Al-Mansour University College

قسم الهندسة المدنية

Civil Eng. Dept.

المرحلة الثالثة

3rd. Stage

Theory of Structures

2022 - 2023

Sheet-3

انشاءات

Rigid Frames

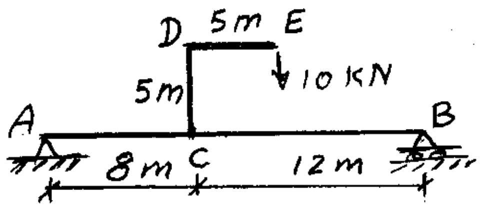
(Solved Problems)

Dr. Maloof Mahmood

Problem Sheet (2) – Structures

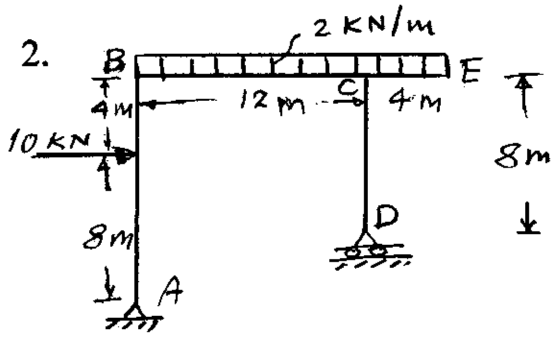
Analyze each of the frames shown below and draw A.F.D., S.F.D. & B.M.D.

1.



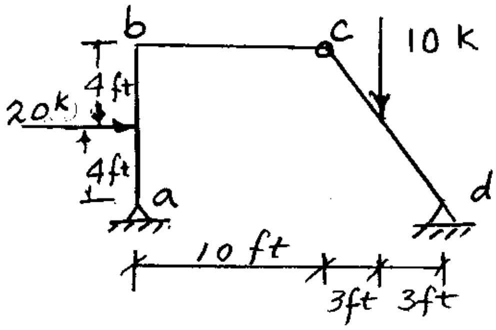
Ans. $(M_c)_{max}$ = 78 kN-m

2.



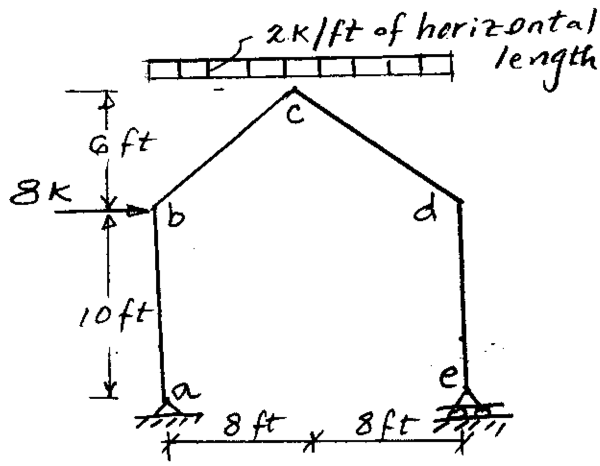
Ans. M_B = 80 kN-m

3.



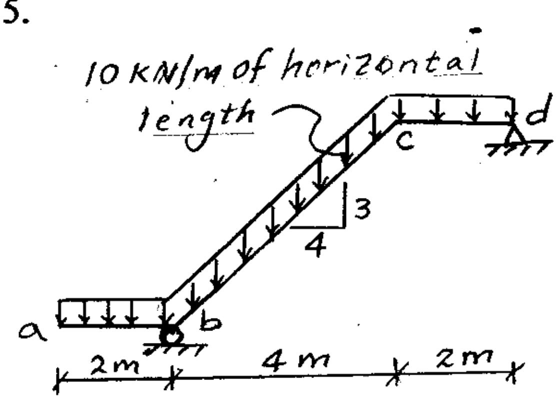
Ans. M_b = 31.25 kip-ft

4.



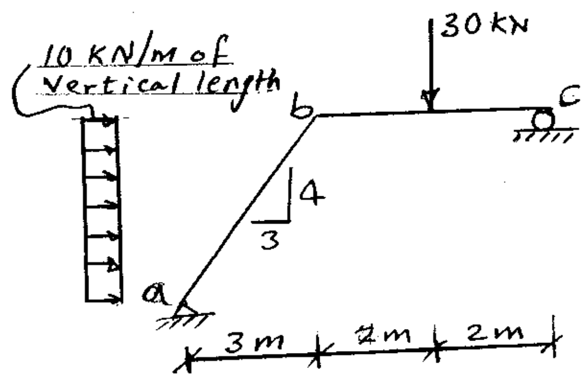
Ans. M_b = 80 kip-ft, M_c = 104 kip-ft

5.



Ans. M_c = 33.3 kN-m

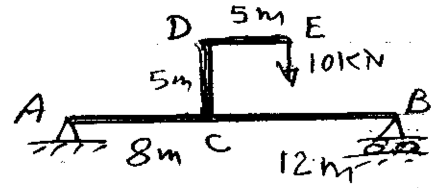
6.



Ans. M_b = 71.4 kN-m

Prob.(1)

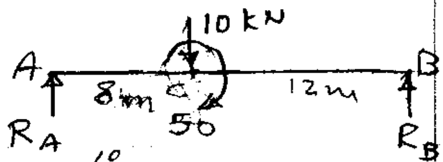
Analyze the frame shown and draw A.F.D., S.F.D. and B.M.D.

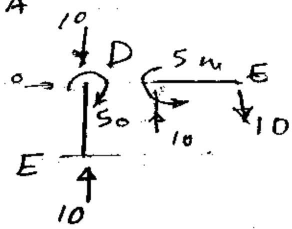


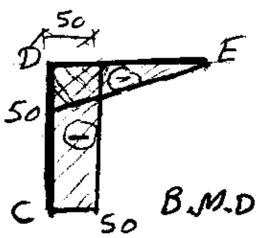
$\overset{+}{\curvearrowleft}$ $\sum M_B = 0$

$R_A(20) + 50 - 10(12) = 0$

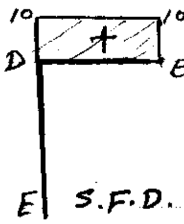
$R_A = \frac{70}{20} = 3.5$ KN , $\sum F_x = 0 \Rightarrow R_B = 6.5$ KN



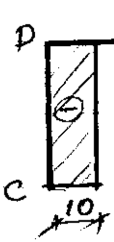




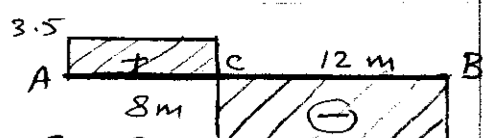
B.M.D



S.F.D.



A.F.D.



S.F.D.

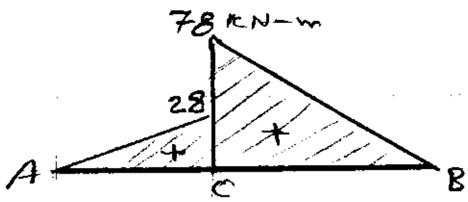
B.M.D.

$AC \Rightarrow M_c = 3.5 \times 8 = 28$ KN-m

$M_c = 28 + 50 = 78$ KN-m

$M_B = 78 - 6.5(12) =$

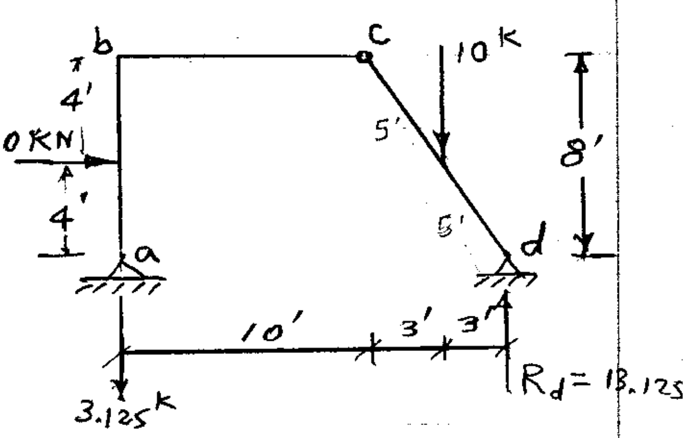
$= 78 - 78 = 0$ ✓



B.M.D.

Prob. (3)

Analyze the frame shown and draw A.F.D., S.F.D. & B.M.D.



—·—

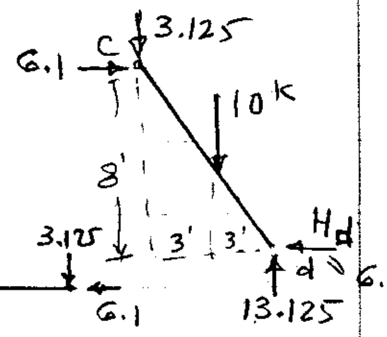
$\overset{+\circlearrowleft}{\sum} M_d = 0$ for the whole frame:

$R_a(16) + \overset{80}{20(4)} - \overset{30}{10(3)} = 0$

$\boxed{R_a} = -\frac{50}{16} = \boxed{-3.125\ k\downarrow}$

$\sum F_y = 0 \Rightarrow \boxed{R_D} = 3.125 + 10 = \boxed{13.125^k \uparrow}$

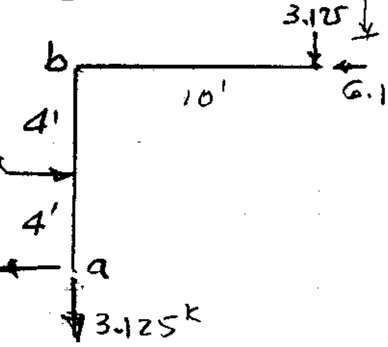
Make a cut at c ; right portion:



$\overset{+\circlearrowleft}{\sum} M_c = 0 \Rightarrow 10(3) + H_d(8) - \underset{=78.75}{13.125(6)} = 0$

$\therefore \boxed{H_d} = \frac{48.75}{8} \cong \boxed{\overleftarrow{6.1}\ k}$

Take Left portion of frame:



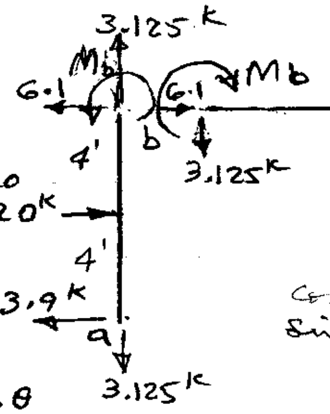
$\overset{+\circlearrowleft}{\sum} M_c = 0 \Rightarrow -3.125(10) + H_a(8) - 20(4) = 0$

$\therefore \boxed{H_a} = \frac{111.25}{8} \cong 13.9^k$

Check $\sum F_x = 0 \Rightarrow H_a + H_d = 20$

$13.9 + 6.1 = 20^k \Rightarrow OK$

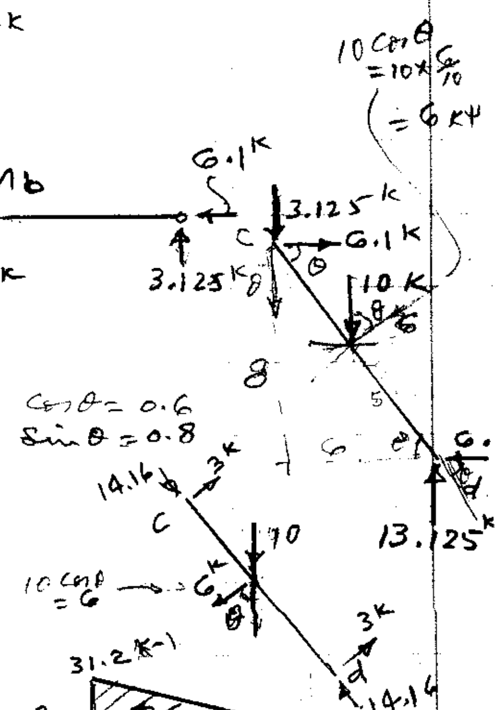
Take a cut at (b):



$\overset{+\circlearrowleft}{\sum} M_b = 0 \Rightarrow \overset{111.2}{13.9(8)} - \overset{80}{20(4)} - M_b = 0$

$\therefore M_b = 31.2^k$

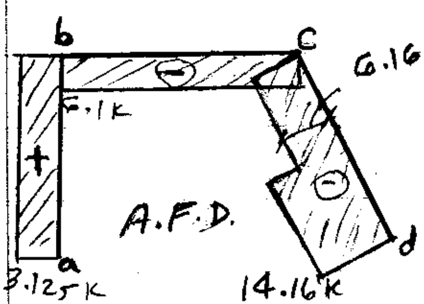
Axial force at c (bar cd):

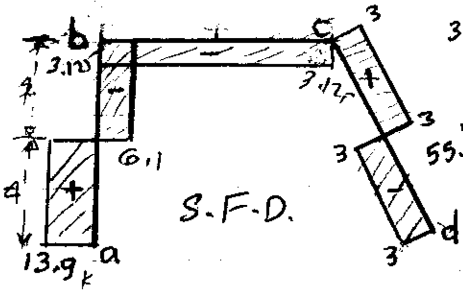


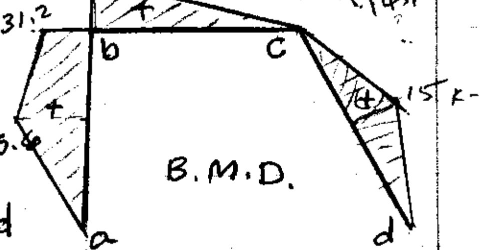
$= 6.1\cos\theta + 3.125\sin\theta + 10\sin\theta$

$= \underbrace{6.1\times(0.6)}_{=3.66} + \underbrace{3.125(0.8)}_{2.5} + \underbrace{10(0.8)}_{8.0} = 14.16^k$

Axial force at (d) $= 6.1\cos\overset{=0.6}{\theta} + 13.125\sin\overset{=0.8}{\theta} = 14.16^k$







$\frac{PL}{4} = \frac{6\times10}{4} = 15$ kN-m

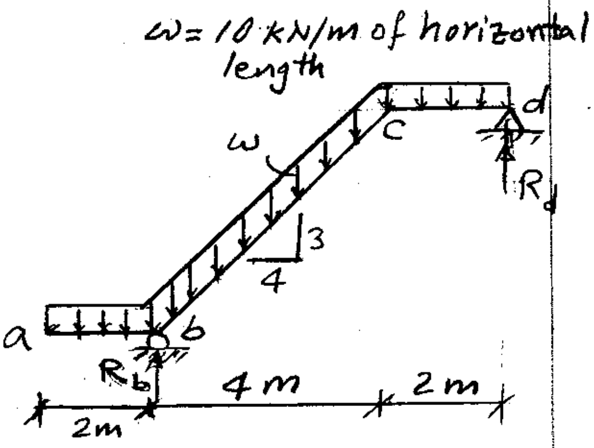
Axial force at 'c' = $6.1\cos\theta$ $3.125\sin\theta$

$= 6.1\times\frac{6}{10} + 3.125\times\frac{8}{10} = 3.66 + 2.50 = 6.16$ k

5

Prob. 5

Analyze the frame shown and draw the A.F.D., S.F.D. & B.M.D.



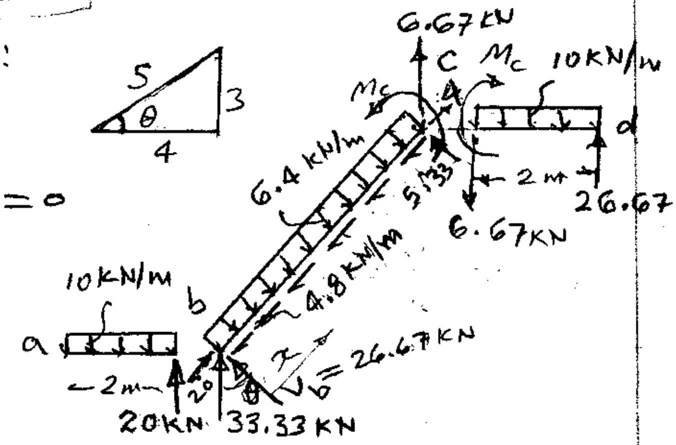
For the whole frame:

$\overset{+}{\circlearrowleft}\ \Sigma M_d = 0 \Rightarrow R_b(6) - 10(8)(4) = 0$

$\therefore \boxed{R_b = 53.33 \text{ KN}}$,

$\Sigma F_y = 0 \Rightarrow \boxed{R_d} = 80 - 53.33 = \boxed{26.67 \text{KN}}$

Take a section at (c) and at (b):



For the portion (cd):

$\overset{+}{\circlearrowleft}\ \Sigma M_c = 0 \Rightarrow 10(2)(1) + M_c - 26.67(2) = 0$

$M_c = \underline{33.3}$ KN-m

For beam (bc):

$(10\times4)(\frac{4}{5}) = 32$ KN

$\frac{32}{5} = \underline{6.4}$ KN/m $\perp$ (bc)

$(10\times4)(\frac{3}{5}) = 24$ KN parallel to bc

$\frac{24}{5} = \underline{4.8}$ KN/m along bc

$V_b = 33.33 \cos\theta = 33.33(0.8) = \underline{26.66}$ KN

$V_c = 6.4\times5 - 26.67 = \underline{5.33}$ KN

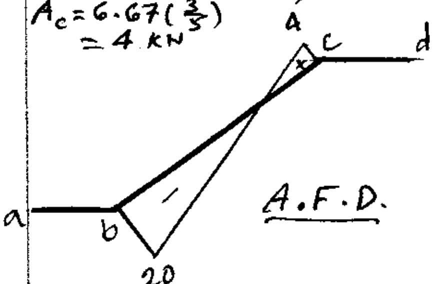
Axial force at (b) $= 33.33(\frac{3}{5}) = 20$ KN

$A_c = 6.67(\frac{3}{5})$
$= 4$ KN

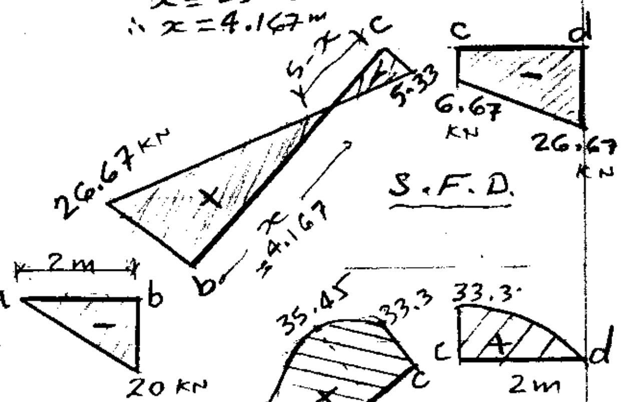
$\frac{26.67}{5.33} = \frac{x}{5-x}$ (≈ 5)

$x = 25 - 5x$

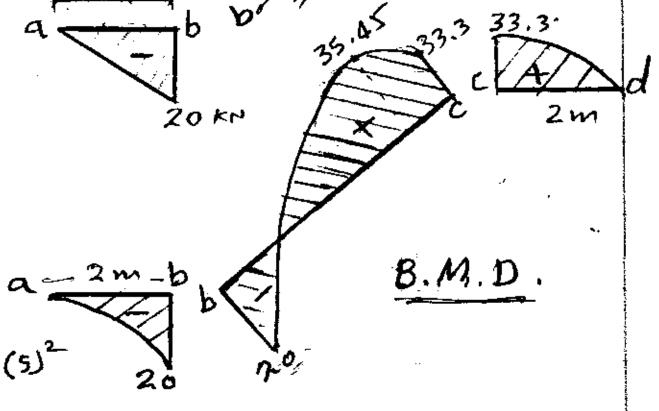
$\therefore x = 4.167$ m



A.F.D.



S.F.D.



B.M.D.

B.M. Beam bc:

$M_x = -20 + 26.66x - \frac{\overset{3.2}{\cancel{6.4}}x^2}{\cancel{2}}$

$M_x]_{x=0} = -20$ KN-m, $M_x]_{x=5} = -20 + 26.66(5) - 3.2(5)^2$

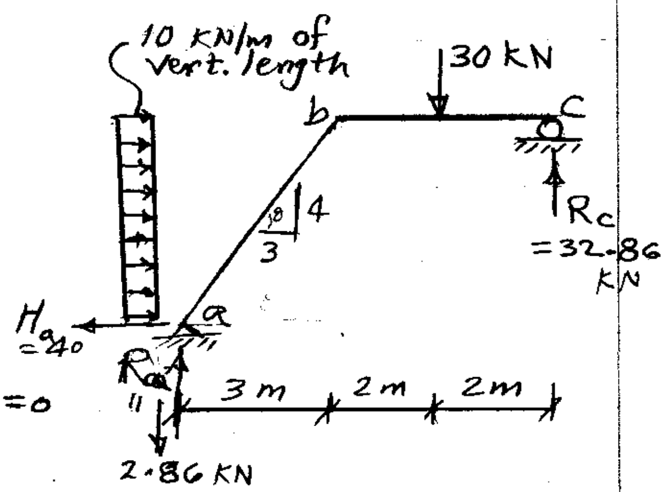
$= -20 + 133.3 - 80 = 33.3$ KN-m

$M_x]_{x=4.167} = \hat{M} = -20 + \underbrace{26.66(4.167)}_{111} - \underbrace{3.2(4.167)^2}_{55.55} = \underline{35.45}$ KN-m

7

Prob. 6

Analyze the frame shown and draw A.F.D., S.F.D. & B.M.D.



For the whole frame,

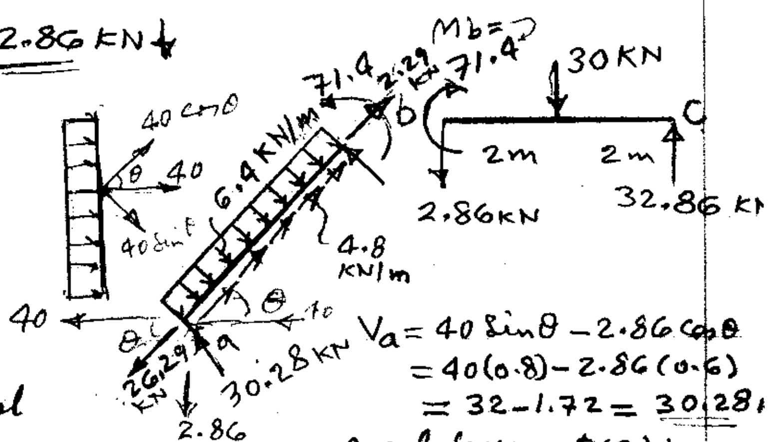
$\Sigma F_x = 0 \Rightarrow H_a = 10 \times 4 = 40$ KN

$\Sigma M_a = 0 \Rightarrow -R_c(7) + 30(5) + 40(2) = 0$

$R_c = \frac{230}{7} = 32.86$ KN ↑

$\Sigma F_y = 0 \Rightarrow R_a = 32.86 - 30 = -2.86$ KN ↓

Take a section at (b):



For beam (ab)

$40 \sin\theta = 40(\frac{4}{5}) = 32$ KN

$\frac{32}{5} = 6.4$ KN/m $\perp^r$ ab

$40 \cos\theta = 40(\frac{3}{5}) = 24$ KN parallel to (ab)

$\frac{24}{5} = 4.8$ KN/m along ab

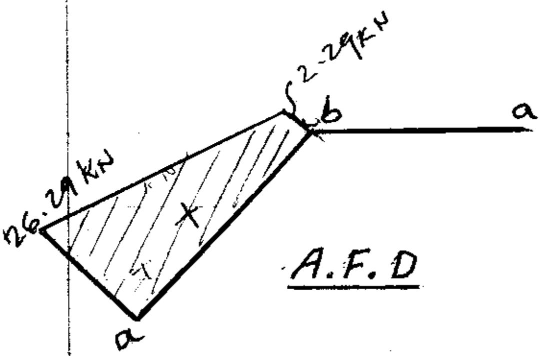
$V_a = 40 \sin\theta - 2.86 \cos\theta$
$= 40(0.8) - 2.86(0.6)$
$= 32 - 1.72 = 30.28$

Axial force at (a):
$= 40(0.6) + 2.86(0.8)$
$= 24 + 2.29 = 26.29$ KN

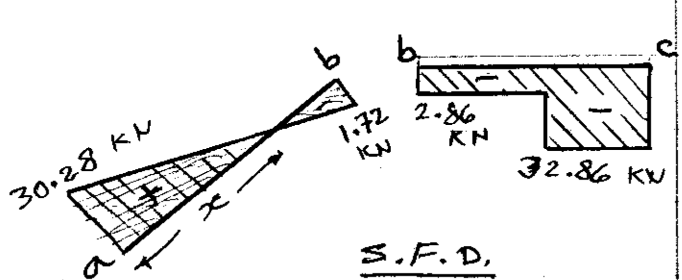
Axial force at (b):
$= 26.29 - 4.8(5) = 2.29$ KN

For beam (b c): $\Sigma M_b = 0 \Rightarrow M_b + \underbrace{30(2)}_{=60} - \underbrace{32.86(4)}_{=131.4} = 0$

$\therefore M_b = 71.4$ KN-m



A.F.D



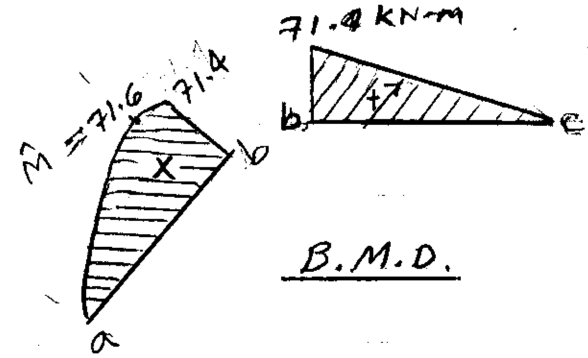
S.F.D.

$\frac{30.28}{1.72} = \frac{x}{5-x}$ ($\frac{30.28}{1.72} = 17.6$)

$x = 88 - 17.6x$

$x = \frac{88}{18.6} = 4.73$ m

$\hat{M} = 30.28 \times (4.73) = 71.6$ KN-m



B.M.D.

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المرحلة الثالثة | 3rd. Stage

Theory of Structures

2022 - 2023

Sheet-4

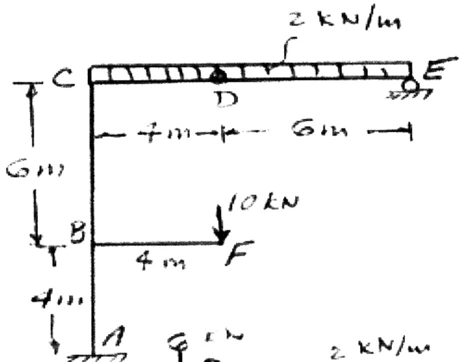
انشاءات

Rigid Frames

Dr. Maloof Mahmood

Ex. 3

For the frame shown below, draw axial force, shear force and bending moment diagrams with internal hinge at D .



Make a cut at hinge D :

For the whole frame $\Sigma F_x = 0 \Rightarrow \boxed{H_A = 0}$

$V_E = V_D = 6$ KN

$V_A = 2 \times 4 + 6 + 10 = 24$ KN ↑

Make a cut at C :

$V_{C_1} = 24 - 10 = 14$ KN ↓

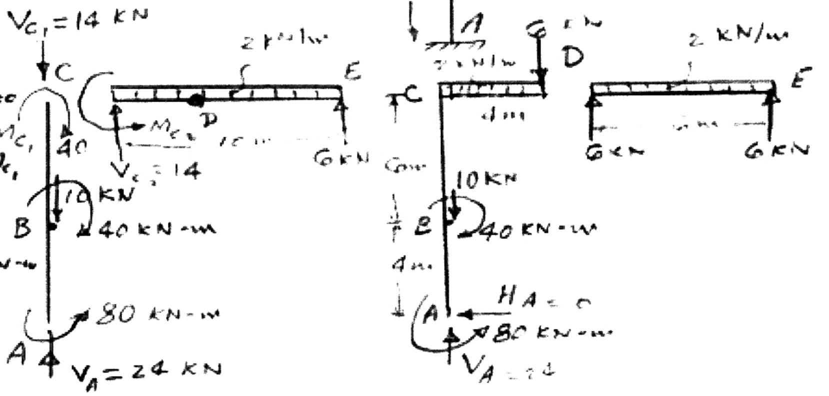
$\Sigma M_{C_2} = 0$

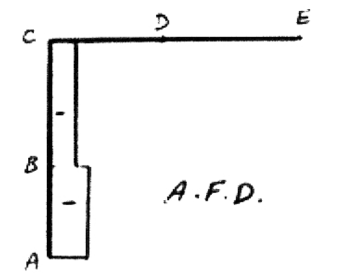
$-M_{C_2} - 6 \times 10 + 20(5) = 0$

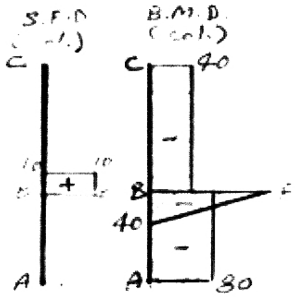
$\therefore M_{C_2} = 40$ kN·m $= M_{C_1}$

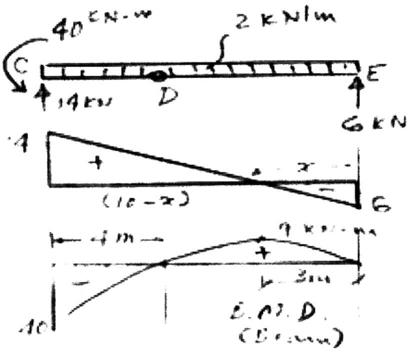
For the Col. ABC :

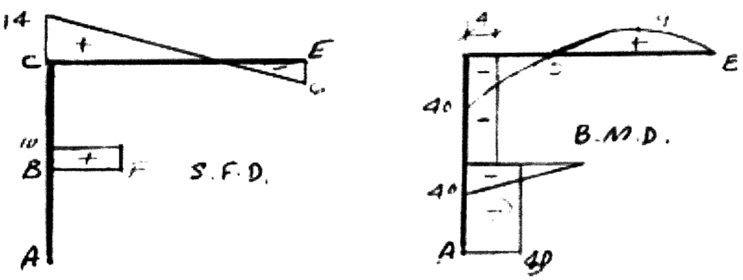
$M_A = 40 + 40 = 80$ kN·m











$\frac{x}{10 - x} = \frac{6}{14}$

$14x = 60 - 6x$

$20x = 60$

$x = 3$ m

Statically Determinate Rigid Frames

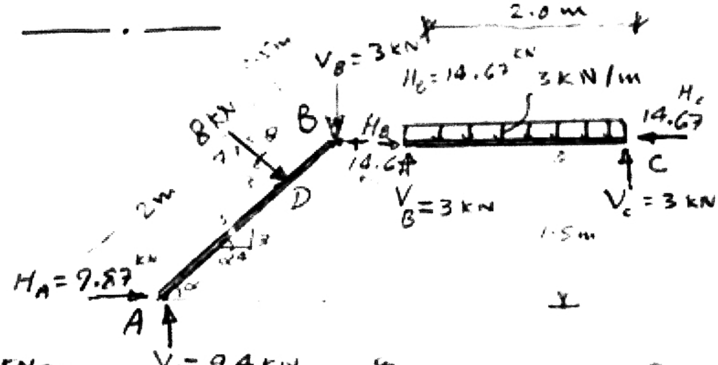
A rigid frame is defined as a structure composed of a number of members connected together by joints some or all are rigid, i.e., capable of resisting force and moment while pin-connected joints cannot resist moment.

To analyze a statically determinate rigid frame, we start by finding the reaction components from statical equations for the entire structure. Once this is done, we'll be able to determine the shear, moment and axial force at any cross section of the frame by taking a free body cut through that section and by using the equ^{m}. eqs.

Based on the above, we can plot the A.F.D., S.F.D. and B.M.D. for the rigid frame.

Ex. 1 (cont'd.)

For the previous problem, draw the A.F.D., S.F.D. and B.M.D of the frame ABC.



Beam AB

Axial Force Diagram (A.F.D.)

at end A

$F_{AB} = H_A \cos\alpha + V_A \sin\alpha$

$= 9.87 \times \frac{4}{5} + 9.4 \times \frac{3}{5}$

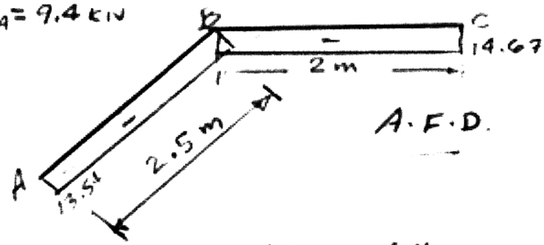
$= 7.9 + 5.64 = \underline{13.54 \text{ kN}}$

Check at end B:

$F_{BA} = H_B \sin\theta + V_B \cos\theta$

$= 14.67 \times \frac{4}{5} + 3 \times \frac{3}{5}$

$= 11.74 + 1.80 = \underline{13.54 \text{ kN}}$

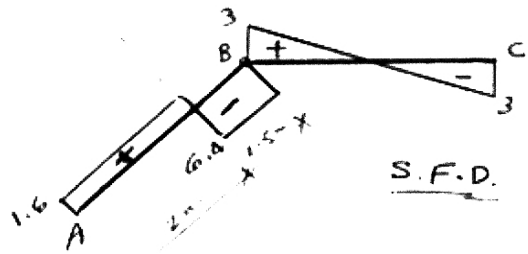


Beam BC : $F_{BC} = 14.67$ kN

S.F.D.

$S_A = V_A \cos\alpha - H_A \sin\alpha$

$= 9.4 \times \frac{4}{5} - 9.87 \times \frac{3}{5} = 1.60$ kN



open the frame: sagging +ve (B.M.)

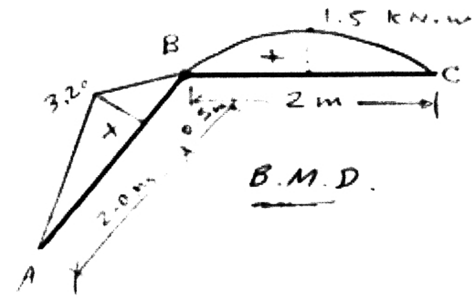
B.M.D.

$M_D = 1.6 \times 2 = 3.20$ kN-m

$M_B = 3.20 - 6.4 \times 0.5 = \underline{0}$

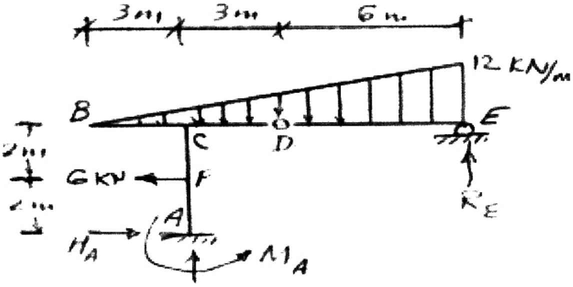
For beam BC:

$M = \frac{1}{2} \times 3 \times 1 = \underline{1.5}$ kN-m



Ex. 2

Analyze the rigid frame shown below and draw the A.F.D, S.F.D. & B.M.D for the frame with the given loadings and hinge at (D).



$b = 2,\ r = 4,\ j = 3,\ c = 1$

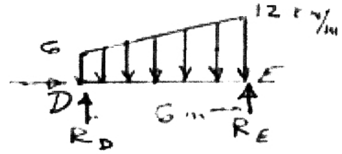
$3b + r = 6 + 4 = 10$

$3j + c = 9 + 1 = 10$ $\Rightarrow$ stable & det.

- For the whole frame:

$\Sigma F_x = 0 \Rightarrow \boxed{H_A = 6\ KN}$

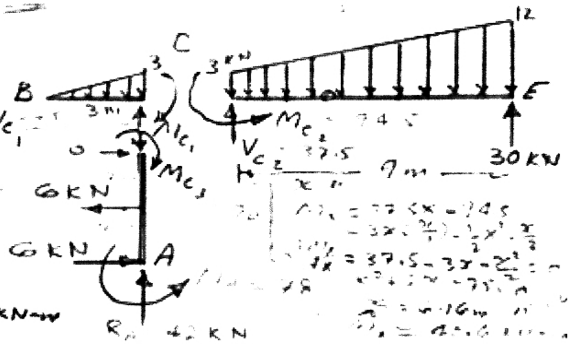
- Make a cut at (D): For the right portion:



$\Sigma M_D = 0 \Rightarrow -R_E(6) + (6\times6\times3) + \frac{1}{2}\times6\times6\left(\frac{2}{3}\times6\right)$

$-6R_E + 108 + 72 \Rightarrow R_E = \frac{180}{6} = 30\ KN$

Make a cut at (C):



$V_{c_1} = \frac{1}{2}\times3\times3 = 4.5\ KN$

$V_{c_2} = \frac{1}{2}(3+12)(9) - 30 = 37.5\ KN$

$V_{c_3} = 4.5 + 37.5 = 42\ KN = R_A\uparrow$

For left part (BC): $\Sigma M_c = 0$:

$M_{c_1} - \frac{1}{2}\times3\times3\left(\frac{1}{3}\times3\right) = 0 \Rightarrow M_{c_1} = 4.5\ KN\text{-}m$

For beam CE: $\Sigma M_c = 0 \Rightarrow (3\times9\times4.5) + \left(\frac{1}{2}\times9\times9\right)\left(\frac{2}{3}\times9\right) - 30(9) - M_{c_2} = 0$

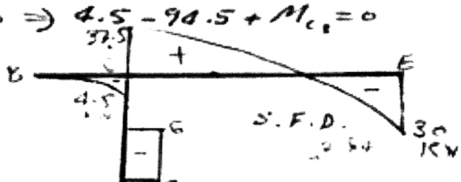
$121.5 + 243 - 270 - M_{c_2} = 0 \Rightarrow M_{c_2} = 94.5\ KN\text{-}m$

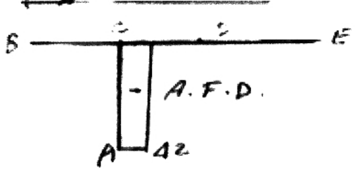
Σ Moment at jt. $= 0 \Rightarrow M_{c_1} + M_{c_2} + M_{c_3} = 0 \Rightarrow 4.5 - 94.5 + M_{c_3} = 0$

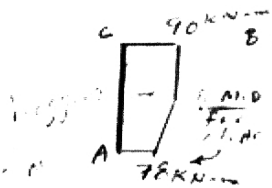
$\therefore M_{c_3} = 90\ KN\text{-}m$

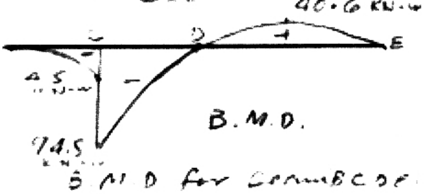
For col. (CA): $\Sigma M_A = 0 \Rightarrow 90 - 6(2) + M_A = 0$

$\therefore M_A = 78\ KN\text{-}m$









Al-Mansour University College

قسم الهندسة المدنية

Civil Eng. Dept.

المرحلة الثالثة

3rd. Stage

Theory of Structures

2022 - 2023

Sheet-5

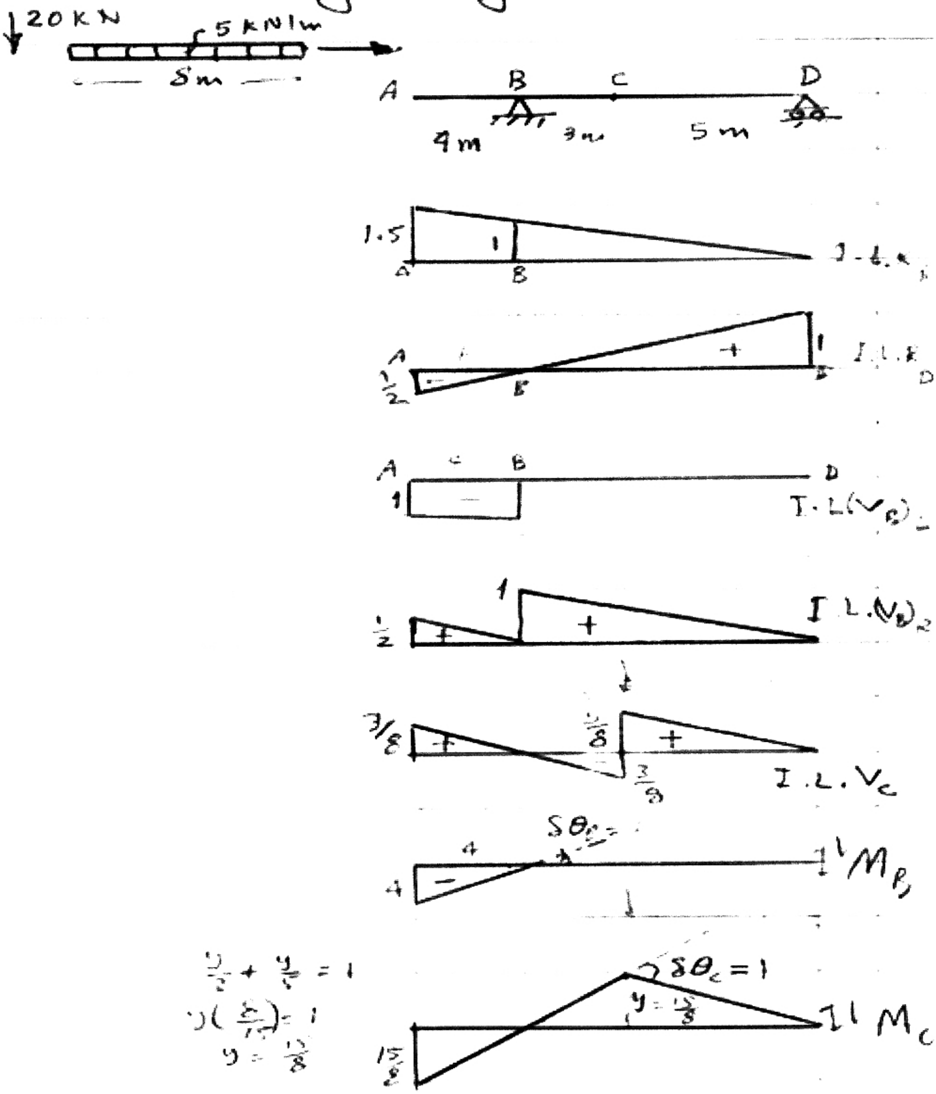
انشاءات

Influence Lines of Statically Determinate Beams

Dr. Maloof Mahmood

H.W. Prob.(4)

By virtual work method, construct the I.L. for R_B, R_D, $(V_B)_L$, $(V_B)_R$, V_C, M_B & M_C. Compute the max. value for each of them due to a u.d.l. of 5 kN/m (length 8m) and a concentrated load of 20kN moving along the beam.



$\frac{y}{3} + \frac{y}{5} = 1$

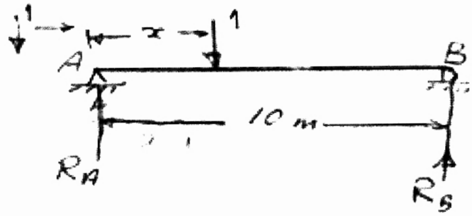
$y\left(\frac{8}{15}\right) = 1$

$y = \frac{15}{8}$

Influence Lines for Statically Determinate Structures

Definition of Influence Line (I.L.)

An influence line is a curve the ordinate (y-value) of which gives the value of (reaction, shear, moment etc.) in a member such as a beam or a truss when a unit load is at the ordinate.



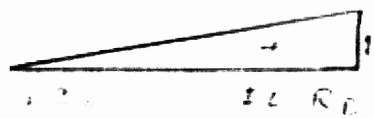
Consider a S.S. beam AB with a unit load moving to the right across the beam. Assume the unit load is placed at distance (x) from support (A), then $\Sigma M_B = 0 \Rightarrow R_A = \frac{1\,(10-x)}{10} = \left(1 - \frac{x}{10}\right)$

$\therefore R_B = \frac{x}{10}$

For $R_A = 1 - \frac{x}{10}$

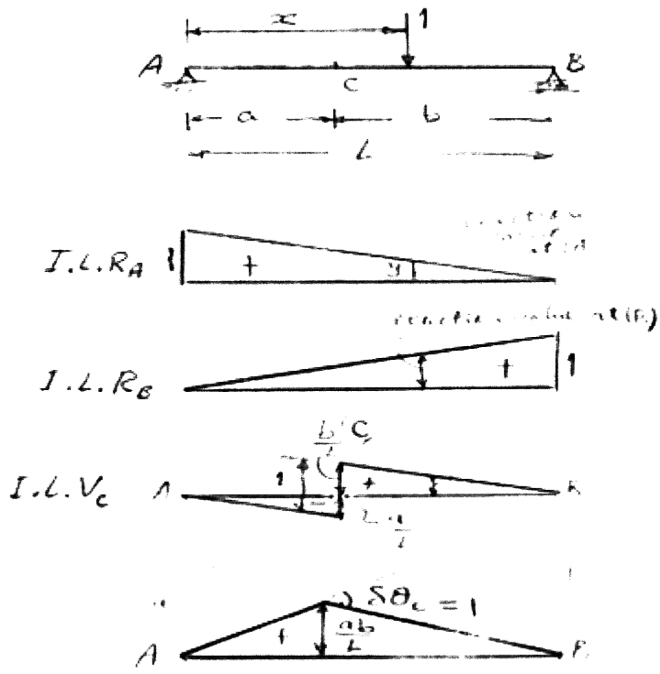
$R_A]_{x=0} = 1$, $R_A]_{x=10} = 0$

$R_B]_{x=0} = 0$, $R_B]_{x=10} = 1$



Influence Lines by Virtual Work

1. To obtain an I.L. for the reaction of a beam, remove the support and make a unit displacement at that support. The deflected beam is the I.L. for the reaction.
2. To obtain an I.L. for the shear at a section of a beam, cut the section and induce a unit relative displacement sliding between the portion to the left of the section and the portion to the right. The deflected beam is the I.L. for the shear at that section.
3. To obtain the I.L. for the moment at a section of any beam, cut the section and induce a unit rotation between the portions to the left and to the right of the section. The deflected beam is the I.L. for the moment at that section.

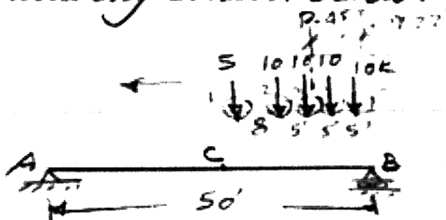


Absolute Maximum Bending Moment

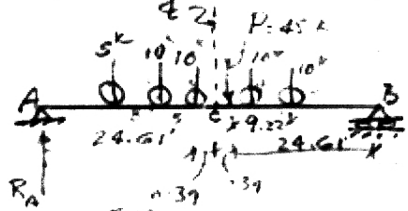
Under a single concen. load or a uniform load the max B.M. in a simple beam occurs at midspan section. However, when a simple beam is subjected to a group of concen. loads, the max. B.M. does not usually occur at midspan.

Ex.

Find the absolute max. B.M. for the beam & loading shown below.



This is same beam & loading of last problem. To find absolute max. B.M., place the loads on the beam in which the centreline of the span bisects wheel (3) and the resultant.



The absolute max B.M. will occur under wheel (3).

$\Sigma M_B = 0$

$R_A(50) = 45(24.61)$

$R_A = \frac{45(24.61)}{50}$

$$M_{③} = \frac{45(24.61)}{50}(24.61) - 10(5') - 5(13')$$

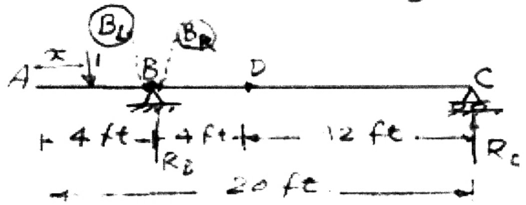
$$(\hat{M}_3)_{Abs.} = 545.09 - 50 - 65 = \underline{430.09} \text{ K.ft} > 429.95 \text{ K.ft}$$

(last prob.)

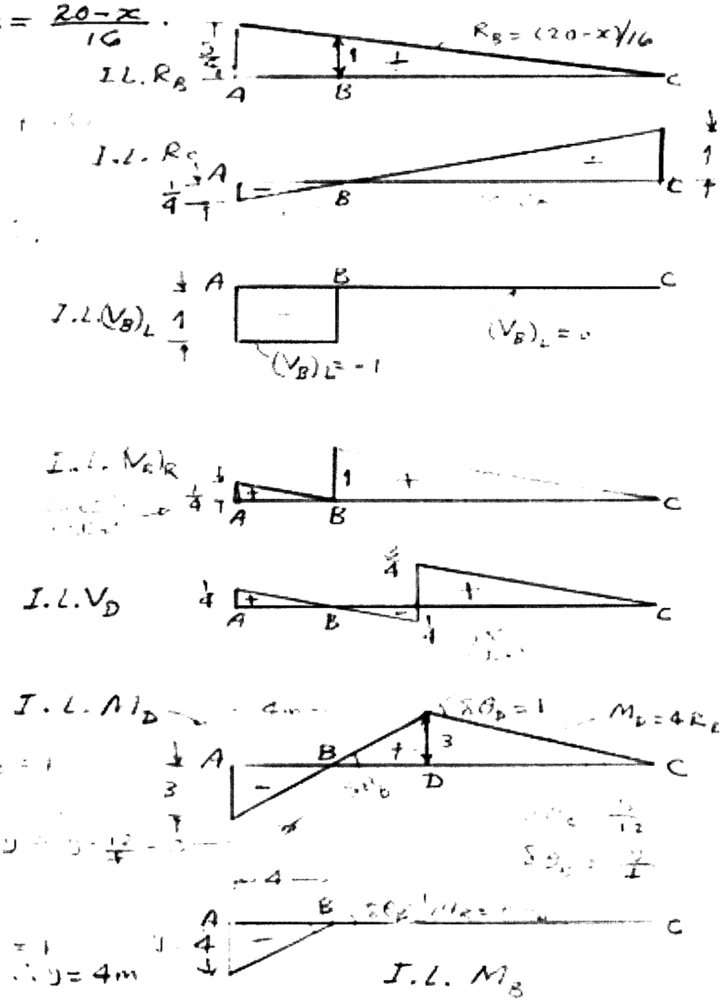
Ex.

The simple beam with an overhang is shown below. Draw the I.L. for R_B, R_C, $(V_B)_L$, $(V_B)_R$, V_D, M_D and M_B by (virtual work) method, where $(V_B)_L$ = I.L. for the shear at the sec. just to the left of B, $(V_B)_R$ = I.L. for the shear at the section just to the right of B.

To construct the I.L. R_B, we place a unit load distance x from end A.



Apply $\Sigma M_C = 0 \Rightarrow R_B = \frac{20-x}{16}$.

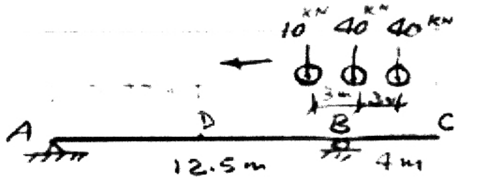


$\delta\theta_D = \frac{y}{4} = 1$

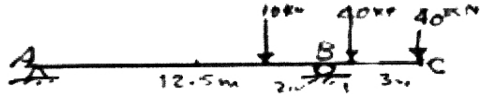
$\therefore y = 4m$

Ex.

For the beam and loads shown below, find the absolute max. moment & also the max moment.



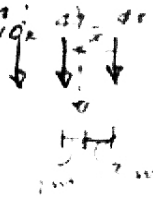
By inspection, the max moment at Cantilever occurs when the loads are as shown →

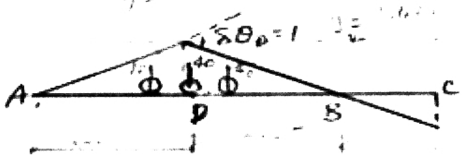


$\hat{M}_B = 40(1) + 40(4) = \underline{200}$ kN-m

Move the wheel loads between A & B

I.L. of M_D of beam as shown:





Res. of loads = 90 kN

$90\bar{x} = 10(6) + 40(3)$

$\bar{x} = \frac{180}{90} = 2$ m

place mid load at ℄ beam:

$\frac{G_1}{a} = \frac{10}{6.25} = 1.6 < \frac{G_2}{b} = \frac{80}{6.25}$

$\frac{G_1}{a} = \frac{50}{6.25} = 8 > \frac{40}{6.25} = 6.4$

∴ Placing mid wheel at D gives max. M_D : from I.L. M_D

$\Sigma M_B = 0$

$R_A(12.5) - 10(9.25) - 40(6.25) - 40(3.25) = 0$

$12.5R_A - 92.5 - 250 - 130 = 0$

$R_A = 37.8$ kN

$M_D = 37.8(6.25) - \ldots$

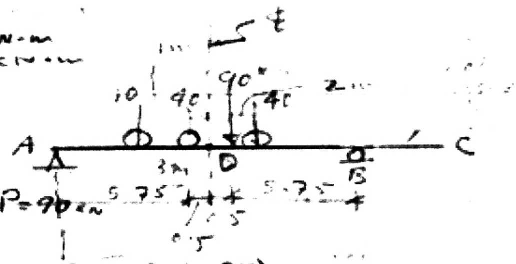
$= 206.25$ kN-m

$\hat{M}_D = 40(3.125) + 40(1.625) + 10(1.625)$

$= 125 + 65 + 16.25 = 206.25$ kN-m > 200 kN-m

To find Absolute B.M.,

place ℄ of beam D at mid distance between mid load & Res. P = 90 kN



$R_A = \frac{90(5.75)}{12.5}$

Max absolute moment occurs under wheel load ②:

$(\hat{M}_{②})_{Abs.} = \frac{90(5.75)}{12.5}(5.75) - 10(3.0) = 238.05 - 30$

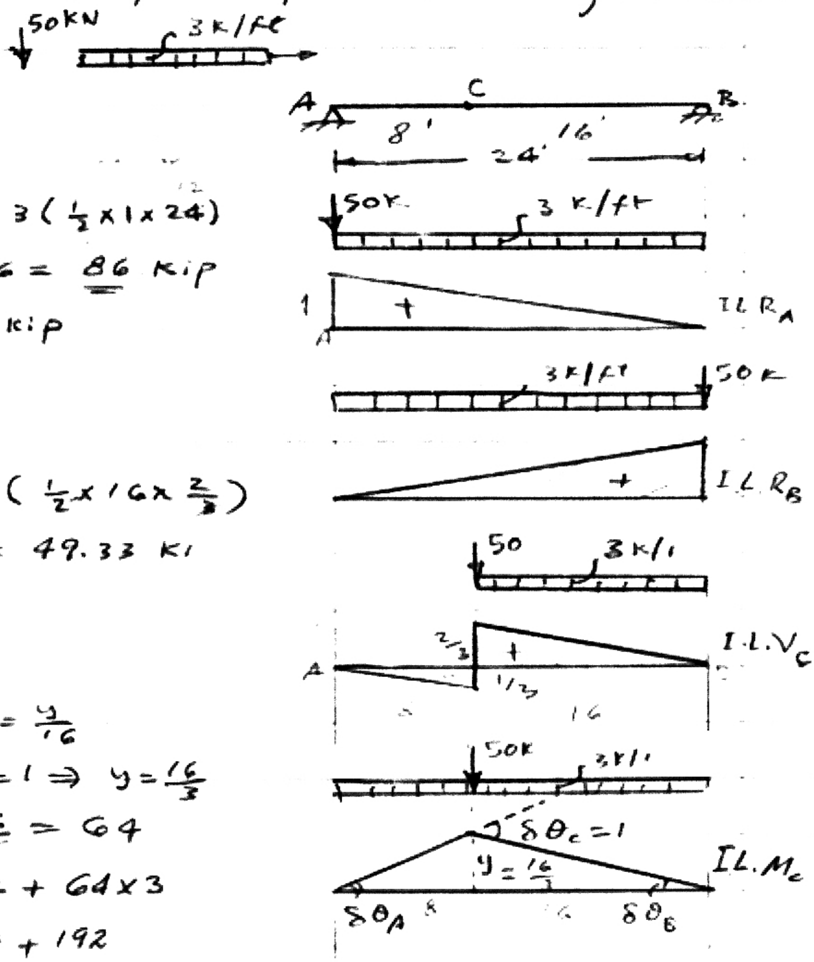
$= \underline{208.05}$ kN-m > 206.25

Abs. max. moment on beam.

Ex.1

Given a simple beam 24 ft long. Construct the I.L. for the Reactions at A & B (R_A & R_B) by virtual work, and for the shear and moment at point (C) 8 ft from A, shown below (1).

2) Find the max value for R_A, R_B, V_c and M_c when a moving uniform load of 3 kN/m (length > 24 ft) and a movable concentrated load of 50 kips travel along the beam.



I.L. R_A & R_B

$$(R_A)_{max} = 50 \times 1 + 3\left(\tfrac{1}{2} \times 1 \times 24\right)$$
$$= 50 + 36 = \underline{\underline{86}} \text{ kip}$$
$$(R_B)_{max} = 86 \text{ kip}$$

I.L. V_c

$$\hat{V}_c = 50 \times \tfrac{2}{3} + 3\left(\tfrac{1}{2} \times 16 \times \tfrac{2}{3}\right)$$
$$= \tfrac{100}{3} + 16 = 49.33 \text{ Ki}$$

I.L. M_c :

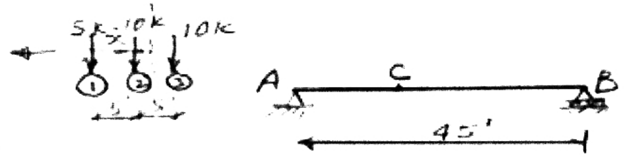
$$\delta\theta_A = \frac{y}{8}, \quad \delta\theta_B = \frac{y}{16}$$
$$\delta\theta = \frac{y}{8} + \frac{y}{16} = \frac{3y}{16} = 1 \Rightarrow y = \frac{16}{3}$$
$$A = \tfrac{1}{2} \times 24 \times \tfrac{16}{3} = 64$$
$$\therefore \hat{M}_c = 50 \times \tfrac{16}{3} + 64 \times 3$$
$$= 266.67 + 192$$
$$= 458.67 \text{ KN-m}$$

Ex. 2

A simple beam 45 ft long carries moving loads of 5 k, 10 k and 10 k spaced 5 ft apart. Calculate:

(a) the max left reaction and max right reaction.

(b) the max Shear & B.M. at a section 15 ft from left end.

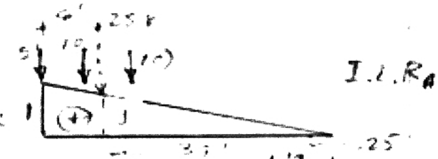


Resultant $= 25^k$

Let $\bar{x}$ = dist. of res. from left load and $\Sigma M_{①}$

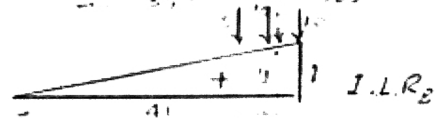
$25\bar{x} = 10(5) + 10(10)$

$\bar{x} = \frac{150}{25} = 6$ ft



$y = \frac{39}{45} \Rightarrow \hat{R}_A = 25\left(\frac{39}{45}\right) = 21.67^k$

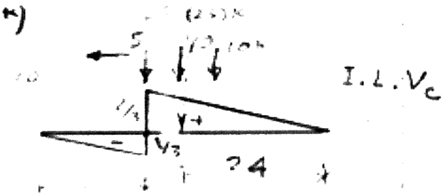
For right reaction:



$y = \frac{41}{45} \Rightarrow \hat{R}_B = 25\left(\frac{41}{45}\right) = 22.78^k$

I.L. V_c: $\frac{Pd}{L} \geq P_1 \Rightarrow \frac{25 \times 5'}{45} = 2.78 < P_1 (=5^k)$

$\therefore$ ok., otherwise move wheels to left until u get $\frac{Pd}{L} < P_2$ or P_3



$(V_c)_{max} = 25 \times \left(\frac{2}{3} \times \frac{24}{30}\right) = \underline{13.33}$ KN

I.L. M_c

place mid-wheel load at c:

Mid wheel just to the right.

$G_1 = 5 \qquad G_2 = 20$

$\frac{G_1}{a} = \frac{5}{15} = \frac{1}{3} < \frac{G_2}{b} = \frac{20}{30} = \frac{2}{3}$

Mid wheel just to the left:

$\frac{G_1}{a} = \frac{15}{15} = 1 > \frac{G_2}{b} = \frac{10}{30} = \frac{1}{3}$

$\therefore$ o k Max moment occurs when wheel (2) is at C.

$\Sigma M_B = 0$

$R_A(45) = 25 \times 29 \Rightarrow R_A = \frac{25 \times 29}{45}$

$\therefore M_c = \frac{25}{45} \times 29(15) - 5(5)$

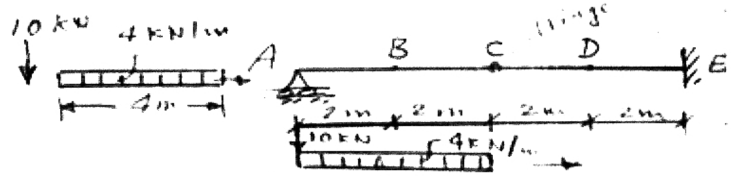
$\hat{M}_c = 216.67$ k.ft

Ex. 3

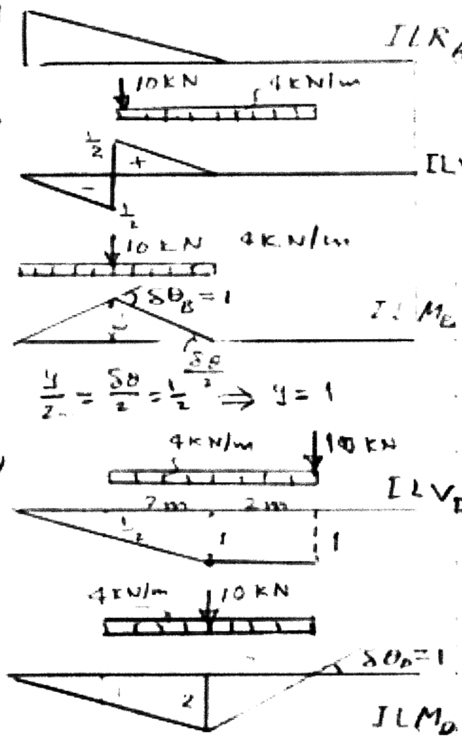
The compound beam ABCDE shown below is subject to a moving u.d.l of 4 KN/m (Length = 4 m) and a single load of 10 kN crossing the beam to the right.

(a) Draw I.L. for R_A, V_B, M_B, V_D & M_D.

(b) Calculate the max values of R_A, V_B, M_B, V_D & M_D.



(a) IL's are drawn as shown.



(b) $R_A = 10(1) + \frac{1}{2}(4)(1)(4)$

$= 10 + 8 = \underline{18}$ kN the max reaction at (A).

Cut the beam at B as shown:

$V_B = 10(\frac{1}{2}) + \frac{1}{2}(\frac{1}{2} \times 2)(4)$

$= 5 + 2 = 7$ KN the max shear at (B).

$M_B = 10(1) + \frac{1}{2} \times 4 \times 1\ (4^{kN/m})$

$= 10 + 8 = 18$ KN-m the max moment at (B)

$V_D = 4\left[\frac{1}{2}(\frac{1}{2}+1)(2) + 1 \times 2\right] + 10(1)$

$= 4 \times 3.5 + 10 = 24$ kN the max shear at (D)

$M_D = 4\left[\frac{1}{2}(1+2)2 + \frac{1}{2} \times 2 \times 2\right] + 10(2)$

$= 20 + 20 = 40$ KN max moment at D

Moving Concentrated Loads
Criteria for Maxima

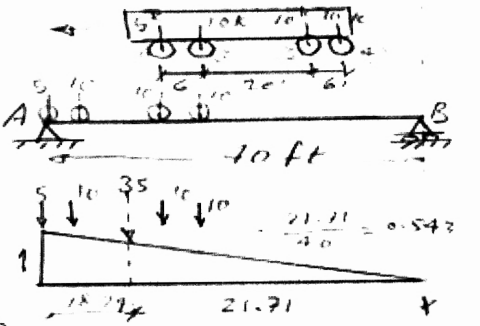
1. Criterion for Max. Value of Reactions:

Procedure:

Draw the I.L. of the reaction. If several concen. loads are moving across the beam, then the support reaction will be maximum when one wheel load is placed over the support.

Ex. 1

Find the max. reaction that will occur in a s.s. girder of 40 ft long under the car wheel loads shown.



place 1st wheel on support (A):

Res. of 4-wheel = 35 k

$\Sigma M_{①} = 0 \Rightarrow$

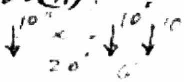
$35\bar{x} = 10(6) + 10(26) + 10(32)$

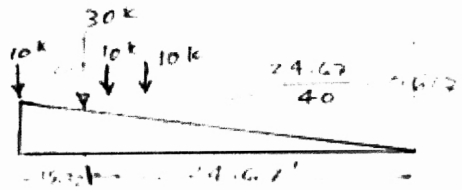
$\bar{x} = \frac{640}{35} = 18.29'$

$R_A(40) = 35(21.71) \Rightarrow R_A = 35\left(\frac{21.71}{40}\right)$

$R_A = 19$ kip

Move wheels to the left and place 2nd wheel on (A):





$30\bar{x} = 10(20) + 10(26)$

$\bar{x} = \frac{460}{30} = 15.33$

$R_A = 30 \times \frac{24.67}{40} = 18.5$ kip < 19 kip

$\therefore$ case (1) yields max. reaction on support.

2. Criterion for Max Shear & Moments

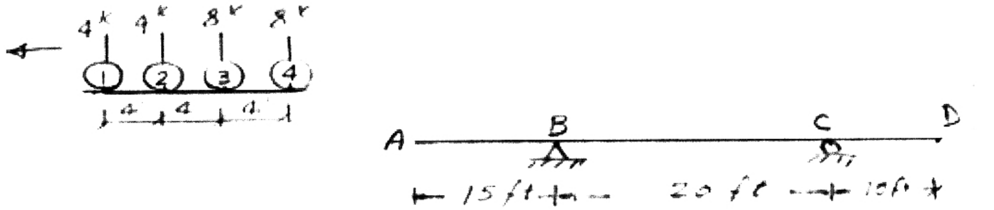
See Ex. 2 to explain the 2-cases.

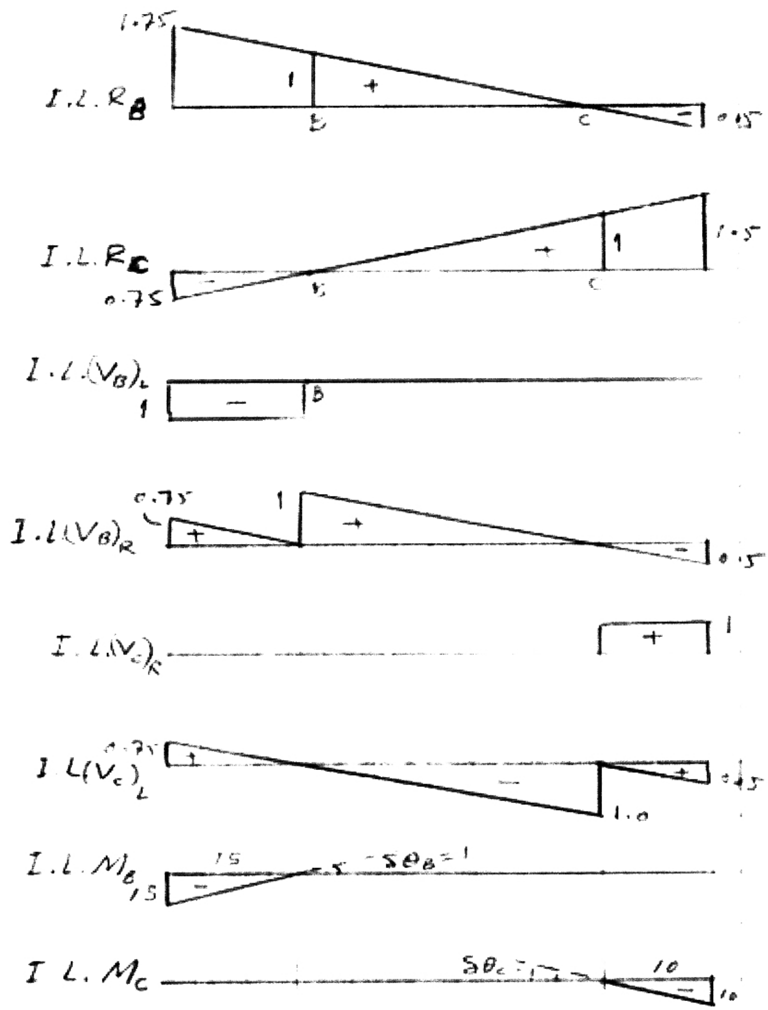
4-wheel loads of a vehicle are travelling along the girder beam shown below. Determine:

(a) The max reaction at the supports B and C.

(b) The max +ve and max -ve shear force on the girder beam

(c) " " " " " " Moment for the girder beam



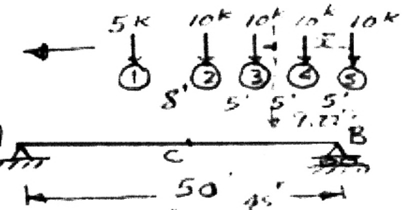


Ex.

The group of wheels shown below are assumed passing over a simple beam of 50 ft span.

(1) Find the max. Shear (+) and minimum shear (-) at the mid span section C.

(2) Find the max. B.M. at mid span section (C).



Determine location of resultant

$45\bar{x} = 5(23) + 10(15) + 10(10) + 10(5)$

$\bar{x} = \frac{415}{45} = 9.22'$

(1) Draw I.L. for S.F. at C $\quad \frac{Pd}{L} \gtrless P_1$

Place 1st wheel at C : $P = 45^k$

$\frac{Pd}{L} = \frac{45 \times 8'}{50} = 7.2 > P_1 (= 5^k)$ N.G.

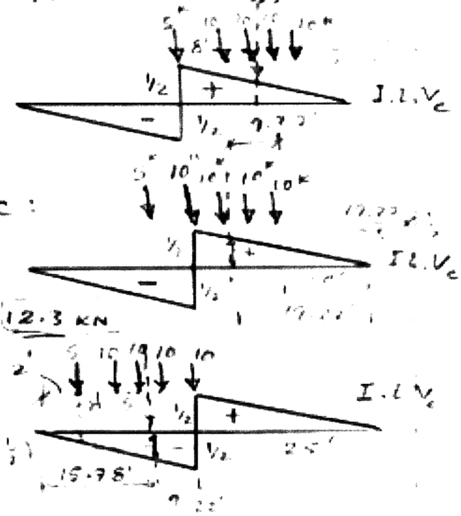
$\therefore$ Move wheels to left & place P_2 at C :

$\frac{Pd}{L} = \frac{45 \times 5'}{50} = 4.5 < 10 \quad \therefore$ ok

$+(V_c)_{max} = \left(\frac{19.22}{25} \times \frac{1}{2}\right)45 - 5 = 17.3 - 5 = 12.3$ KN

For $\min(V_c)(-) = -45\left(\frac{15.78}{25} \times \frac{1}{2}\right)$

$(V_c)_{min}(-) = -14.20$ KN



(2) Draw I.L. for M_c

Place mid wheel at C (mid-span) 25' from A

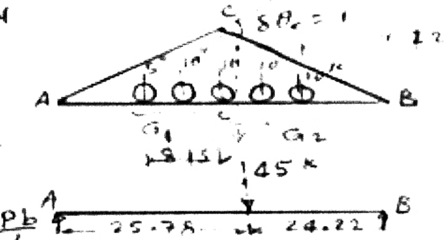
Check:

$\frac{G_1}{a} = \frac{15}{25} = \frac{3}{5} \quad , \quad \frac{G_2}{b} = \frac{30}{25} = \frac{6}{5}$

(1) $\quad \frac{G_1}{a} < \frac{G_2}{b} \quad$ ∟ just to left

$\frac{G_1}{a} = \frac{25}{25} = 1 \quad , \quad \frac{G_2}{b} = \frac{20}{25} = \frac{4}{5}$

(2) $\quad \frac{G_1}{a} > \frac{G_2}{b} \quad \therefore$ ok Mid load at C gives $(M_c)_{max}$



$R_A = \frac{Pb}{L} = \frac{45 \times 24.22}{50}$ KN

$M_c = \left(\frac{45 \times 24.22}{50}\right)(25) - 5(8+5) - 10(5)$

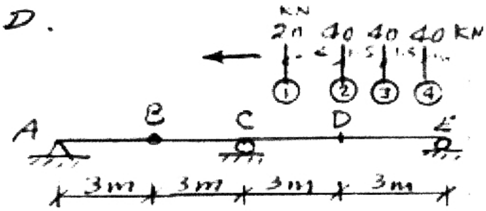
$= 544.95 - 65 - 50 = 429.95$ KN-m

Ex.

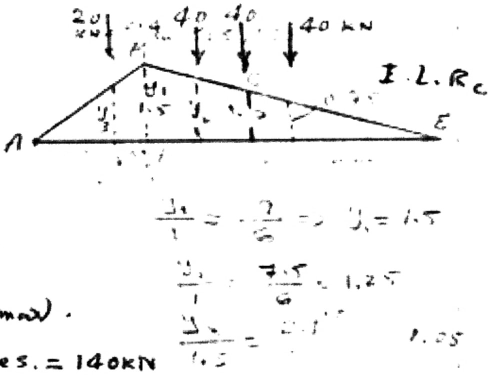
For the compound beam and wheel loads shown below, find:

(a) The max. reaction at support C

(b) The max. moment at point D.



Note: For compound beams (i.e. not s.s. beam where I.L. for reaction is right angle triangle) we don't need to find location of res. of loads but follow the method below.



Draw I.L. by V.W. method for R_c

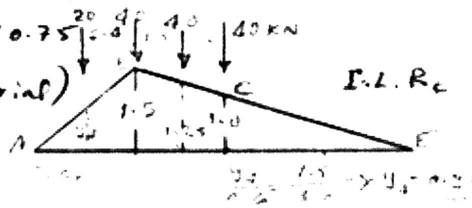
Remove support at (C), displace by 1 unit

Always place one wheel at supp. for R_{max}.

place 3rd wheel at (C) as 1st trial. Res. = 140 KN

$(R_c)_{max} = 20 \times 1.05 + 40 \times 1.25 + 40 \times 1.0 + 40 \times 0.75$

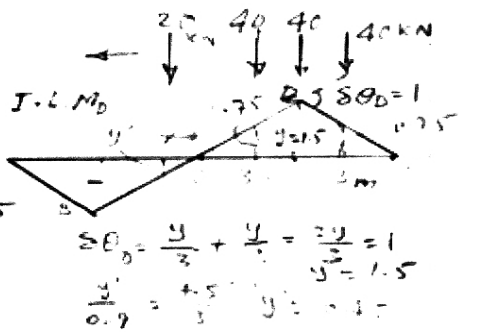
$= 21 + 50 + 40 + 30 = \underline{141}$ KN (1st trial)



2nd trial place, place 4th wheel at (C):

$(R_c)_{max} = 20 \times 0.3 + 40 \times (1.5 + 1.25 + 1.0)$

$(R_c)_{max} = 6 + 150 = \underline{156}$ KN > 141 KN $\leftarrow R_{c\,max}$



I.L. M_D:

1st trial, place wheel (3) at (C):

$(M_D)_{max} = 40(0.75 + 1.5 + 0.75) - 20 \times 0.45$

$= 120 - 9 = \underline{111}$ KN·m

By inspection, this gives the max moment at (D).