to. These are:

2.2.1 Dead Loads

Represents the structure selfweight in addition to all other loads that applied along the structure life like finishings and flooring and roofing loads. Those loads can be calculated by multiplying the volume by the material density. Table 2-1 illustrates the typical densities of some construction materials.

Table 2-1: Typical densities for some construction materials.

Material	**Typical Density (kg/m^3)**
Construction Bricks	20
Cement	14
Gypsum	12
Plain Concrete	23
Reinforced Concrete	24
Dry Soil	16
Hollow Blocks	14
Sand	17
Reinforcing Bars	78
Thermostone	9
Waterproofing	14
Cement Paste	20
Gypsum Plastering	20

Fig. 10 shows typical stress-strain curves for steel reinforcement. Reinforcing bars are available in three grades; 40, 60 and 80 steels. They have corresponding yield strengths of 40,000, 60,000 and 80,000 psi (276, 414 and 552 MPa) respectively and generally have well-defined yield points as shown in Fig. 11.

For steel that lack a well-defined yield point, the yield -strength value is taken as the strength corresponding to a unit strain of 0.005 for grades 40 and 60 steel and 0.0035 for grade 80 steel. The ultimate tensile strengths corresponding to the 40, 60 and 80 grade steel are 70,000, 90,000 and 100,000 psi (483, 621 and 690 MPa), and some steel types are given in the Table below.

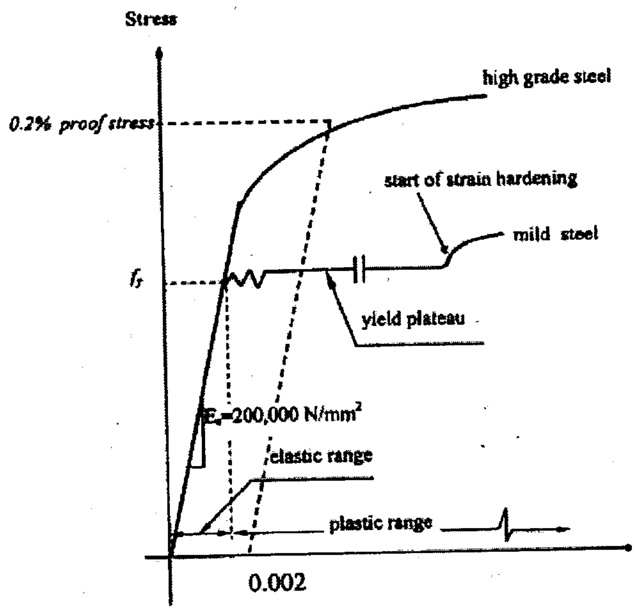


Fig. 10: Typical stress-strain curve for mild and high grade steels.

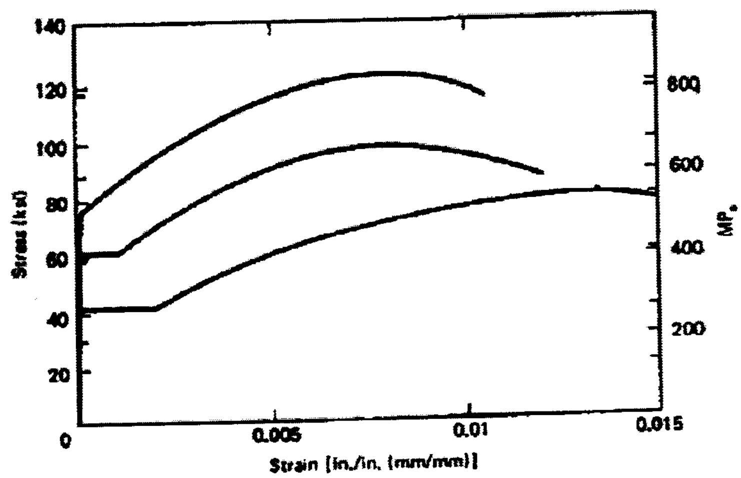


Fig. 11: Typical stress-strain diagram for various steel grades.

Lecture # 03: Analysis and Design Methods

2.1 Introduction

Structural engineering includes two main issues; anaylsis and design of structures. The analysis consists of evaluating the member end forces, joints displacements, shear and bending moment diagrams, and member bending strength to determine the maximum forces for design purposes. While the design includes evaluating the section dimensions, reinforcement amount and details for the concrete members.

Both analysis and design of concrete members are interacting each other and can not be separated. Since, in the analysis, the moment of inertia and the section area need to be given, i.e., the member can not be analyzed befor the design stage. On the other hand, The design stage requires that the acting forces on the member to be known, that is mean, the member can not be designed before analyzing it.

Based on the above, the analysis-design process represents a closed cycle and in order to start it, a conceptual (priliminary) design to be made for estimating the section size or assuming values for the member section area and the moment of inertia. As the preliminary design finished, the structure has to be analyzed to evaluate the forces and displacements.

The analysis-design process may continue untile an appropirate design follow the basic requirements to be obtained. The basic requirements include that the induced stresses and displacements are less than the maximum stresses and displacements respecively while the cost of the structure to be minimized.

Before starting the analysis process, the applied loads on the structure has to be estimated as illustrated in the next item.

Let $n = E_s / E_c \rightarrow \therefore$ $\boxed{f_s = n . f_{cs}}$

Where:

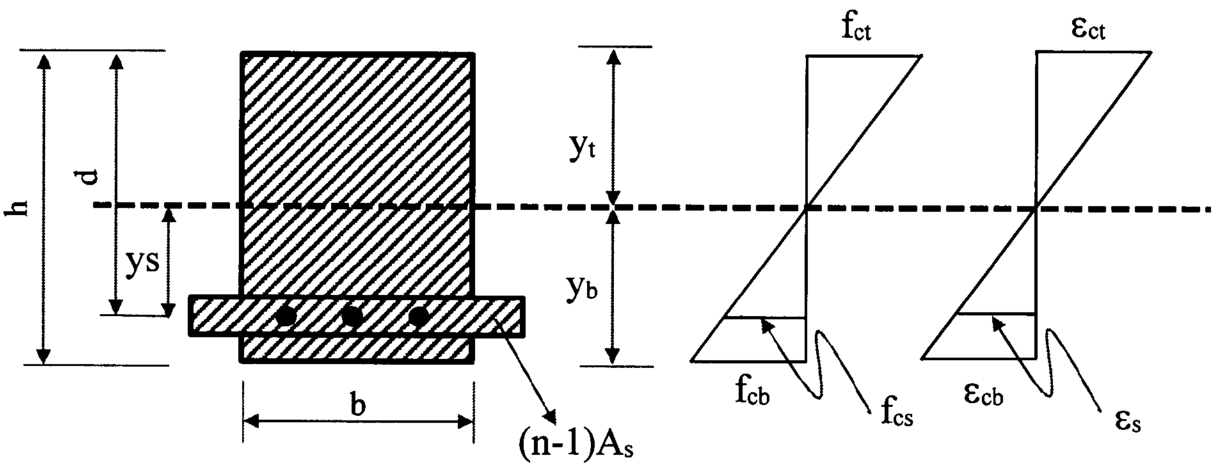
n = modular ration

f_{cs} = concrete stress at reinforcement level

f_s = reinforcement stress

ε_{cs} = concrete strain at reinforcement level

ε_s = reinforcement strain



Transformed Section *Stress Distribution* *Strain Distribution*

- Using the second assumption;

$F_c = F_s \rightarrow A_e . f_{cs} = A_s . f_s \rightarrow A_e . f_{cs} = A_s . n . f_{cs} \rightarrow$ $\boxed{A_e = n . A_s}$

Where:

F_s = steel force

F_c = equivalent concrete force

A_e = concrete area equivalent to steel

A_s = steel area

2.2.2 Live Loads

Represents all loads that can be moved like human being or have the ability to move like furnatures...etc. The live loads can be estimated based on the "ASCE/SEI 7" standards that represents the *Minimum Design Loads for Buildings and Other Structures,* Chapter 4, Table 4-1.

TABLE 4-1 MINIMUM UNIFORMLY DISTRIBUTED LIVE LOADS, L_o, AND MINIMUM CONCENTRATED LIVE LOADS

Occupancy or Use	Uniform psf (kN/m²)	Conc. lb (kN)
Apartments (see *Residential*)		
Access floor systems Office use Computer use	 50 (2.4) 100 (4.79)	 2,000 (8.9) 2,000 (8.9)
Armories and drill rooms	150 (7.18)	
Assembly areas and theaters Fixed seats (fastened to floor) Lobbies Movable seats Platforms (assembly) Stage floors	 60 (2.87) 100 (4.79) 100 (4.79) 100 (4.79) 150 (7.18)	
Balconies (exterior) On one- and two-family residences only, and not exceeding 100 ft² (9.3 m²)	100 (4.79) 60 (2.87)	
Bowling alleys, poolrooms, and similar recreational areas	75 (3.59)	
Catwalks for maintenance access	40 (1.92)	300 (1.33)
Corridors First floor Other floors, same as occupancy served except as indicated	 100 (4.79)	
Dance halls and ballrooms	100 (4.79)	
Decks (patio and roof) Same as area served, or for the type of occupancy accommodated		
Dining rooms and restaurants	100 (4.79)	
Dwellings (see *Residential*)		
Elevator machine room grating (on area of 4 in.² [2,580 mm²])		300 (1.33)
Finish light floor plate construction (on area of 1 in.² [645 mm²])		200 (0.89)
Fire escapes On single-family dwellings only	100 (4.79) 40 (1.92)	
Fixed ladders	See Section 4.4	
Garages (passenger vehicles only) Trucks and buses	40 (1.92)^{a,b}	
Grandstands (see *Stadiums and arenas, Bleachers*)		
Gymnasiums—main floors and balconies	100 (4.79)	
Handrails, guardrails, and grab bars	See Section 4.4	
Hospitals Operating rooms, laboratories Patient rooms Corridors above first floor	 60 (2.87) 40 (1.92) 80 (3.83)	 1,000 (4.45) 1,000 (4.45) 1,000 (4.45)

2.2.3 Wind Loads

Can be calculated as force/m^2 from the building elevation according to "ASCE/SEI 7" standards that represents the *Minimum Design Loads for Buildings and Other Structures,* Chapter 6. The wind loads can be estimated using:

1-Simplified Method.

2- Analytical Procedure, and

3- Wind Tunnel Procedure.

2.2.4 Other Loads

Like soil pressure, fluid loads, thermal loads, seismic loads and others.

2.3 Analysis Methods

There are two types of structures; *simple limited structures* that can be analyzed using equlibirum equations only. *Complex unlimited structures* that need consistant-displacement equations in addition to equilibirum equations for their analysis. Most structures are complex umlimited ones and there are many analysis methods can be used.

2.3.1 Classical Methods

Accurate methods like:

1. Consistance Deformation Method
2. Slope-Deflection Method
3. Least work Method

Approximate methods like Moment-distribution method. It was very important analysis method before computer development age because it needs to solve a lote of simultenous equations.

Other methods specialized with laterial loads (like wind loads) that include:

1. Portal Method
2. Cantilever Method

2.3.2 Advanced Methods

In these methods, matrices to be used for structural analysis. Advaned methods include:

1. Force or Flexibilty Method
2. Displacement or Stiffnes Method

Using the displacement method, many structural programs have been developed for structural analysis like SAP2000 (**S**tructural **A**nalysis **P**rogram) and STAAD III (**S**tructural **A**nalysis **a**nd **D**esign in **3**-Dimensions)

2.4 ACI-Code Coefficient Method

Represents the simplest methods for analyzing continous beams and one-way slabs (ACI-Code 8.3.3). Using this method, the ultimate bending moment can be calculated as folllows:

$$M_u = C\, W_u\, {L_u}^2$$

Where:

C = factor obtained from Table 2-2 or ACI-Code 8.3.3.
W_u = the estimated ultimated loads
L_n = the clear span face-to-face of supports for positive moments and shear, while it represents the average length of two adjacement spans for the negative moments.

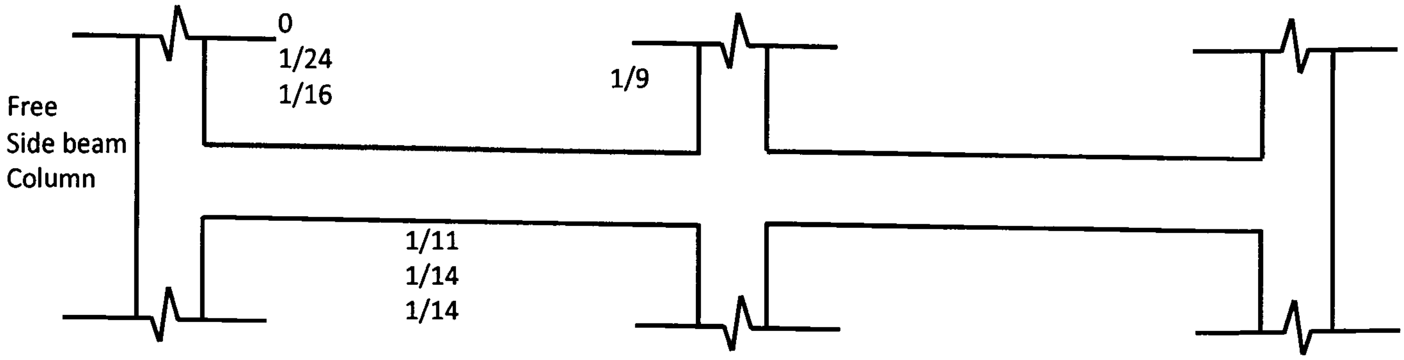
For the shear calculation, the ultimate shear at the face of support is:

$$V_u = C\, W_u\, L_u$$

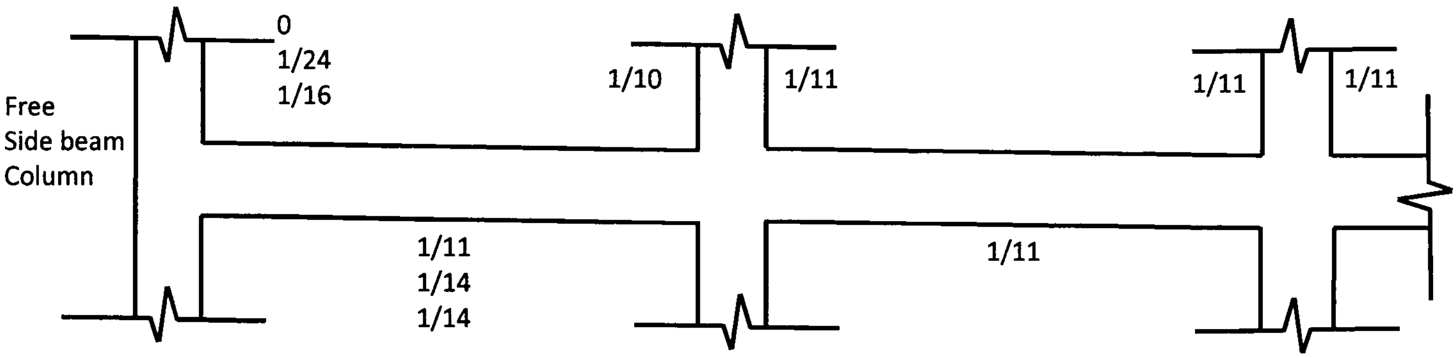
Table 2-2: Approximate factor values for moment and shear calculations.

Positive moment	
End spans	
Discontinuous end unrestrained	$w_u \ell_n^2/11$
Discontinuous end integral with support	$w_u \ell_n^2/14$
Interior spans	$w_u \ell_n^2/16$
Negative moments at exterior face of first interior support	
Two spans	$w_u \ell_n^2/9$
More than two spans	$w_u \ell_n^2/10$
Negative moment at other faces of interior supports	$w_u \ell_n^2/11$
Negative moment at face of all supports for Slabs with spans not exceeding 3 m; and beams where ratio of sum of column stiffnesses to beam stiffness exceeds 8 at each end of the span	$w_u \ell_n^2/12$
Negative moment at interior face of exterior support for members built integrally with supports	
Where support is spandrel beam	$w_u \ell_n^2/24$
Where support is a column	$w_u \ell_n^2/16$
Shear in end members at face of first interior support	$1.15 w_u \ell_n/2$
Shear at face of all other supports	$w_u \ell_n/2$

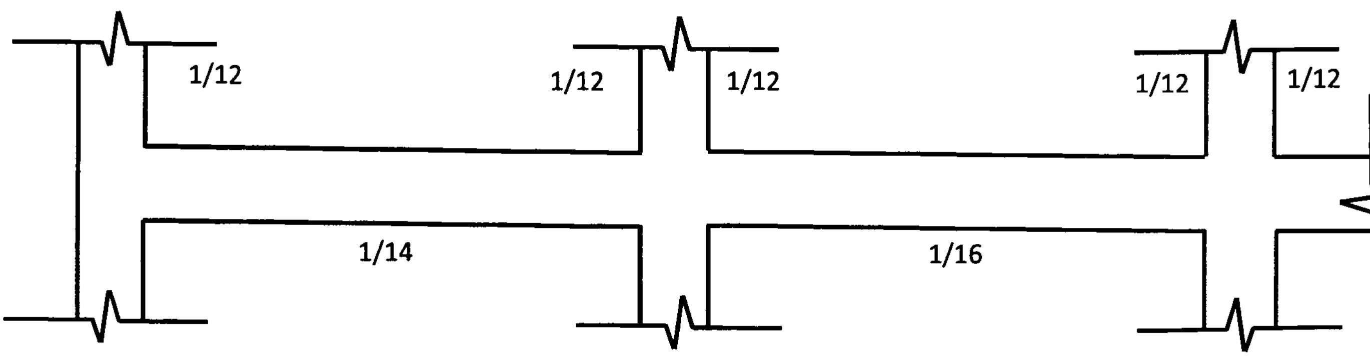
1- **Two Spans**



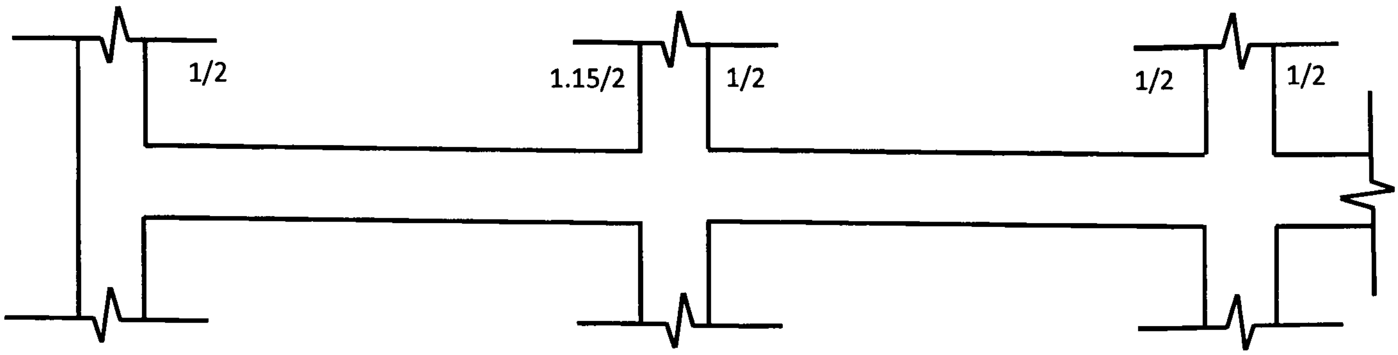
2- **More than to Spans**



3- **Slabs with summation of I (columns) to the I (beam) at each end more than 8 and span less than 3 m.**



4- Shear



This method can be used under the following conditions:

1- There are at least two spans.
2- The spans to be approximately equal and no ore than 20% difference between two adjacement spans.
3- The loads to be uniformly distributed.
4- The live loads to be less than three times the dead loads.
5- All members have the same cross-sections.

2.5 Design Methods

Generally, there are two method for design of reinforced concrete members. These are:

2.5.1 Working Stress Method

This method assumes linear elastic behavior in analyzing the structures and linear relation between stresses and strains. Therefore, the concrete stress (*fc*) is limited to 0.45 *f'c,* i.e., 45% from the concrete ultimate strength. While steel stresses (*fs*) are limited to be

140 MPa for steel types (fy = 300 MPa) & (fy = 350 MPa) and 170 MPa for steel type (fy = 400 MPa), i.e., less than or equal half the yield stress.

The above stresses are called "Allowable stresses" or "Working stresses". The reason for calling it working stresses because the structure is designed under the working loads (The actual loads on it).

In this method, the safety factors are the ratio between the ultimate stresses and the allowable stresses which is 1/0.45 = 2.22 for concrete and greater than or equal to 2 for steel reinforcement. In the present time, the usage of this method becomes undesirable. In spite ot this, it is necessary and important to understand it in order to check the structure serviceability (control of deflection and cracks) because the serviceability requirements are always checked under working loads or service loads.

2.5.2 Strength Method

In this method, the structure has to be analyzed assuming linear elastic behavior, i.e, same as the analysis method in case of working stress method. While sections design and analysis have been conducted under the ultimate loading (using safety factors more than 1 multiplied by the working loads).

For the stresses, the ultimate ones have to be used for both concrete and steel reinforcement. The ultimate strength calculation depends on the real stress distribution at failure and its relation with strains will be non-linear. This method becomes more common and popular than the working stress method during the last decades.

If the moment is known, the stresses will be:

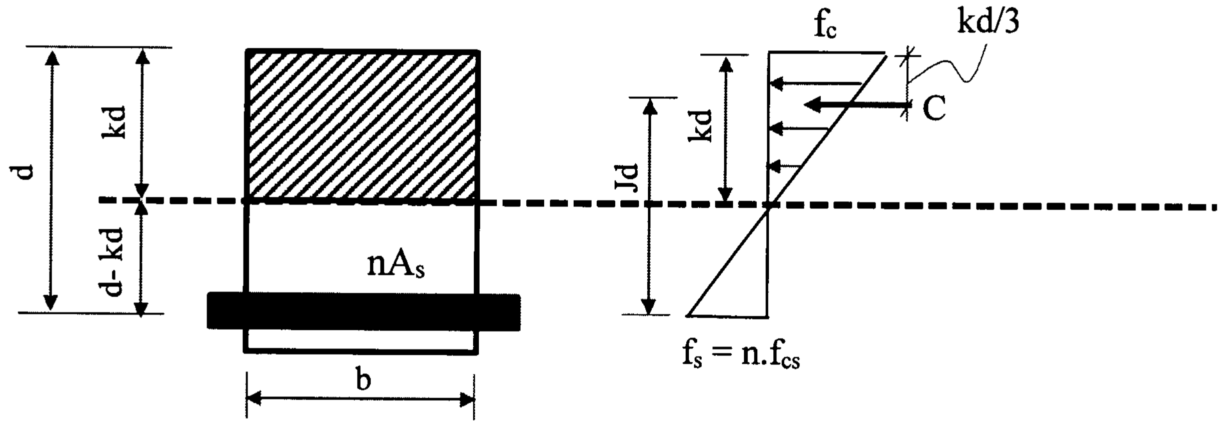
$f_c = \frac{2M}{kJbd^2}$ .... concrete compressive stress from equation (5)

$f_s = \frac{M}{A_s.Jd}$ .... steel tensile stress from equation (6)

If one of the stresses is known, the other can be found from the linear interpolation.

2- *Second Method : Transformed Section Moment.*

In this method, the neutral axis location to be determined after transforming the steel area into an equivalent concrete as shown below.



Transformed Section *Stress Distribution*

$A_t = n. A_s$, $n = E_s/E_c$

The value of kd is calculated by taking moments of area about the neutral axis. Then a 2nd degree equation in terms of k is developed. Solve for k.

k can be calculated based on the principle:

$$b\frac{(kd)^2}{2} = nA_s(d - kd) \rightarrow b\frac{(kd)^2}{2} - \rho nbd(d - kd) = 0$$

2.6 Serviceability Requirements

After completing the member design regarding the bending, shear and other applied forces, serviceability requirements have to be checked as:

1- Deflection: to be limited to certain value due to its disadvantageous.
2- Cracks: to be under control to avoid reinforcement oxidation.

If Serviceability of the member does not comply with the standards and limitations under workin stresses, the member must be re-dseigned including section size and reinforcement amount.

Lecture # 04: Working Stress Method

3.1 Introduction

Working stress method assumes linear proportionality of stresses and strain for concrete and steel reinforcement. For steel reinforcement, linear proportionality will be when the stresses is less than the yield stress (f_y). While for concrete, the stress-strain relation can be assumed linear when the stress is less than half of concrete compressive strength, i.e., less than ($0.5\,f'_c$).

During the last decades, the use of this method becomes little and the ACI-code put it in the appendices and call it the ***alternate method*** since 1977. So, in this chapter, only rectangular reinforced concrete sections will be analyzed and designed using this method.

Before analyzing and designing the reinforced concrete beams using this method, the behavior of reinforced concrete beams under applied loads should be studies to get more understanding of working stress and ultimate stress methods.

3.2 Behavior of Reinforced Concrete Beams under Load Effect

The behavior of the beams can be divided into three stages up to failure:

3.2.1 Before Cracking Stage

This stage happened with little loads when stress and strain proportionality is linear. For the analysis, the steel reinforcement has to be transformed to an equivalent concrete to get the transformed section. Then the beam will be homogenous and the mechanics of material rules can be applied to it.

The area of the transformed section (A_t) will be:

$A_t = A_c + A_e = A_g - A_s + n.A_s \rightarrow A_t = A_g + (n-1) A_s$

Where:

A_t = transformed section area

A_g = gross area = b x h

A_c = concrete area

After converting the steel to its equivalent of concrete, mechanics of material rules can be applied to the homogeneous section to evaluate the stresses as follows:

$$\sigma = \frac{M.y}{I}$$

Where:

σ = bending stress

M = applied bending moment

y = distance from the neutral axis

I = moment of inertia

Thus, the maximum concrete and steel stresses will be:

$$f_{ct} = \frac{M.y_t}{I},\ f_{cb} = \frac{M.y_b}{I},\ f_s = n.f_{ct} = n.\frac{M.y_s}{I}$$

Where:

f_{ct} = concrete compressive stress

y_t = distance of concrete top fiber to the neutral axis

f_{cb} = Concrete tensile stress

y_b = distance of concrete bottom fiber to the neutral axis

f_s = steel stress

y_s = distance of steel reinforcement to the neutral axis

To find the neutral axis location, the moment of the total area about the top axis will be equal to the moment of the concrete rectangle and the concrete area equivalent to steel about the same axis as:

$A_t \,.\, y_t = A_g \,.\, (h/2) + (n-1)A_s\, d \rightarrow$ Find y_t

Then find y_b & y_s as follows:

$y_b = h - y_t$, $y_s = d - y_t$

The moment of inertia can be calculated as:

$I = (bh^3/12) + b.h.e^2 + (n-1)\, A_s.\, y_s^2 \rightarrow I = (b.\, y_t^3/3) + (b.\, y_b^3/3) + (n-1)\, A_s\, y_s^2$

Where:

e = distance of rectangular area to the neutral axis

d = is the distance from the concrete compression fiber to the steel axis

3.4 Cracking Moment Calculation

When analyzing reinforced concrete beams subjected to relatively light loads, the section has to be checked for cracking. This can be done through evaluating the moment required to cause cracking and comparing it with the applied moment. In cracking moment calculation, the concrete stress has to be equal to the modulus of rapture (f_r).

The cracking moment can be calculated as follows:

$$M_{cr} = \frac{f_r.I}{y_b}$$

Where f_r is the modulus of rapture of concrete.

I = is the moment of inertia of the given section

y_b = is the distance between the tension fiber of concrete to the neutral axis.

When the cracking moment is greater than the applied one, the previous procedure can be followed. Otherwise, both the analysis and design has to be done with either the working stress or ultimate methods.

Example #01: *Determine the cracking moment for a simply supported reinforced concrete beam whose size (width x depth) is 300 x 600 mm with reinforcement amount of 3200 mm² as shown below. The beam has a modular ratio (n) of 8 and modulus of rapture (f_r) of 3.1 MPa. Find also the concrete & steel stresses if the applied bending moment is 33.9 kN.m.*

Solution:

$A_c = 300 \times 600 + (8-1)(3200) = 202400\ mm^2$

$202400\ (y_t) = (300 \times 600)\ 300 + (7 \times 3200)\ 500$

$y_t = 322$ mm, $y_b = 600 - 322 = 278$ mm, $y_s = 278 - 100 = 178$ mm

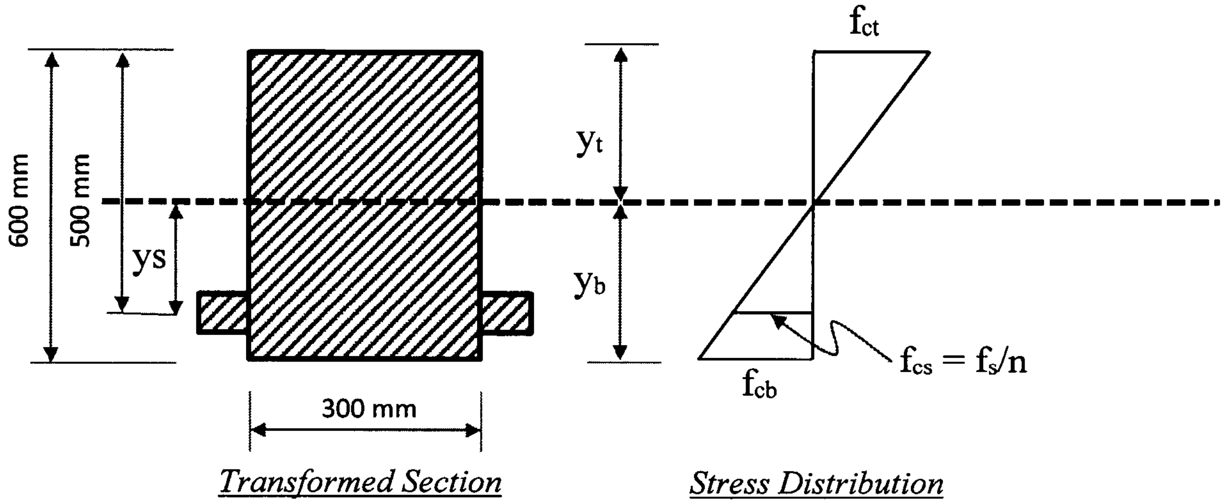
$I = (300 \times 600^3)/12 + 300 \times 600 \times 22^2 + 3200 \times (8-1) \times 178^2 = 697 \times 10^6\ mm^4$

$M_{cr} = fr.\ I/yb = ((3.1 \times 6197 \times 10^6)/278) \times 10^{-6} = 69.1$ kN.m

Since $M_{cr} = 69.1 > 33.9 \rightarrow$ un-cracked section, find concrete and steel stresses.

$f_{ct} = \frac{M.yt}{I} = (33.9 \times 10^6 \times 322)/(6197 \times 10^6) = 1.76\ N/mm^2$

$f_s = n.f_{cs} = n\frac{M.ys}{I} = (8 \times 33.9 \times 178 \times 10^6)/(6197 \times 10^6) = 7.79\ N/mm^2$



3.5 Analysis of Reinforced Concrete Beams by Working Stress Method

The analysis includes the evaluation of concrete and steel reinforcement stresses when the applied moment, section size and reinforcement amount are known. The analysis also involves the maximum bending moment calculation that the section can withstand if the allowable stresses, section size and reinforcement amount are known.

The allowable stresses based on the ACI-Code represent a percentage of material strengths and used as safety factors for the stresses as;

- For concrete: $f_c = 0.45 f'_c$
- For steel reinforcement

  $f_s = 140$ N/mm^2 for steel with $f_y = \begin{cases} 300 \text{ N/mm}^2 \\ 350 \text{ N/mm}^2 \end{cases}$

  $f_s = 170$ N/mm^2 for steel with $f_y = 400$ N/mm^2

- Modulus of elasticity of concrete; $E_c = 4700\sqrt{f'_c}$ N/mm^2
- Modulus of elasticity of steel; $E_s = 200{,}000$ N/mm^2

Where f'_c is the cylinder compressive strength at 28 day

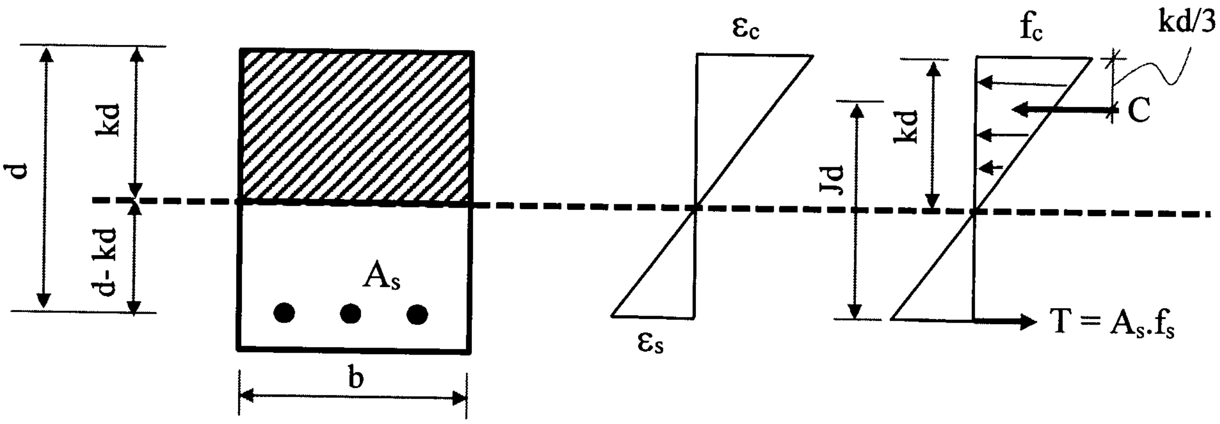
The above method depends on several assumptions like:

1- Plane section before bending remain plane after bending, i.e., the strains are linearly proportional with the distance from the neutral axis.
2- The stress is linearly proportional with the strain.
3- Concrete tensile strength is neglected and the tensile forces are assumed to be carried by steel reinforcement only.
4- Full bond between concrete and steel reinforcement, i.e., there is no slip between them.

There are two methods of analysis of reinforced concrete beams as follows:

1- First Method : Internal Couple Moment.

In this method, the neutral axis location to be determined first using the equilibrium of forces as shown below:



Rein. Con. Section *Strain Distribution* *Stress Distribution*

C = T

$0.5\ f_c\ b\ kd = A_s.\ f_s$ ....(1)

From strain distribution:

$$\frac{\varepsilon_c}{kd} = \frac{\varepsilon_s}{d-kd} \quad , \frac{f_c}{E_c kd} = \frac{f_s}{E_s(d-kd)} \ , \ \therefore \ \frac{f_s}{d-kd} = n.\frac{f_c}{kd}$$

Where

d is the effective depth = distance from steel center to concrete compression fiber.

kd = is the distance from the concrete compression fiber to the neutral axis.

From the above equation; $f_s = \frac{1-k}{k}\ n.f_c$ ....(2)

Find f_s in terms of f_c from equation (2) and substitute into equation (1), a 2nd degree equation in terms of k will be developed. Solve for k.

$\frac{k^2}{2} - \rho n(1-k) = 0 \;\; \rightarrow \therefore$ k^2 - 2ρn -2ρnk = 0

$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$ .... Same as equation (4).

The moment of inertia can be calculated as:

$$I = \frac{b(kd)^3}{3} + nA_s\,(d-kd)^2$$

The stresses can be calculated as:

$f_c = \frac{M\,kd}{I}$ ....(7)

$f_s = n.f_{cs} = \frac{nM\,(d-kd)}{I}$ ....(8)

<u>Example #02</u>: *Determine both the concrete and steel stresses for a reinforced concrete beam with size b = 300 mm and d (effective depth) = 500 mm and reinforcement amount of 1500 mm² and a modular ratio of 8. The applied bending moment is 70 kN.m.*

Solution:

<u>Method 1: internal couple</u>

$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$, ρ =(1500)/(300 x500) = 0.01

ρn = 0.01 x 8 = 0.08, $\therefore\ k = \sqrt{2\,x0.08 + (0.08)^2} - 0.08$ = 0.328

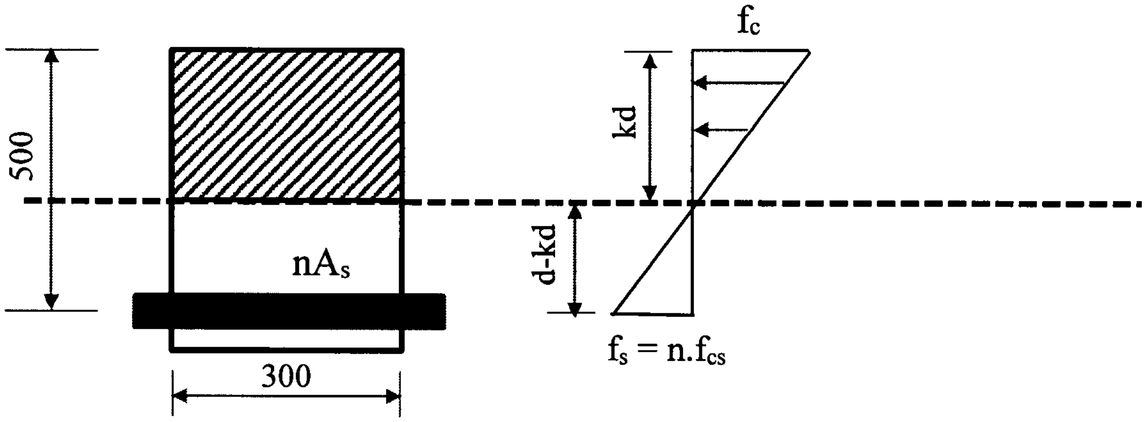
kd = 0.328 x 500 = 164 mm

J = 1-(k/3) = 1-(0.328/3) = 0.89

Jd = 0.89 x 500 = 445 mm

$f_c = \frac{2M}{kJbd^2} = \frac{2x70x10^6}{0.328x0.89x300x500^2}$ = 6.39 N/mm²

$f_s = \frac{M}{A_s.Jd} = \frac{70x10^6}{1500x445}$ = 104.9 N/mm²



Transformed Section *Stress Distribution*

Method 2: Transformed section

The value of k is calculated as done in method1.

$I = (300x164^3)/3 + 8x1500x(500-164)^2 = 1795 \times 10^6 \ mm^4$

$f_c = (M.kd)/I = (70 \times 10^6 \times 164)/(1795 \times 10^6) = 6.39 \ N/mm^2$

$$f_s = n.f_{cs} = \frac{nM\,(d-kd)}{I} = \frac{8x70x10^6(500-164)}{1795x10^6} = 104.8 \ N/mm^2$$

6.3 تصميم العتبات الخرسانية المسلحة المستطيلة المقطع :-

Design of R.C Rectangular Beams

يقصد بالتصميم إيجاد إبعاد المقطع ومساحة وتفاصيل الحديد حيث تكون الاجهادات المسموح بها والعزوم المسلط معلومة . ويتم التصميم على اعتبار ان الحديد والخرسانة يصلان إلى الاجهادات المسموح بها في آن واحد (يسمى المقطع في هذه الحالة متوازن التسليح) وسبب ذلك هو الحصول على مقطع اقتصادي باستغلال كل خواص الخرسانية والحديد .

يتم إيجاد المعامل (k) بأخذ تناسب الاجهادات (شكل 4.3) :-

$$\frac{f_c}{kd} = \frac{f_s / n}{d - kd} \qquad \therefore \frac{f_s}{f_c} = \frac{n(1-k)}{k}$$

حيث

$$r = \frac{n(1-k)}{k} \qquad r = \frac{f_s}{f_c}$$

$$rk = n(1-k) = n - nk$$

$$\therefore k = \frac{n}{n+r} \quad \text{...(14.3)}$$

ثم نستخدم المعادلة (10.3)

$$M = 0.5 f_c K J b d^2$$

وإذا رمزنا إلى :-

$$R = 0.5 f_c K J$$

فان

$$M = R b d^2 \quad \text{...(15.3)}$$

إن R تعتمد على خواص المواد وليس لها علاقة بإبعاد المقطع .

من المعادلة السابقة نجد قيمة bd^2 ثم نجد الأبعاد وهناك ثلاث احتمالات .

1- الأبعاد غير معلومة نفرض احدها ونجد الثاني فنحصل على أزواج من القيم نختار العملية منها .

2- قد تكون النسبة بين الأبعاد معلومة نعوض ونجد الأخرى .

3- قد يكون احد الأبعاد معلوماً نعوض لإيجاد الأخر .

ثم يتم حساب الحديد من المعادلة

$$A_s = \frac{M}{f_s J d}$$

ثم نكمل التصميم بحساب عدد القضبان ورسم تفاصيل المقطع .

مثال 4.3 :- صمم العتبة المبينة في الشكل (7.3) لتقاوم عزم انحناء ناتج عن حمل خدمي منتشر منتظم مقداره 24KN /m استخدم $f'c = 25Mpa$ ، $fy = 300Mpa$
$b = 250mm$

الحل :-

$f_c = 0.45 \times 25 = 11.25Mpa$

$f_s = 140Mpa \quad for \quad fy = 300Mpa$

$E_c = 4700\sqrt{25} = 23500\ Mpa \qquad n = E_s / E_c = 8.51$

استخدم n = 9 حيث قيمة (n) حسب الكود تقرب إلى اقرب عدد صحيح .

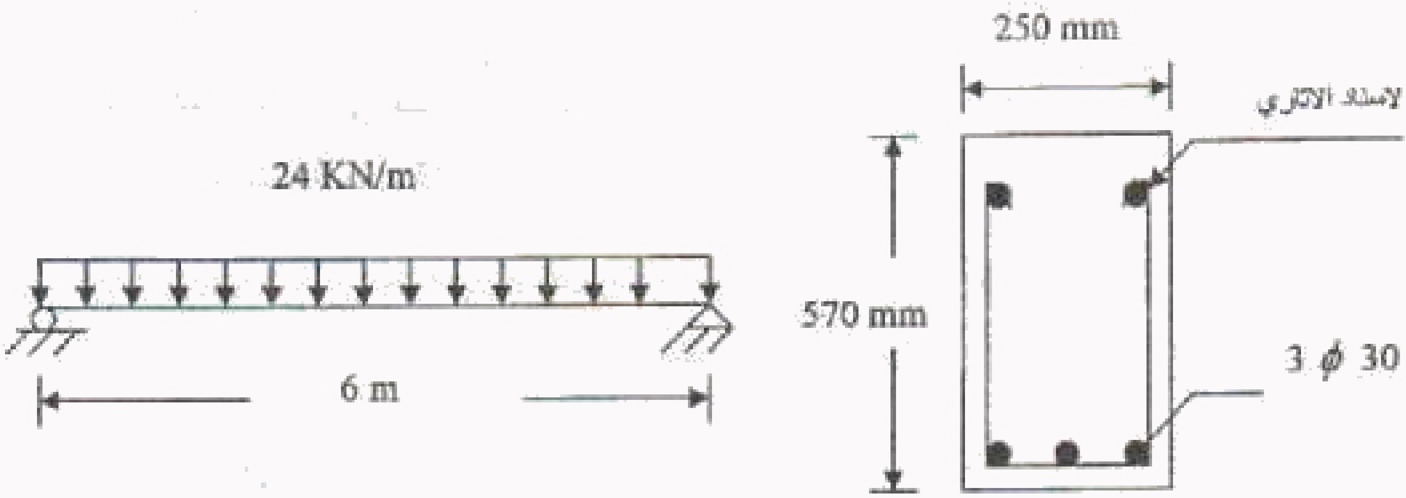
لإيجاد وزن العتبة نفرض أن العمق الكلي = 600 mm عليه فأن :-

$W_b = 0.6 \times 0.25 \times 24 = 3.6KN / m$

استخدم $W_b = 4KN / m$

$$M = \frac{WL^2}{8} = \frac{28 \times 6^2}{8} = 126KN.m$$

$$k = \frac{n}{n+r} = \frac{9}{9 + \frac{140}{11.25}} = 0.42 \qquad J = 1 - \frac{k}{3} = 0.86$$



شكل (7.3)

تفاصيل العتبة للمثال (4.3)

Assume ϕ 8 mm stirrups.

Beam total height (h) = 498+40+8+16 = 562 mm……⇒ say h = 570 mm

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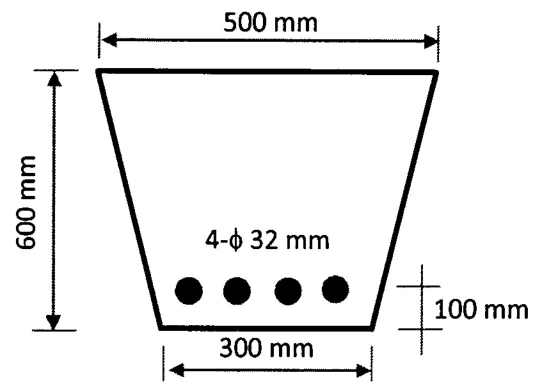
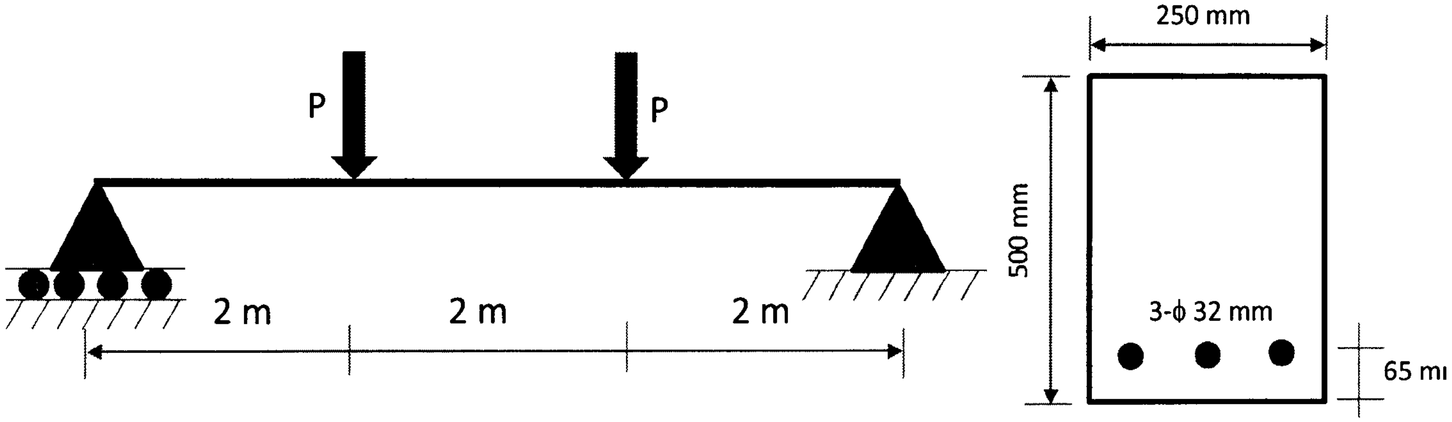
Home works:

1- Find the cracking moment for the section below if the modulus of rapture, f_r is equal to 3.13 N/mm^2 and f'_c = 20 N/mm^2.



2- Find the greatest service point load (P) that can be applied on the given beam so that no cracking happened. Knowing that n = 8 and f_r = 4 N/mm^2.



$R = 0.5 \times 11.25 \ kJ = 2.03$

$M = Rbd^2 = 2.03bd^2$

$bd^2 = \frac{126 \times 10^6}{2.03} = 62.07 \times 10^6$

$b = 250mm \qquad \therefore d = 498mm \qquad Jd = 428mm$

$A_s = \frac{M}{f_s Jd} = \frac{126 \times 10^6}{140 \times 428} = 2102mm^2$

استخدم قضبان نوع (($\phi 30$) حيث $A_b = 707 \text{ mm}^2$ عليه يكون عدد القضبان مساوياً إلى

$n = \frac{2102}{707} = 2.97$

استخدم قضبان ($3\phi 30$).

$h = d + \frac{\phi}{2} + \phi_s + 40$

العمق الكلي يصبح

$h = 498 + 15 + 10 + 40 = 563mm$

استخدم $h = 570 \ mm$

الرموز التي استخدمت لايجاد h هي :-

ϕ = قطر القضيب الرئيسي .

ϕ_s - قطر الاتاري .

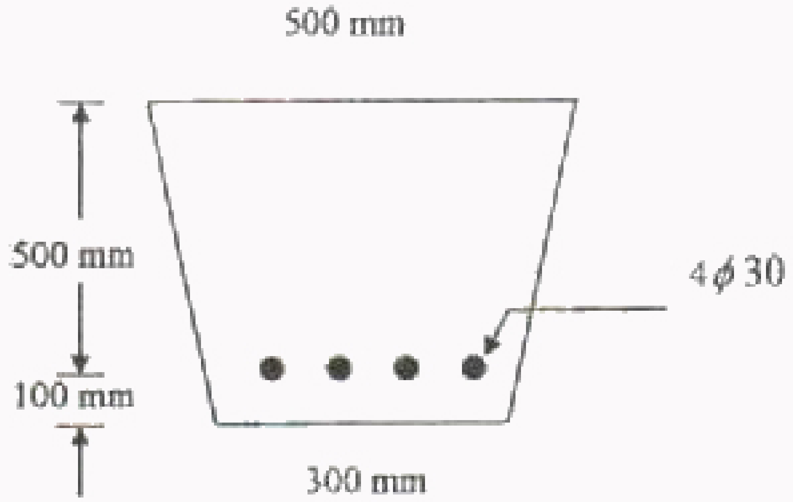
وتم استخدام غطاء خرساني صافي - 40 mm

تفاصيل المقطع موضحة على الشكل (7.3)

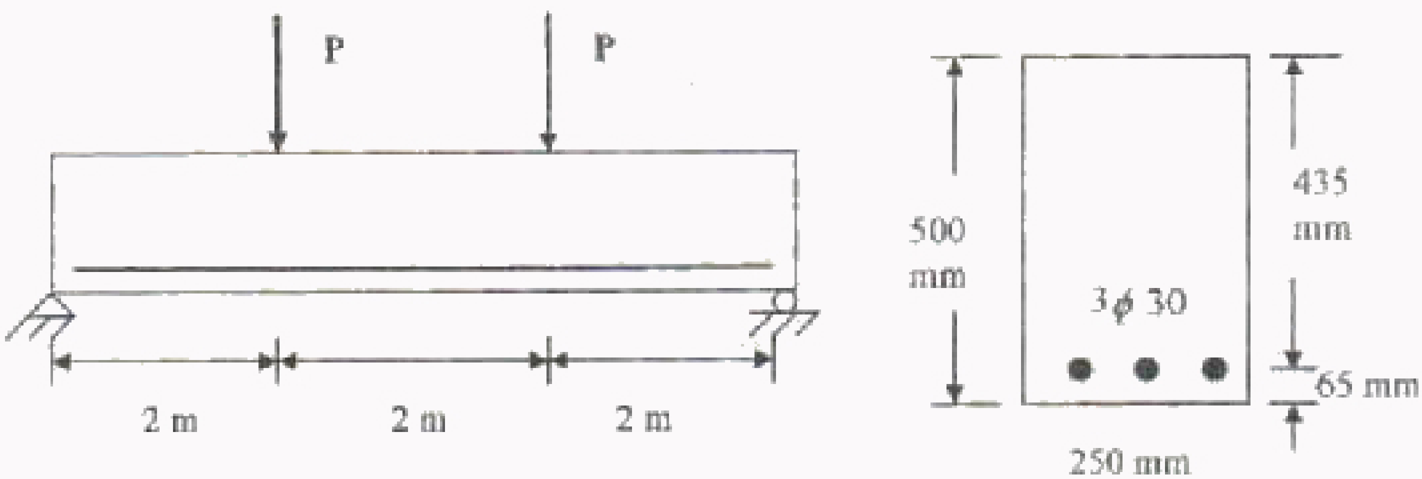
مسائل :-

1.3 جد عزم التشقق للمقطع أدناه إذا كان معامل الكسر يساوي $3.13Mpa$

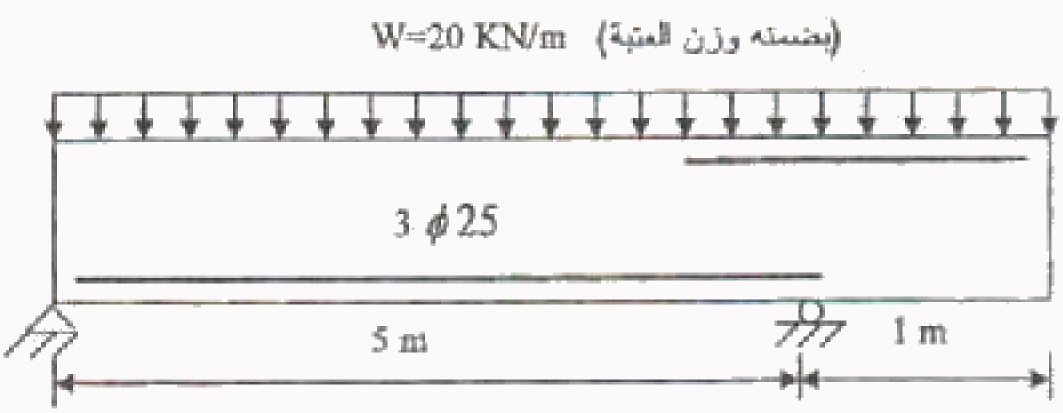
$f'c = 20Mpa$



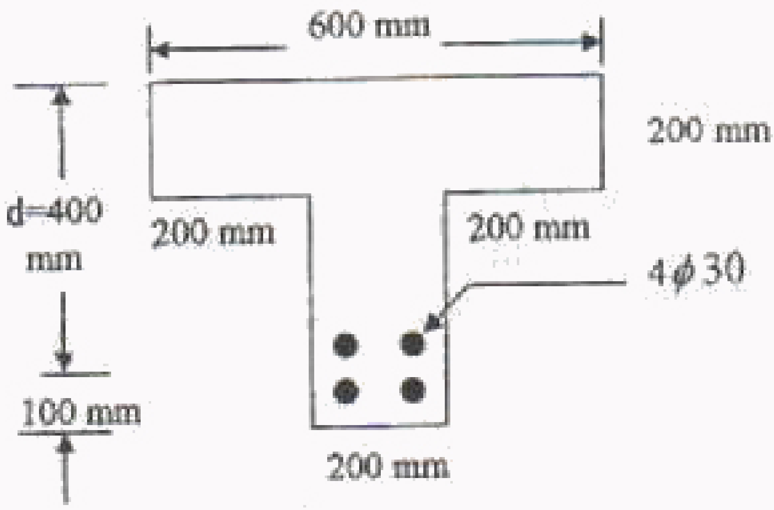
2.3 جد أقصى حمل خدمي مركز (p) يمكن تسليطه على العتبة أدناه بحيث لايحدث تشقق في العتبة علماً أن $n = 8$ ، $f_r = 4Mpa$



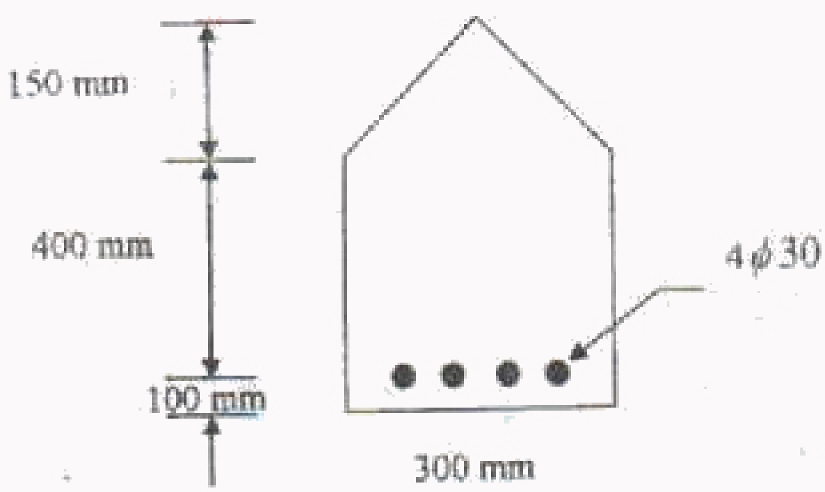
3.3 للعتبة أدناه جد اجهادات الحديد والخرسانة للعزم الموجب الأقصى فقط.استخدم $n = 10$ علماً أن المقطع للعزم الموجب هو نفسه للسؤال (2.3) .



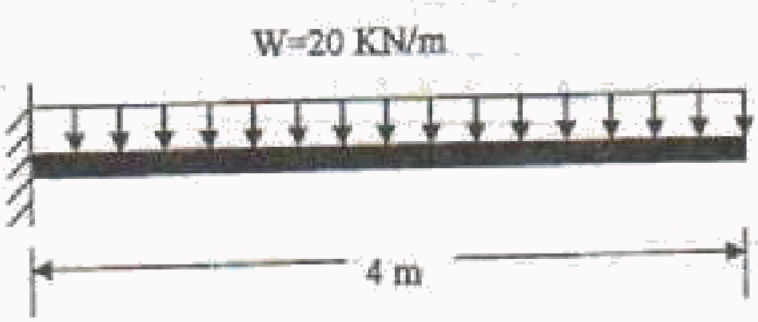
4.3 لمقطع العتبة أدناه احسب اجهادات الحديد والخرسانة إذا كان العزم المسلط $M = 100 KN.m$ ، ومعامل المعيارية $n = 10$.



5.3 جد اجهادات الحديد والخرسانة للمقطع أدناه إذا كان العزم الخدمي المسلط $M = 100 KN.m$ ، ومعامل المعيارية $n = 10$



6.3 صمم العتبة أدناه إذا كان الحمل المنتشر المنتظم يساوي (20 KN/m) ، $n = 10$
عرض المقطع $b = 300mm$ ، والاجهادات المسموح بها $fc = 10Mpa$
$fs = 140Mpa$



Example #03: *Determine the maximum service moment that can be applied to reinforced concrete beam with size b = 250 mm and d (effective depth) = 400 mm and reinforcement amount of 1000 mm² and a modular ratio of 8. Knowing that the allowable stresses for concrete and steel are 12 N/mm² and 140 N/mm² respectively.*

Solution:

Method 1: internal couple

$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$, ρ =(1000)/(250 x 400) = 0.01

ρn = 0.01 x 8 = 0.08, $\therefore$ $k = \sqrt{2\,x0.08 + (0.08)^2} - 0.08$ = 0.328

kd = 0.328 x 400 = 131 mm

J = 1-(k/3) = 1-(0.328/3) = 0.89

Jd = 0.89 x 400 = 356 mm

- Assume that the concrete will reaches its allowable stress. Thus:

$f_c = \frac{2M}{kJbd^2} \rightarrow M = \frac{f_c\, kJbd^2}{2}$ = 12 (250)(400)² x 0.328 x 0.89 x 10^{-6}/2 = 70 kN.m ⇐

- Assume that the steel reinforcement reaches its allowable stress as:

$f_s = \frac{M}{A_S{\cdot}Jd}$ → M = A_s f_s Jd = 1000 x 140 x 0.89 x 400 = 49.8 x 10^6 = 49.8 kN.m ⇐

∴ The largest service moment is 49.8 kN.m

Method 2: Transformed section

The value of k is calculated as done in method1.

I = 250 (131)³/3+ 8 x 1000 (400-131)² = 766.23 x 10^6 mm⁴

- Assume that the concrete will reaches its allowable stress. Thus:

$f_c = \frac{M\,kd}{I} \rightarrow M = \frac{f_c.\,I}{kd}$ = (12 x 766.23 x 10^6)/ 131 = 70 kN.m ⇐

Let $r = \frac{f_s}{f_c} \Rightarrow r = \frac{n(1-k)}{k} \Rightarrow$ rk=n (1-k) = n-nk $\Rightarrow \therefore k = \frac{n}{n+r}$ ....(9)

Using Equ. (5);

$M = 0.5\, f_c\, k\, J\, b\, d^2$, let $R = 0.5\, f_c\, k\, J$, $\Rightarrow$ $M = R\, b\, d^2$.................(10)

It is noted that the parameter R depends on material properties rather than the section size.

From Equ. (10), the value of bd^2 is evaluated. To determine the size:

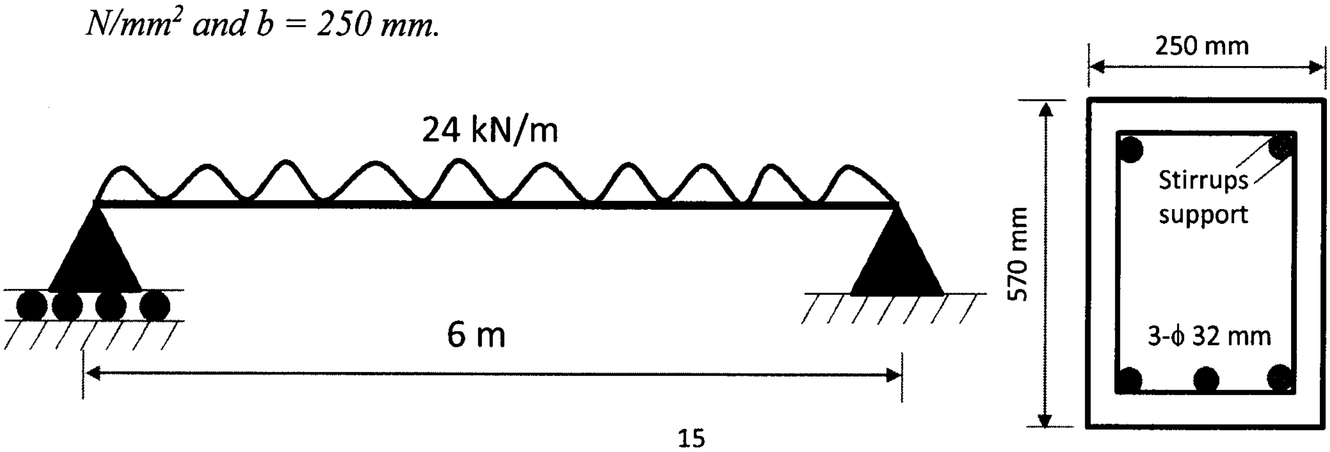
1- Assume one dimension and find the other. This will continue until reaching practical values.
2- The ratio of b/d may be known from the given data.
3- One of the dimensions may be known from the given data.

The reinforcement amount is then evaluated from Equ. (6):

$$A_s = \frac{M}{f_s . Jd}$$

Then the design will be completed by calculating the number of the required steel bars and the detailing of reinforcement.

Example #04: *Design a 6m span simply supported beam to resist the bending results from a service uniformly distributed load of 24 kN/m. Use f'_c = 25 N/mm², f_y = 300 N/mm² and b = 250 mm.*



Solution:

$f_c = 0.45\ (f'_c) = 0.45\ (25) = 11.25$ N/mm^2

$f_s = 140$ N/mm^2 for $f_y = 300$ N/mm^2

$E_c = 4700\sqrt{f'_c} = 4700\sqrt{25} = 23{,}500$ N/mm^2

$E_s = 200{,}000$ N/mm^2

$n = E_s / E_c = 200{,}000/23{,}500 = 8.51 \ldots\ldots \Rightarrow$ say $n = 9$ (to the nearest integer number)

To find the beam self-weight, assume the beam height = 600 mm

$W_b = 0.6 \times 0.25 \times 24 = 3.6$ kN/m..... $\Rightarrow$ say $W_b = 4$ kN/m

Total weight (W) = 24 + 4 = 28 kN/m

Mid-span moment = $M = W L^2/8 = 28 \times (6)^2/8 = 126$ kN.m

$$k = \frac{n}{n+r} = \frac{9}{9+\frac{140}{11.25}} = 0.42$$

$R = 0.5\ f_c\ k\ J = 0.5\ (11.25)(0.42)(1-(0.42/3)) = 2.03$

$M = R.bd^2 = 2.03\ bd^2 \Rightarrow bd^2 = 126 \times 10^6/2.03 = 62.07 \times 10^6$

Use b = 250 mm, d = 498 mm, Jd = 0.86 (498) = 428 mm

To find the amount of reinforcement,

$$A_s = \frac{M}{f_s.Jd} = \frac{126 \times 10^6}{140 \times 428} = 2102 \text{ mm}^2$$

Use ϕ 32 mm steel bars.

No. of bars = 2102/804 = 2.61.....$\Rightarrow$ use 3- ϕ 32 mm steel bars

كلية المنصور الجامعة

Al-Mansour University College

قسم
الهندسة المدنية
المرحلة الثالثة

Civil Engineering
Department
3rd. Stage

Reinforced Concrete Design Lectures

2022 – 2023

Lec.2

خرسانة مسلحة

Flexurul Analysis and Design of Brams

Lec. Basher Faisal

For f'_c > 30 MPa.......⇒ β_1 will be reduced by 0.05 for every 7 MPa increase in f'_c but not less than 0.65.

$\beta_1 = 0.85 - 0.05\,(f'_c - 30)/7$ ....(4-17)

$0.325 \le \beta \le 0.425$

4.4 Beams Classifications based on Reinforcement Amount

The theory of material strength depends on the fact that the failure of beams occurs either by yielding of steel, reaching the yield strength, f_y, and this type of failure is called ***Tension Failure***. Or crushing of concrete before yielding of steel and this is called ***Compression Failure***. It noted from experimental work that the crushing of concrete usually occurs at strain, ε_u, ranging between 0.003 and 0.004 and a value of 0.003 has been adopted as conservative value for crushing of concrete.

In case of reaching concrete to its ultimate strain (ε_u = 0.003) at the same time of yielding of steel reinforcement, this kind of failure called ***Balanced failure***. The type of failure depends mainly on the reinforcement amount if the section size and material strength is limited.

4.4.1 Balanced Reinforced Concrete Beams

As mentioned before, this kind of failure occurs when the concrete reaches its crushing strain (ε_u = 0.003) at the same time the steel reinforcement reaches yielding stress ($f_s = f_y$). The balanced steel ratio (ρ_b) equation can be derived from equilibrium of forces as:

$C = T, \Rightarrow A_s f_y = 0.85\, f'_c\, ab = 0.85\, f'_c\, \beta_1 cb \;\Rightarrow \rho bd\, f_y = 0.85\, f'_c\, \beta_1 cb$ ....(4-18)

$$\therefore \rho = \frac{0.85\beta_1 c\, f'_c}{f_y\, d} \quad \text{..........(4-19)}$$

From strain distribution and taking (ε_u =0.003) as a condition of concrete crushing;

$\frac{c_b}{d} = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} \Rightarrow c_b = \frac{\varepsilon_u}{\varepsilon_u + \frac{f_y}{E_s}} d$, using E_s = 200, 000 MPa.

Flexural Analysis and Design of Beams

4.1 Introduction

Design and analysis of reinforced concrete beams can be conducted through two main methods:

1- Working Stress Method (as discussed in the previous chapter).

2- Strength (Ultimate) Method.

4.2 Strength (Ultimate) Method

In this method, design and analysis of structural members will be based on the ultimate loads that the structure can carry at failure. That's mean, both concrete and steel stresses reach its maximum values. The ultimate loads can be calculated through multiplying the expected service load along the structure life by certain factors called "*safety factors*".

The section analysis includes evaluating the section ultimate strength when the stresses reach its maximum values. While, the section design includes finding some or all section size and reinforcement amount when the structure is subjected to ultimate loads.

The stresses distribution at failure is nonlinear. While the strains distribution will remain linear. There is no accurate analysis to predict the stress distribution shape and the shape proposed by the ACI-Code researchers has been adopted. The proposed stress distribution shape is parabolic and depends on a lot of experimental tests.

4.2.5 Strength Reduction Factors

The design strength is normally calculated through multiplying the nominal strength by factor less than 1.0 called strength reduction factor and denoted by "ϕ". The value of ϕ varies based on the type of strength required (ACI-Code 9.3.1).

1- Tension –controlled sections....... $\phi = 0.9$
2- Compression-controlled sections:
   - Members with spiral reinforcement.............$\phi = 0.75$
   - Other reinforced members.......................$\phi = 0.65$
3- Shear & Torsion......................$\phi = 0.75$

Generally, the required strength should be less than or equal to the design strength.

$M_u \leq \phi M_n$, $P_u \leq \phi P_n$, $V_u \leq \phi V_n$.

4.3 Stress & Strain Distributions

Using the ultimate (strength) method, it is assumed that the strain distribution will be linear and this is approved experimentally even in failure state. While the stress distribution continue linear until approximately $0.5 f'_c$. After that it will be non-linear.

The actual stress distribution is unknown up to now because the concrete stress-strain curve depends on several factors like the concrete strength, loading speed,....etc. The ACI-code adopts parabolic shape for stress distribution as shown below.

The most important thing in the analysis and design is:

1- The total concrete compressive force, C.
2- The location of C with respect to the compressive fiber.

4.2.1 Basic Concepts

- **Service Loads**: The loads that actually applied to the structure like dead load, live loads, wind loads,....etc.

- **Factored (Ultimate) Loads**: The loads result from multiplying the service loads by the *safety factors*.

- **Design Forces & Moments**: The forces and moments result from the ultimate loads like the design bending moments (M_u), design shear force (V_u), design axial force (P_u) and design torsional moment (T_u) and also can be called *factored forces and moments*.

- **Required Strength**: It is the strength to be provided by the section and equal to the design forces and moments. So, the required strength for bending will be M_u and the same for other strengths.

- **Nominal Strength**: The section capacity calculated based on the strength of material theory and code requirements. It represents the maximum section strength and theoretically equal the strength at which the section fails and denoted by the nominal strength (M_n, V_n, P_n, T_n).

- **Design Strength**: The section capacity that has been adopted in both analysis and design and is equal to the nominal strength multiplied by a factor less than one called *strength reduction factor*. This is because the approximation in analysis and design and the Uncertainty of materials strengths.

4.2.4 Load Factors

The required strength U to be provided to resist both dead and live loads is (ACI-Code 9.2.1):

U = 1.2 D + 1.6 L ....(4-3)

U = 1.2 D + 1.6 L + 0.5 (L_r or S or R)....................(4-4)

U = 1.2 D + 1.6 (L_r or S or R) + (1.0 L or 0.5 W).......(4-5)

U = 1.2 D + 1.0 W + 1.0 L + 0.5 (L_r or S or R).........(4-6)

U = 1.2 D + 1.0 E + 1.0 L + 0.2 S.......................(4-7)

U = 0.9 D + 1.0 W..(4-8)

U = 0.9 D + 1.0 E..(4-9)

Where :

U = Required Strength
D = Dead Load
L = Live Load
L_r = Roof Live Load
S = Snow Load
R = Rain Load
W = Wind Load
E = Earthquake Load

Notes:

1. The live load factor in Equs. (4-5) to (4-7) shall be permitted to be reduced to ***0.5*** except for garages, area occupied as places of public assembly, and all area where **L** is greater than 4.8 kN/m^2.
2. Where ***W*** is based on service level wind loads, ***1.6 W*** shall be used in place of ***1.0 W*** in Equs. (4-6) & (4-8).
3. Where E is based on service load forces, ***1.4 E*** shall be used in place of ***1.0 E*** in Equs. (4-7) & (4-9).

The total concrete compressive force can be expressed as:

$C = f_{av.}.b.c$ ....(4-10)

Where:

$f_{av.}$ = average concrete compressive stress.

b = section width.

c = neutral axis depth.

Equ. (4-10) can be written as:

$C = \alpha f'_c\, b\, c$..(4-11)

Where α is the ratio between average compressive stress and the compressive strength (f_{av}/f'_c)

The location of the Total concrete compressive force can be designated as βc, where β is the ratio between the compressive force depth and neutral axis depth. From experimental results, the values of α as follows:

For $f'_c \leq 30$ MPa,.............$\Rightarrow \alpha = 0.72$

For $f'_c > 30$ MPa.......$\Rightarrow$ α will be reduced by 0.04 for every 7 MPa increase in f'_c but not less than 0.56.

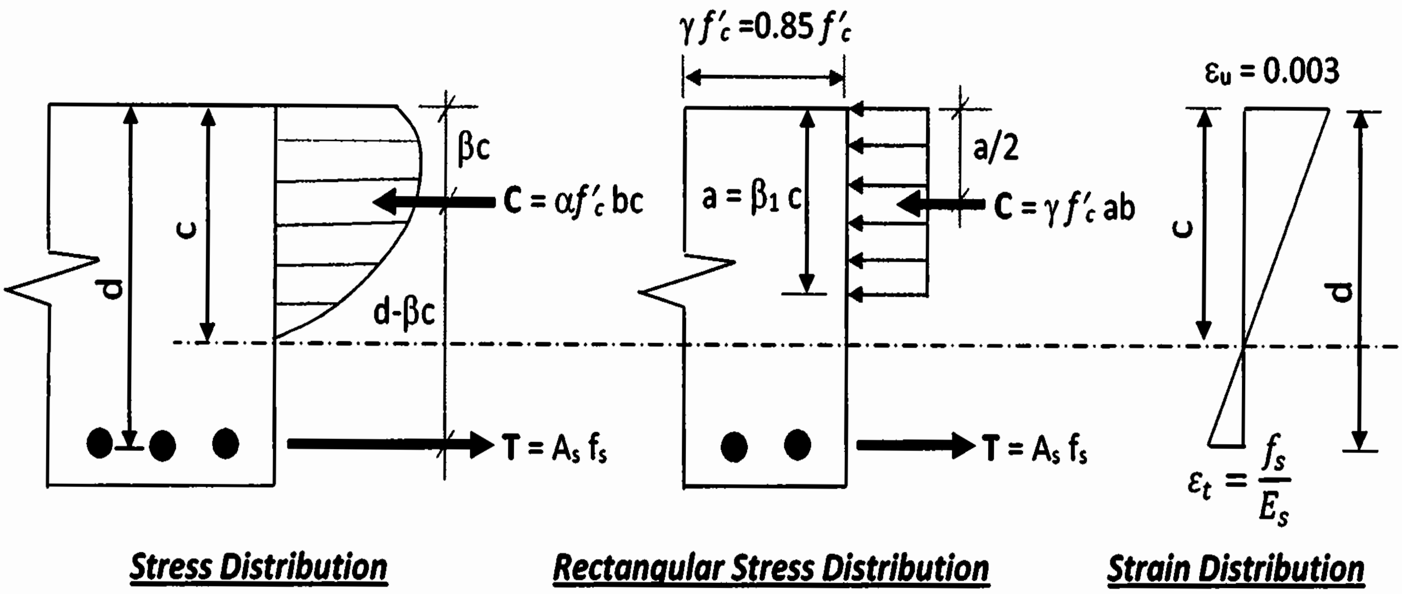
$$0.56 \leq \alpha \leq 0.72$$

Also, from experimental results, the values of β as follows:

For $f'_c \leq 30$ MPa,.............$\Rightarrow \beta = 0.425$

For $f'_c > 30$ MPa.......$\Rightarrow$ β will be reduced by 0.025 for every 7 MPa increase in f'_c but not less than 0.325.

$$0.325 \leq \beta \leq 0.425$$



The analysis of some sections become complex using the above (parabolic) stress distribution. Therefore, a uniform shapes shall be adopted to approximate the stress distribution. The ACI-code proposed a rectangular distribution previously proposed by *Whitney* in condition that the compressive force and its location is the same.

If the depth of the Whitney stress block is a, then:

$C = \alpha f'_c\, b\, c = \gamma f'_c\, a\, b$ ....(4-12)

$a = \beta_1 c$..(4-13)

The values of both γ and β_1 can be determined in terms of α and β and since the location of the resultant force is the same for both cases;

$a / 2 = \beta c \Rightarrow \therefore a = 2 \beta c \Rightarrow \beta_1 c = 2 \beta c \Rightarrow$ $\boxed{\beta_1 = 2 \beta.}$....(4-15)

Substitute in Equ. (4-12) results:

$\alpha f'_c\, b\, c = \gamma f'_c\, \beta_1\, c\, b \Rightarrow$ $\boxed{\gamma = \alpha / \beta_1 = 0.85 \text{(constant)}}$...........(4-16)

Also, from experimental results, the values of β_1 as follows:

For $f'_c \leq 30$ MPa,.............$\Rightarrow \beta_1 = 0.85$

$$c_b = \frac{600}{600+f_y} d \quad \text{.......................................(4-20)}$$

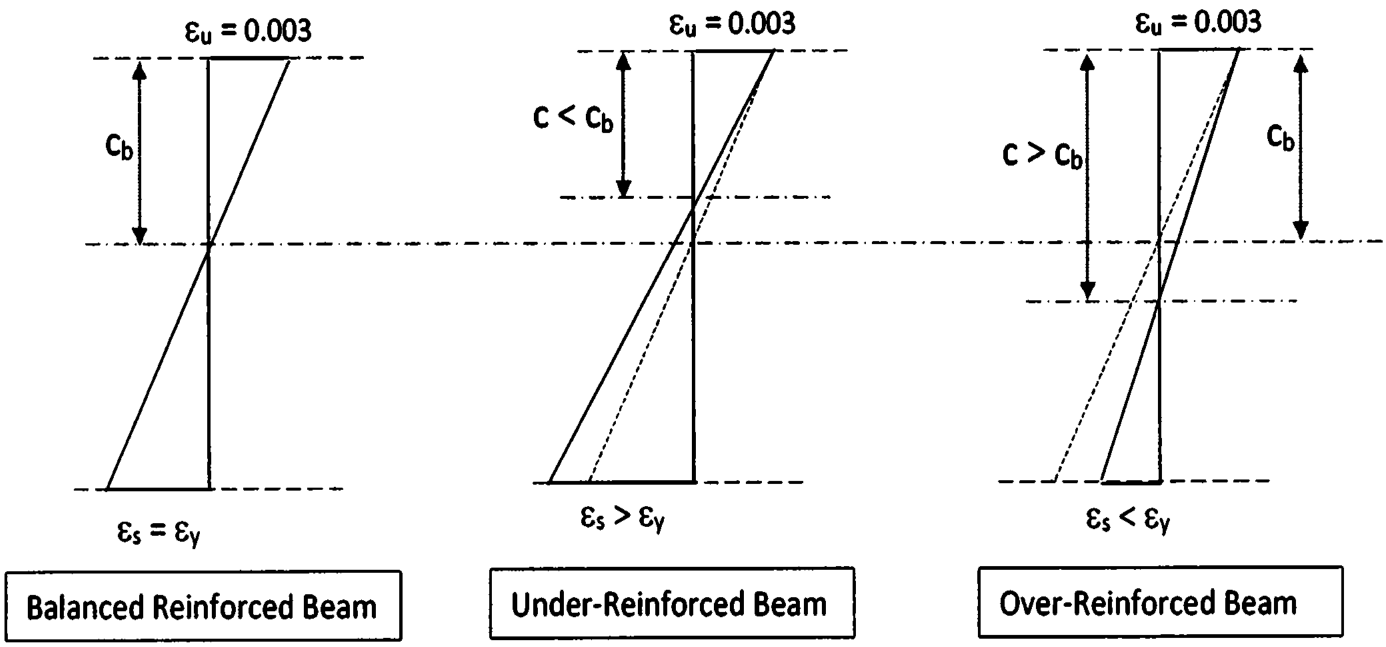
Substitute in Equ. (4-19) results:

$$\boxed{\rho_b = 0.85\beta_1 \frac{f'_c}{f_y} \cdot \frac{600}{600+f_y}} \quad \text{..............................(4-21)}$$

4.4.2 Under-Reinforced Concrete Beams

Beams in which the steel reinforcement is less than those causing the balanced failure, i.e., $\rho = A_s / bd < \rho_b$, here......... $c < c_b$ and $\varepsilon_s > \varepsilon_y$

This failure occurs when the steel reinforcement reaches f_y first. After that the neutral axis lower and the concrete strains increases until reaches 0.003 followed by crushing of concrete as shown in the figure below. This kind of failure is called "***Tension Failure***".



Strain distributions for different types of reinforced beams.

In order to find the ultimate design strength, ϕM_n, the nominal design moment, M_n calculated using Equs. (4-22) to (4-24) is multiplied by the strength reduction factor, ϕ, calculated based on the value of steel strain, ε_t.

1- Over-Reinforced Beams:

When $\rho > \rho_b$, the beams will be over-reinforced and the steel stress, f_s, will equal unknown. The equilibrium equation will be:

$C = T, \Rightarrow 0.85 f'_c\, ab = A_s f_s, \Rightarrow 0.85 f'_c\, \beta_1\, bc$ ....(4-25)

In the above equation (4-25), there are two unknowns (c and f_s), so, it is important to develop another equation relating c and f_s. From strain distribution:

$$\frac{\varepsilon_u}{c} = \frac{\varepsilon_s}{d-c}, \rightarrow \therefore\ \varepsilon_s = \frac{d-c}{c}\varepsilon_u$$

$$f_s = \varepsilon_s E_s = \frac{d-c}{c}\varepsilon_u E_s = 600\frac{d-c}{c}$$, where $\varepsilon_u = 0.003$ and $E_s = 200{,}000$ MPa.

Substituting f_s in equilibrium equation (4-25) to get 2nd order equation in terms of c. Then solve for c and then find $a = \beta_1 c$ and $f_s = 600\frac{d-c}{c}$. Then the design moment strength (M_n) is calculated using Equs. (4-22) to (4-24) in condition that f_y to be replaced by f_s.

In this case, an equation to calculate the value of c can be derived in terms of:

| $c = K_u\, d,$ |

K_u can be calculated as follows:

$C = T, \Rightarrow 0.85 f'_c\, ab = A_s f_s, \Rightarrow 0.85 f'_c\, \beta_1\, bc = \rho bd\, f_s = 0.85 f'_c\, \beta_1\, bK_u\, d$

$$\varepsilon_s = \frac{d-c}{c}\varepsilon_u = \frac{d-K_u d}{K_u d}\varepsilon_u = \frac{1-K_u}{K_u}\varepsilon_u$$

$$f_s = \varepsilon_s E_s = \frac{1-K_u}{K_u}\varepsilon_u E_s = 600\frac{1-K_u}{K_u}$$, where $\varepsilon_u = 0.003$ and $E_s = 200{,}000$ MPa.

4.4.3 Over-Reinforced Concrete Beams

Beams in which the steel reinforcement is greater than those causing the balanced failure, i.e., $\rho = A_s / bd > \rho_b$, here......... $c > c_b$ and $\varepsilon_s < \varepsilon_y$.

Normally, the failure occurs by crushing of concrete as it reaches ultimate strain (ε_u = 0.003) before the steel reaching the yield strength, f_y. This kind of failure is known as "***Compression Failure***". This failure is happened suddenly without previous warnings. Thus, the ACI-code does not allow this kind of failure to happen by designing the beams to be under-reinforced always.

4.5 Practical Consideration in Beams Design

1- *Beam Selfweight Estimation*: If the section size is assumed for analysis purposes, it will be used then for selfweight estimation. For design purposes, if there is no information about the section size, it will be assumed based on experience and used to calculate the beam selfweight. This assumption has to be checked after getting design results.

2- *Beam Section Aspect Ratio*: If there are no architectural requirements or other restrictions, the favorite economical aspect ratio (effective depth/width) is between 2 to 3.

3- *Concrete Cover*: Concrete cover must be provided to protect steel reinforcement from the surrounding exposure, fire and to provide enough bond between concrete and steel. The minimum concrete cover based on ACI-Code (7.7.1) are:

(a): Concrete in cast against & permanently exposed to soil...........75 mm

(b): Concrete exposed to earth or weather:

- Wall panel, slabs & Joists..25 mm
- Other members...40 mm

<u>3- Minimum Steel Ratio</u>: In addition to tension and compression failure, there is another type of failure happened in under-reinforced beams. If the beam bending moment capacity is less than the cracking moment, a sudden beam failure will occur without any advanced warnings.

To avoid the above kind of failure, the ACI-Code (10.5) specify minimum steel ratio to be:

$$\rho_{min} = \frac{As_{min}}{b_w d} = \frac{0.25\sqrt{fc}\, b_w d}{fy} \cdot \frac{1}{b_w d} = \boxed{\frac{\sqrt{fc}}{4\, fy} \geq \frac{1.4}{fy}} \quad \text{......................(4-19)}$$

Where:

b_w = the web width for T-section and equals to **b** for rectangular section.

Generally, to ensure tension failure rather than compression or cracking failure, the reinforcement ratio must be:

$\rho_{min} < \rho < \rho_{max}$ ....(4-20)

Based on the ACI-Code, the above limitation can be neglected if the provided steel reinforcement is greater than the required amount by one-third and this will results to less reinforcement for large sections.

The minimum steel ratio derived above is not applicable to structural slabs and footings. The minimum steel ratio for them will be the temperature and shrinkage one.

4.6 ACI-Code Provisions for Under-Reinforced Beams

There are three main provisions:

1- **Maximum Steel Ratio:** This ratio can be calculated by two methods:

(a): Direct Method: by adopting steel ratio less than ρ_b that results in a net tensile strain in extreme tension steel at normal strength of 0.00376. The limit of 0.004 is slightly more conservative (ACI-Code 10.3.5).

$$\rho_{max} = 0.75\,\rho_b \quad \text{.........(4-10)}$$

(b): Indirect Method: by limiting the minimum strain in extreme tension steel, (ε_t). The extreme distance designated as (d_t). From the strain distribution:

$$\varepsilon_t = \varepsilon_u \frac{d_t - c}{c} \quad \text{.........(4-11)}$$

If the steel reinforcement lies in one layer, $d_t = d$. For more than one layer, $d_t > d$. In this derivation, it is assumed that $d_t \approx d$.

To ensure tension failure, ACI-Code (10.3.5) states that for members with factored axial compressive load < 0.1 $f'_c A_g$, ε_t at nominal strength shall not be less than 0.004.

$$\varepsilon_t \geq 0.004$$

To develop equation for ρ_{max} that cause steel strain of 0.004 using the same principle for derivation of ρ_b.

$$\rho = \frac{0.85\beta_1 c \acute{f_c}}{f_y d} \quad \text{.........(4-12)}$$

From strain distribution and by substituting ε_t instead of ε_s

$$c = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t} d \quad \text{.........(4-13)}$$

Substitute c in equ (4-13) in to equ. (4-12):

$$\rho = 0.85\beta_1 \frac{f'_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t} \quad \text{.......(4-14)}$$

Substitute the minimum strain value ($\varepsilon_t = 0.004$) results:

$$\boxed{\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + 0.004}} \quad \text{.......(4-15)}$$

The least of equs. (4-10) & (4-15) to be the control.

2- **<u>Strength Reduction Factor (ϕ) Calculation:</u>** This factor depends on the type of failure. In tension-controlled members, $\phi = 0.9$, while for compression-controlled members, $\phi = 0.75$ for members with spiral reinforcement and $\phi = 0.65$ for other reinforced members (ACI- Code 9.3.2).

<u>(a) Tension – Controlled Members</u>

Members with net tensile strain in the extreme tension steel, $\varepsilon_t \geq 0.005$. In this case, the strength reduction factor, $\phi = 0.9$. The steel ratio that cause steel strain of 0.005 can be calculated the same way as ρ_{max} and will be denoted as ρ_t:

$$\rho_t = 0.85\beta_1 \frac{f'_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + 0.005} \quad \text{.......(4-16)}$$

<u>(b) Compression – Controlled Members</u>

Members with net tensile strain in the extreme tension steel, $\varepsilon_t \leq 0.002$. In this case, the strength reduction factor, $\phi = 0.75$ for members with spiral reinforcement and $\phi = 0.65$ for other reinforced members. ε_t can be calculated by finding the compression zone depth, a, and then the neutral axis depth, c using equ. (4-11).

4.7 Analysis & Design of Singly Reinforced Concrete Rectangular Beams

4.7.1 Analysis of Singly Reinforced Concrete Rectangular Beams

The analysis is to determine the nominal or ultimate moment strength as both the section size of the beam together with the material properties are known or given. The analysis can be carried out through evaluating the steel ratio, ρ, and compare it with the balanced steel ratio, ρ_b, and then classify the beams in to two kinds based on steel amount:

1- Balanced or Under-Reinforced Beams:

When $\rho \leq \rho_b$, the beams will be either under-reinforced or balanced reinforced as mentioned before. Here, the steel stress, f_s, will equal to yielding strength, f_y. The analysis can be done by calculating the depth of the Whitney stress block, a, from equilibrium of forces:

$$C = T, \Rightarrow 0.85 f'_c\, ab = A_s f_y, \Rightarrow \boxed{a = \frac{A_s f_y}{0.85 f'_c\, b}} \quad \text{.............................(4-21)}$$

The nominal moment strength can be calculated by taking moment about compression force center:

$$\boxed{M_n = A_s f_y \left(d - \frac{a}{2}\right)} \quad \text{...(4-22)}$$

Or by taking moment about the tension force center:

$$\boxed{M_n = 0.85 f'_c\, ab \left(d - \frac{a}{2}\right)} \quad \text{..(4-23)}$$

The nominal moment strength can be calculated also in terms of the actual steel ratio (ρ) as:

$$a = \frac{A_s f_y}{0.85 f'_c\, b} = \frac{\rho\, bd f_y}{0.85 f'_c\, b} = \frac{\rho\, d f_y}{0.85 f'_c}, M_n = A_s f_y \left(d - \frac{a}{2}\right) = \rho bd f_y \left(d - \frac{\rho\, d f_y}{(2)0.85 f'_c}\right)$$

$$\boxed{\therefore M_n = \rho b d^2 f_y \left(1 - 0.59 \rho \frac{f_y}{f'_c}\right).} \quad \text{..(4-24)}$$

$\rho bd\, f_s = 0.85 f'_c\, \beta_1\, bK_u\, d \Rightarrow \rho f_s = 0.85 f'_c\, \beta_1\, K_u \Rightarrow \rho\, 600 \frac{1-K_u}{K_u} = 0.85 f'_c\, \beta_1\, K_u$

$K_u = \frac{600}{0.85 f'_c\, \beta_1} . \rho \frac{1-K_u}{K_u}$, let $m = \frac{600}{0.85 f'_c\, \beta_1}$, $K_u = m.\rho \frac{1-K_u}{K_u}$, $K_u^2 = \rho m\,(1 - K_u)$

$$K_u = \sqrt{\left(\frac{\rho m}{2}\right)^2 + \rho m} - \frac{\rho m}{2} \quad \text{...(4-26)}$$

4.7.2 Design of Singly Reinforced Concrete Rectangular Beams

The design is to determine the appropriate section size and the required reinforcement amount as both the applied service load together with the material properties are known or given.

CASE # 01: Section size or part of it is unknown

1- Calculate the design bending moments (M_u) from structural analysis after including the beam selfweight by assuming the section size.

2- Calculate both the maximum steel ratio (ρ_{max}) and the minimum steel ratio (ρ_{min}).

3- Select the required steel ratio (ρ) such: $\rho_{min} < \rho < \rho_{max}$. It is recommended to select $\rho \le \rho_t$ to get the maximum strength reduction factor ($\phi = 0.9$) allowed.

4- Determine the section dimensions (bd^2) using: $M_n = \rho b d^2 f_y \left(1 - 0.59 \rho \frac{f_y}{f_c}\right)$.

5- Based on experience, assume either b or d to find the other dimension.

6- Calculate the required steel ratio ($As = \rho\, bd$) and the number of appropriate size steel bars and spacing according to ACI-Code provisions.

7- Find the total beam depth (h) as follows:

h = d + 40 + (assumed stirrup size) + (assumed main bars size/2).......... If one layer.

h = d + 40 + (assumed stirrup size) + (assumed main bars size) + (spacing between main bars/2)............. If two layer.

CASE # 02: Section size is known

1. Calculate the design bending moments (M_u) from structural analysis after including the beam selfweight.
2. Calculate both the maximum steel ratio (ρ_{max}) and the minimum steel ratio (ρ_{min}).
3. The strength reduction factor (ϕ) will primarily assumed to be 0.9.
4. Determine the required steel ratio (ρ) as: $M_u = \emptyset M_n = \emptyset \rho b d^2 f_y \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$. This will give 2nd order equation in terms of ρ and can be solved as:

$$M_u = \emptyset \rho b d^2 f_y \left(1 - \frac{\rho f_y}{2*0.85 f_c'}\right) \div \phi b d^2 \Rightarrow \frac{M_u}{\emptyset b d^2} = \rho f_y \left(1 - \frac{\rho f_y}{2*0.85 f_c'}\right)$$

Let $\boxed{\frac{M_u}{\emptyset b d^2} = R \quad \text{and} \quad \frac{f_y}{0.85 f_c'} = m,}$ then:

$$R = \rho f_y \left(\frac{2-\rho m}{2}\right) \rightarrow \therefore 2R = 2\rho f_y - \rho^2 f_y m$$

$$\boxed{\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{f_y}}\right)} \quad \text{.......................................(4-27)}$$

5. The assumed value of strength reduction factor (ϕ) to be checked as:
   - If $\rho \le \rho_t$ .... The assumed value is OK ($\phi = 0.9$).
   - If $\rho > \rho_t$ .... The value of ϕ will be less than 0.9. Therefore, the design moment strength (ϕM_n) will be less the design moment (M_u). In this case, the reinforcement has to be increased and calculate the value of ϕ form Equs. (4-17) and (4-18) and then calculate ϕM_n to make sure that ($\phi M_n \ge M_u$).

6. Calculate the required steel ratio (As = ρ bd) and the number of appropriate size steel bars and spacing according to ACI-Code provisions.

Example (4-1):

A reinforced concrete rectangular beam of width b = 250 mm and effective depth of d = 460 mm has a compressive strength f'_c = 20 MPa and reinforcement tensile strength f_y = 300 MPa. Calculate the ultimate design bending moment for:

(a) : Steel area A_s = 2000 mm².

(b) : Steel area A_s = 5160 mm².

(c) : the maximum design moment based on ACI-Code provisions.

Solution:

(a): Steel ratio A_s = 2000 mm²

$$\rho_b = 0.85\beta_1 \frac{f_c}{f_y} \cdot \frac{600}{600+f_y} = 0.85\,(0.85)\frac{20}{300} \cdot \frac{600}{600+300} = 0.032\,,\ \beta_1 = 0.85 \text{ since } f'_c \leq 30 \text{ MPa.}$$

$$\rho = \frac{A_s}{bd} = \frac{2000}{250\,(460)} = 0.0174 < \rho_b = 0.032 \Rightarrow \boxed{\text{the section is under-reinforced.}}$$

$$\rho_t = 0.85\beta_1 \frac{f'_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u+0.005} = 0.85(0.85)\frac{20}{300} \cdot \frac{0.003}{0.003+0.005} = 0.018$$

Since $\rho = 0.0174 < \rho_t = 0.018$ ....$\Rightarrow$ $\boxed{\phi = 0.9}$

$$a = \frac{A_s f_y}{0.85 f'_c\, b} = \frac{2000\,(300)}{0.85\,(20)(250)} = 141 \text{ mm}$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = 2000\,(300)\left(460 - \frac{141}{2}\right) * 10^{-6} = 233.7 \text{ kN.m}$$

Also, M_u can be calculated using Equs. (4-23) and (4-24):

$$M_n = 0.85 f'_c\, ab\left(d - \frac{a}{2}\right) = 0.85(20)(141)(250)\left(460 - \frac{141}{2}\right) * 10^{-6} = 233.4 \text{ kN.m}$$

$$M_n = \rho b d^2 f_y\left(1 - 0.59\rho\frac{f_y}{f_c}\right) = 0.0174(250)(460)^2(300)(1 - 0.59(0.0174))\frac{300}{20} * 10^{-6}$$

$$= 233.6 \text{ kN.m}$$

$$M_u = \phi\, M_n = 0.9\,(233.7) = \boxed{210 \text{ kN.m}}$$

(c): the maximum design moment based on ACI-Code provisions.

The maximum design moment based on ACI-Code provision will be determined based on maximum steel ratio allowed by the code.

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + 0.004} = 0.85(0.85)\frac{20}{300} \cdot \frac{0.003}{0.003+0.004} = \boxed{0.0206}$$

A_s (max) = ρ_{max} bd = 0.0206 (250)(460) = 2369 mm^2

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2369\,(300)}{0.85\,(20)(250)} = \boxed{167\text{ mm}}\quad c = a/\beta_1 = 167/0.85 = 196.5\text{ mm}$$

Since $\rho = 0.0206 > \rho_t = 0.018$ ....⇒ ε_t must be calculated using Equ. (4-11)

$$\varepsilon_t = \varepsilon_u \frac{d_t - c}{c} = 0.003\frac{460-196.5}{196.5} = 0.0040$$

Since $0.002 < \varepsilon_t = 0.0040 < 0.005$...⇒ ϕ must be calculated using Equ. (4-18)

$$\phi = 0.483 + 83.3\,\varepsilon_t = 0.483 + 83.3\,(0.0040) = \boxed{0.816}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 2369\,(300)\left(460 - \frac{167}{2}\right) * 10^{-6} = 267.6\text{ kN.m}$$

Also, M_u can be calculated using Equs. (4-23) and (4-24):

$$M_n = 0.85 f_c' ab \left(d - \frac{a}{2}\right) = 0.85(20)(167)(250)\left(460 - \frac{167}{2}\right) * 10^{-6} = 267.2\text{ kN.m}$$

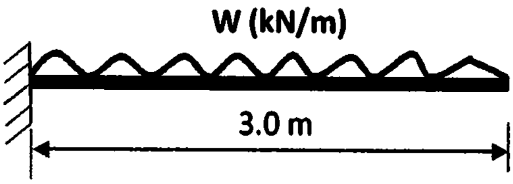
$$M_n = \rho b d^2 f_y \left(1 - 0.59\rho \frac{f_y}{f_c'}\right) = 0.0206(250)(460)^2(300)(1 - 0.59(0.0206))\frac{300}{20} * 10^{-6}$$

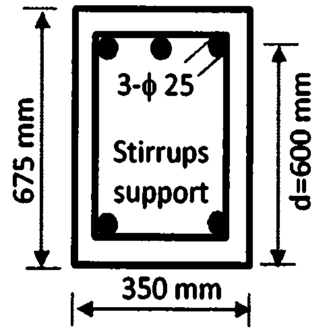
$= 267.3$ kN.m

$$M_u = \phi\, M_n = 0.816\,(267.6) = \boxed{218.36\text{ kN.m}}$$

Example (4-2):

Check the adequacy of the given beam based on bending requirements if the service dead load (including beam selfweight) = 8 kN/m and the service live load = 18 kN/m. Note that the concrete compressive strength f'_c = 35 MPa and reinforcement tensile strength f_y = 300 MPa.





Solution:

Since f'_c = 35 MPa > 30 MPa, ⇒ β_1 can be calculated from Equ. (4-17) as:

$\beta_1 = 0.85 - 0.05\,(f'_c - 30)/7 = 0.85 - 0.05\,(35 - 30)/7 =$ **0.81**

$A_s = 3\,(\pi(25)^2/4) = 1473\ mm^2$

$$\rho = \frac{A_s}{bd} = \frac{1473}{350\,(600)} = 0.007$$

$$\rho_b = 0.85\beta_1 \frac{f'_c}{f_y} . \frac{600}{600+f_y} = 0.85\,(0.81)\frac{35}{300} . \frac{600}{600+300} = 0.0536$$

Since $\rho = 0.007 < \rho_b = 0.0536$ ⇒ **the section is under-reinforced.**

$$\rho_t = 0.85\beta_1 \frac{f'_c}{f_y} . \frac{\varepsilon_u}{\varepsilon_u + 0.005} = 0.85(0.81)\frac{35}{300} . \frac{0.003}{0.003+0.005} = 0.0\ 3$$

Since $\rho = 0.007 < \rho_t = 0.003$ ....⇒ **$\phi = 0.9$**

$$M_n = \rho b d^2 f_y \left(1 - 0.59\rho \frac{f_y}{f'_c}\right) = 0.007(350)(600)^2(300)(1 - 0.59(0.007))\frac{300}{35} * 10^{-6}$$

$= 255.2$ kN.m

- Section capacity (ϕM_n) = 0.9 (255.2) = **229.7 kN.m**

$W_u = 1.2\,D + 1.6\,L = 1.2\,(8) + 1.6\,(18) = 38.4$ kN/m

- Design moment (M_u) = $W_u\,L^2/2 = 38.4\,(3)^2/2 =$ **178.8 kN.m**

Since $M_u = 178.8 < \phi M_n = 229.7$ ⇒ The given section is adequate.

- $M_u = \emptyset \rho b d^2 f_y \left(1 - 0.59\rho \frac{f_y}{f_c}\right)$

$129.6\,x10^6 = 0.9\,(0.016) bd^2 (400) \left(1 - 0.59(0.016)\frac{400}{30}\right) = 5.035\ bd^2$

$\therefore\ bd^2 = 25.74 \times 10^6$

Select beam width to be 250 mm, $d^2 = 25.74 \times 10^6 / 250 = 102.96 \times 10^3$

d = 321 mm ⇒ say 320 mm

- Find the required steel area, $A_s = \rho\, bd = 0.016\,(250)(320) = 1280\ mm^2$

Use ϕ 25 mm diameter bars, (A_b= 491 mm^2)

No. of bars = 1280/491 = 2.61⇒ say 3

∴ use 3-ϕ 25 mm diameter bars

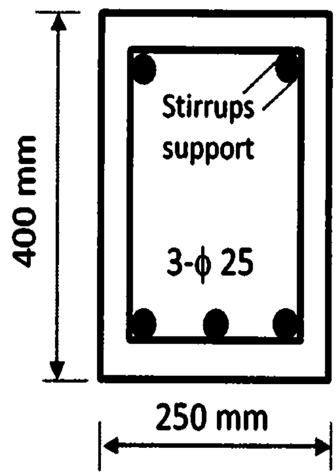
Check the adequacy of the calculated beam width for the number of steel bars.

Required width = 40+40+10+10+3 (25)+(3-1)(25) = 225 mm

Provided width = 250……OK ☺

Calculate the total beam depth (h) as: (assume one layer of reinforcement)

h = d + 40 + 10 + 12.5 = 320 + 62.5 = 383 mm ⇒ say 400 mm



Example (4-4):

Determine the reinforcement required for a beam of width b = 300 mm and total depth h = 700 mm under service dead load moment (including beam selfweight) = 100 kN.m and service live load moment = 150 kN.m. Note that the concrete compressive strength f'_c = 20 MPa and reinforcement tensile strength f_y = 400 MPa.

Solution:

- Calculate the ultimate design moment (M_u):

$M_u = 1.2\, M_d + 1.6\, M_L = 1.2\,(100) + 1.6\,(150) = \boxed{360 \text{ kN.m}}$

- Calculate both ρ_{max} and ρ_{min}:

$\rho_b = 0.85\beta_1 \frac{f'_c}{f_y} \cdot \frac{600}{600+f_y} = 0.85\,(0.85)\frac{20}{400} \cdot \frac{600}{600+400} = 0.0217$, $\beta_1 = 0.85$ since $f'_c \leq 30$ MPa.

$$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + 0.004} = 0.85(0.85)\frac{20}{400} \cdot \frac{0.003}{0.003+0.004} = \boxed{0.0155 \text{ (control)}}$$

$$\rho_{min} = \frac{\sqrt{f'_c}}{4\,f_y} \geq \frac{1.4}{f_y} = \frac{\sqrt{20}}{4\,(400)} \geq \frac{1.4}{400} = 0.00280 \geq \boxed{0.0035 \text{ (control)}}$$

- The strength reduction factor (ϕ) will primarily assumed to be 0.9.
- Calculate steel ratio as :

d = 700-40-10-12.5 = 637.5 mm⇒ say d = 635 mm

$$R = \frac{M_u}{\phi b d^2} = \frac{360x10^6}{0.9(300)(635)^2} = 3.3, \qquad m = \frac{f_y}{0.85\, f'_c} = \frac{400}{0.85(20)} = 23.53$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{f_y}}\right) = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2(23.53)(3.3)}{400}}\right) = \boxed{0.0093}$$

Since $\rho_{min} = 0.0035 < \rho = 0.0093 < \rho_{max} = 0.0155$.....⇒ Design is OK ☺

$A_s = \rho\, bd = 0.0093\ (300)(635) = 1772\ mm^2$

- Re-check the assumed value of ϕ as :

$$\rho_t = 0.85\beta_1 \frac{f_c'}{f_y} . \frac{\varepsilon_u}{\varepsilon_u + 0.005} = 0.85(0.85)\frac{20}{400} . \frac{0.003}{0.003+0.005} = 0.0136$$

Select $\rho = 0.0093 < \rho_t = 0.0136$.....⇒ Assumption is OK ☺

- Find the number and arrangement of steel bars.

Use ϕ 25 mm diameter bars, ($A_s = 491\ mm^2$)

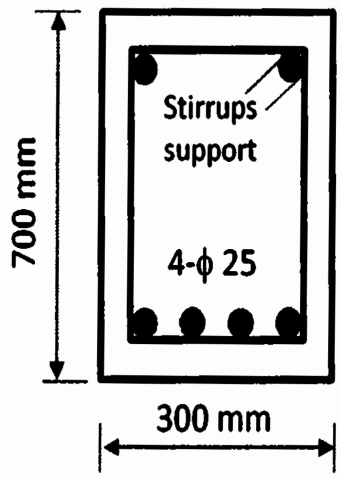
No. of bars = 1772/491 = 3.61⇒ say 4

∴ use 4-ϕ 25 mm diameter bars

Check the adequacy of the calculated beam width for the number of steel bars.

Required width = 40+40+10+10+4 (25)+(4-1)(25) = 275 mm

Provided width = 300......OK ☺



4.8 Analysis & Design of Singly Reinforced Concrete Rectangular Beams Using Tables

Sometimes, in order to make easy the analysis and design of singly reinforced concrete rectangular beams, special Tables can be used that represent the solutions of design and analysis equations. Those Tables can be constructed using Equ. (4-24) as follows:

$$M_n = \rho b d^2 f_y \left(1 - 0.59\rho \frac{f_y}{f_c}\right) \div f'_c\,\mathrm{bd}^2$$

$$\frac{M_n}{f_c b d^2} = \rho \frac{f_y}{f_c}\left(1 - 0.59\rho \frac{f_y}{f_c}\right) \qquad (4\text{-}28)$$

Let $\frac{M_n}{f_c b d^2} = k$ and $\rho \frac{f_y}{f_c} = \omega$

Equ. (4-28) becomes:

$k = \omega\,(1 - 0.59\,\omega)$ .... (4-29)

By selecting values for ω, the corresponding values of k can be calculated. This will results in a relation between the nominal strength and the steel ratio as listed in the Table below.

The first row and column represent the values of ω, while the remaining cells represent the values of k. Knowing that the value of ω consists of three digits composed of the values in the 1st column plus those in the 1st row.

Tables have been prepared for ρ ≤ ρ only (tension failure) and cannot be used for balanced or compression failures.

ω	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
	$k = M_n / f'_c bd^2 = M_u / \phi f'_c bd^2$									
0	0	0.0010	0.0020	0.0030	0.0040	0.0050	0.0060	0.0070	0.0080	0.0090
0.01	0.0099	0.0109	0.0119	0.0129	.0139	0.0149	0.0159	0.0168	0.0178	0.0188
0.02	0.0197	0.0207	0.0217	0.0226	0.0236	0.0246	0.0256	0.0266	0.0275	0.0285
0.03	0.0295	0.0304	0.0314	0.0324	0.0333	0.0343	0.0352	0.0362	0.0372	0.0381
0.04	0.0391	0.0400	0.0410	0.0420	0.0429	0.0438	0.0448	0.0457	0.0467	0.0476
0.05	0.0485	0.0495	0.0504	0.0513	0.0523	0.0532	0.0541	0.0551	0.0560	0.0569
0.06	0.0579	0.0588	0.0597	0.0607	0.0616	0.0625	0.0634	0.0643	0.0653	0.0662
0.07	0.0671	0.0680	0.0689	0.0699	0.0708	0.0717	0.0726	0.0735	0.0744	0.0753
0.08	0.0762	0.0771	0.0780	0.0789	0.0798	0.0807	0.0816	0.0825	0.0834	0.0843
0.09	0.0852	0.0861	0.0870	0.0879	0.0888	0.0897	0.0906	0.0915	0.0923	0.0932
0.10	0.0941	0.0950	0.0959	0.0967	0.0976	0.0985	0.0994	0.1002	0.1011	0.1020
0.11	0.1029	0.1037	0.1046	0.1055	0.1063	0.1072	0.1081	0.1089	0.1098	0.1106
0.12	0.1115	0.1124	0.1133	0.1141	0.1149	0.1158	0.1166	0.1175	0.1183	0.1192
0.13	0.1200	0.1209	0.1217	0.1226	0.1234	0.1243	0.1251	0.1259	0.1268	0.1276
0.14	0.1284	0.1293	0.1301	0.1309	0.1318	0.1326	0.1334	0.1342	0.1351	0.1359
0.15	0.1367	0.1375	0.1384	0.1392	0.1400	0.1408	0.1416	0.1425	0.1433	0.1441
0.16	0.1449	0.1457	0.1465	0.1473	0.1481	0.1489	0.1497	0.1506	0.1514	0.1522
0.17	0.1529	0.1537	0.1545	0.1553	0.1561	0.1569	0.1577	0.1585	0.1593	0.1601
0.18	0.1609	0.1617	0.1624	0.1632	0.1640	0.1648	0.1656	0.1664	0.1671	0.1679
0.19	0.1687	0.1695	0.1703	0.1710	0.1718	0.1726	0.1733	0.1741	0.1749	0.1756
0.20	0.1764	0.1772	0.1779	0.1787	0.1794	0.1802	0.1810	0.1817	0.1825	0.1832
0.21	0.1840	0.1847	0.1855	0.1862	0.1870	0.1877	0.1885	0.1892	0.1900	0.1907
0.22	0.1914	0.1922	0.1929	0.1937	0.1944	0.1951	0.1959	0.1966	0.1973	0.1981
0.23	0.1988	0.1995	0.2002	0.2010	0.2017	0.2024	0.2031	0.2039	0.2046	0.2053
0.24	0.2060	0.2067	0.2075	0.2082	0.2089	0.2094	0.2103	0.2110	0.2117	0.2124
0.25	0.2131	0.2138	0.2145	0.2152	0.2159	0.2166	0.2173	0.2180	0.2187	0.2194
0.26	0.2201	0.2208	0.2215	0.2222	0.2229	0.2236	0.2243	0.2249	0.2256	0.2263
0.27	0.2270	0.2277	0.2284	0.2290	0.2297	0.2304	0.2311	0.2317	0.2324	0.2331
0.28	0.2337	0.2344	0.2351	0.2357	0.2364	0.2371	0.2377	0.2384	0.2391	0.2397
0.29	0.2404	0.2410	0.2417	0.2423	0.2430	0.2437	0.2443	0.2450	0.2456	0.2463
0.30	0.2469	0.2475	0.2482	0.2488	0.2495	0.2501	0.2508	0.2514	0.2520	0.2527
0.31	0.2533	0.2539	0.2546	0.2552	0.2558	0.2565	0.2571	0.2577	0.2583	0.2590
0.32	0.2596	0.2602	0.2608	0.2614	0.2621	0.2627	0.2633	0.2639	0.2645	0.2651
0.33	0.2657	0.2664	0.2670	0.2676	0.2682	0.2688	0.2694	0.2700	0.2706	0.2712
0.34	0.2718	0.2724	0.2730	0.2736	0.2742	0.2748	0.2754	0.2760	0.2766	0.2771
0.35	0.2777	0.2783	0.2789	0.2795	0.2801	0.2807	0.2812	0.2818	0.2824	0.2830
0.36	0.2835	0.2841	0.2847	0.2853	0.2858	0.2864	0.2870	0.2875	0.2881	0.2887
0.37	0.2892	0.2898	0.2904	0.2909	0.2915	0.2920	0.2926	0.2931	0.2937	0.2943
0.38	0.2948	0.2954	0.2959	0.2965	0.2970	0.2975	0.2981	0.2986	0.2992	0.2997
0.39	0.3003	0.3008	0.3013	0.3019	0.3024	0.3029	0.3035	0.3040	0.3045	0.3051
0.40	0.3056	0.3061	0.3066	0.3072	0.3077	0.3082	0.3087	0.3093	0.3098	0.3103

Example (4-5):

Repeat Example (4-1) when the provided steel reinforcement = 2000 mm^2 using tables.

Solution:

After calculating the value of steel ratio: ρ = 0.0174

$\omega = \rho \frac{f_y}{f_c} = 0.0174 \; \frac{300}{20}$ = 0.261 = 0.26 + 0.001

Using the table, the corresponding value of k = 0.2208

$\frac{M_n}{f_c b d^2} = k \; , M_n = k f_c b d^2$ = 0.2208 (20) (250)$(460)^2$ x 10^6 = 233.6 kN.m

The rest can be calculated as in Example (4-1).

Example (4-6):

Repeat Example (4-3) using tables.

Solution:

After calculating the value of steel ratio: ρ = 0.016 and the design moment: M_u = 129.6 kN.m

$\omega = \rho \frac{f_y}{f_c} = 0.016 \; \frac{400}{30}$ = 0.213 = 0.21 + 0.003

Using the table, the corresponding value of k = 0.1862

$\frac{M_n}{f_c b d^2} = k \; , bd^2 = \frac{M_n}{k f_c} = \frac{129.6x10^6/0.9}{0.1862\,(30)}$ = 25.78 x 10^6 mm^3

The rest can be calculated as in Example (4-3).

<u>Example (4-8)</u>:

Repeat Example (4-4) using tables.

<u>Solution:</u>

After calculating the value of the design moment: $M_u = 360$ kN.m

$k = \frac{M_n}{f_c' bd^2} = \frac{360x10^6/0.9}{20(300)(635)^2} = 0.1653$, the nearest value is 0.1656

Using the table, the corresponding value of $\omega = 0.186$

$\omega = \rho \frac{f_y}{f_c'} \rightarrow \rho = \frac{\omega f_c'}{f_y} = \frac{0.186(20)}{400} = 0.0093$

The rest can be calculated as in Example (4-4).

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