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المرحلة الثالثة

Al-Mansour University College

Civil Engineering Department
3rd. Stage

Reinforced Concrete Design Lectures

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Lec.2

خرسانة مسلحة

Flexurul Analysis and Design of Brams

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For f'_c > 30 MPa.......⇒ β_1 will be reduced by 0.05 for every 7 MPa increase in f'_c but not less than 0.65.

$\beta_1 = 0.85 - 0.05\,(f'_c - 30)/7$ ....(4-17)

$0.325 \leq \beta \leq 0.425$

4.4 Beams Classifications based on Reinforcement Amount

The theory of material strength depends on the fact that the failure of beams occurs either by yielding of steel, reaching the yield strength, f_y, and this type of failure is called ***Tension Failure***. Or crushing of concrete before yielding of steel and this is called ***Compression Failure***. It noted from experimental work that the crushing of concrete usually occurs at strain, ε_u, ranging between 0.003 and 0.004 and a value of 0.003 has been adopted as conservative value for crushing of concrete.

In case of reaching concrete to its ultimate strain (ε_u = 0.003) at the same time of yielding of steel reinforcement, this kind of failure called ***Balanced failure***. The type of failure depends mainly on the reinforcement amount if the section size and material strength is limited.

4.4.1 Balanced Reinforced Concrete Beams

As mentioned before, this kind of failure occurs when the concrete reaches its crushing strain (ε_u = 0.003) at the same time the steel reinforcement reaches yielding stress ($f_s = f_y$). The balanced steel ratio (ρ_b) equation can be derived from equilibrium of forces as:

$C = T, \Rightarrow A_s f_y = 0.85\, f'_c\, ab = 0.85\, f'_c\, \beta_1 cb \Rightarrow \rho bd\, f_y = 0.85\, f'_c\, \beta_1 cb$ ....(4-18)

$$\therefore \rho = \frac{0.85\beta_1 c\, f'_c}{f_y\, d}$$ ....(4-19)

From strain distribution and taking (ε_u =0.003) as a condition of concrete crushing;

$\frac{c_b}{d} = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} \Rightarrow c_b = \frac{\varepsilon_u}{\varepsilon_u + \frac{f_y}{E_s}} d$, using E_s = 200, 000 MPa.

Flexural Analysis and Design of Beams

4.1 Introduction

Design and analysis of reinforced concrete beams can be conducted through two main methods:

1- Working Stress Method (as discussed in the previous chapter).

2- Strength (Ultimate) Method.

4.2 Strength (Ultimate) Method

In this method, design and analysis of structural members will be based on the ultimate loads that the structure can carry at failure. That's mean, both concrete and steel stresses reach its maximum values. The ultimate loads can be calculated through multiplying the expected service load along the structure life by certain factors called "*safety factors*".

The section analysis includes evaluating the section ultimate strength when the stresses reach its maximum values. While, the section design includes finding some or all section size and reinforcement amount when the structure is subjected to ultimate loads.

The stresses distribution at failure is nonlinear. While the strains distribution will remain linear. There is no accurate analysis to predict the stress distribution shape and the shape proposed by the ACI-Code researchers has been adopted. The proposed stress distribution shape is parabolic and depends on a lot of experimental tests.

4.2.5 Strength Reduction Factors

The design strength is normally calculated through multiplying the nominal strength by factor less than 1.0 called strength reduction factor and denoted by "ϕ". The value of ϕ varies based on the type of strength required (ACI-Code 9.3.1).

1- Tension –controlled sections....... $\phi = 0.9$
2- Compression-controlled sections:
   - Members with spiral reinforcement.............$\phi = 0.75$
   - Other reinforced members.......................$\phi = 0.65$
3- Shear & Torsion.......................$\phi = 0.75$

Generally, the required strength should be less than or equal to the design strength.

$M_u \leq \phi M_n$, $P_u \leq \phi P_n$, $V_u \leq \phi V_n$.

4.3 Stress & Strain Distributions

Using the ultimate (strength) method, it is assumed that the strain distribution will be linear and this is approved experimentally even in failure state. While the stress distribution continue linear until approximately $0.5 f'_c$. After that it will be non-linear.

The actual stress distribution is unknown up to now because the concrete stress-strain curve depends on several factors like the concrete strength, loading speed,....etc. The ACI-code adopts parabolic shape for stress distribution as shown below.

The most important thing in the analysis and design is:

1- The total concrete compressive force, C.
2- The location of C with respect to the compressive fiber.

4.2.1 Basic Concepts

- **Service Loads**: The loads that actually applied to the structure like dead load, live loads, wind loads,....etc.

- **Factored (Ultimate) Loads**: The loads result from multiplying the service loads by the *safety factors*.

- **Design Forces & Moments**: The forces and moments result from the ultimate loads like the design bending moments (M_u), design shear force (V_u), design axial force (P_u) and design torsional moment (T_u) and also can be called *factored forces and moments*.

- **Required Strength**: It is the strength to be provided by the section and equal to the design forces and moments. So, the required strength for bending will be M_u and the same for other strengths.

- **Nominal Strength**: The section capacity calculated based on the strength of material theory and code requirements. It represents the maximum section strength and theoretically equal the strength at which the section fails and denoted by the nominal strength (M_n, V_n, P_n, T_n).

- **Design Strength**: The section capacity that has been adopted in both analysis and design and is equal to the nominal strength multiplied by a factor less than one called *strength reduction factor*. This is because the approximation in analysis and design and the Uncertainty of materials strengths.

4.2.4 Load Factors

The required strength U to be provided to resist both dead and live loads is (ACI-Code 9.2.1):

$U = 1.2\,D + 1.6\,L$ ....(4-3)

$U = 1.2\,D + 1.6\,L + 0.5\,(L_r \text{ or } S \text{ or } R)$....................(4-4)

$U = 1.2\,D + 1.6\,(L_r \text{ or } S \text{ or } R) + (1.0\,L \text{ or } 0.5\,W)$.......(4-5)

$U = 1.2\,D + 1.0\,W + 1.0\,L + 0.5\,(L_r \text{ or } S \text{ or } R)$.........(4-6)

$U = 1.2\,D + 1.0\,E + 1.0\,L + 0.2\,S$.......................(4-7)

$U = 0.9\,D + 1.0\,W$..(4-8)

$U = 0.9\,D + 1.0\,E$..(4-9)

Where :

U = Required Strength
D = Dead Load
L = Live Load
L_r = Roof Live Load
S = Snow Load
R = Rain Load
W = Wind Load
E = Earthquake Load

Notes:

1. The live load factor in Equs. (4-5) to (4-7) shall be permitted to be reduced to ***0.5*** except for garages, area occupied as places of public assembly, and all area where **L** is greater than 4.8 kN/m^2.
2. Where ***W*** is based on service level wind loads, ***1.6 W*** shall be used in place of ***1.0 W*** in Equs. (4-6) & (4-8).
3. Where E is based on service load forces, ***1.4 E*** shall be used in place of ***1.0 E*** in Equs. (4-7) & (4-9).

The total concrete compressive force can be expressed as:

$C = f_{av.}.b.c$ ....(4-10)

Where:

$f_{av.}$ = average concrete compressive stress.

b = section width.

c = neutral axis depth.

Equ. (4-10) can be written as:

$C = \alpha f'_c\, b\, c$..(4-11)

Where α is the ratio between average compressive stress and the compressive strength (f_{av}/f'_c)

The location of the Total concrete compressive force can be designated as βc, where β is the ratio between the compressive force depth and neutral axis depth. From experimental results, the values of α as follows:

For $f'_c \leq 30$ MPa,.............$\Rightarrow \alpha = 0.72$

For $f'_c > 30$ MPa.......$\Rightarrow$ α will be reduced by 0.04 for every 7 MPa increase in f'_c but not less than 0.56.

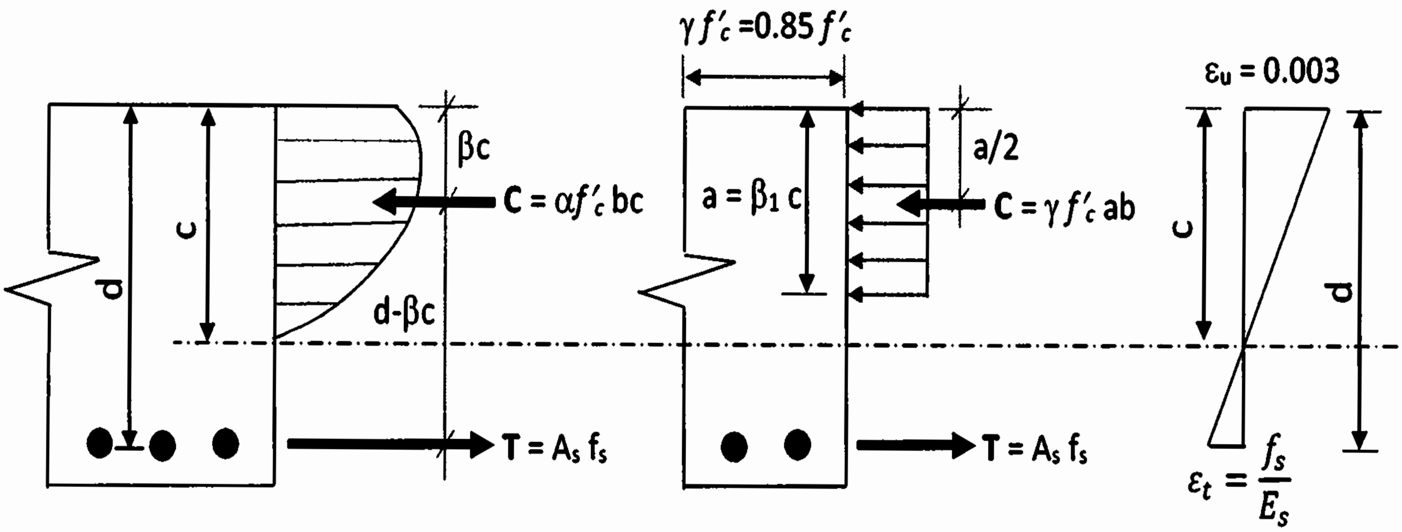
$$0.56 \leq \alpha \leq 0.72$$

Also, from experimental results, the values of β as follows:

For $f'_c \leq 30$ MPa,.............$\Rightarrow \beta = 0.425$

For $f'_c > 30$ MPa.......$\Rightarrow$ β will be reduced by 0.025 for every 7 MPa increase in f'_c but not less than 0.325.

$$0.325 \leq \beta \leq 0.425$$



Stress Distribution **Rectangular Stress Distribution** **Strain Distribution**

The analysis of some sections become complex using the above (parabolic) stress distribution. Therefore, a uniform shapes shall be adopted to approximate the stress distribution. The ACI-code proposed a rectangular distribution previously proposed by *Whitney* in condition that the compressive force and its location is the same.

If the depth of the Whitney stress block is a, then:

$C = \alpha f'_c \, b \, c = \gamma f'_c \, a \, b$ ....(4-12)

$a = \beta_1 c$..(4-13)

The values of both γ and β_1 can be determined in terms of α and β and since the location of the resultant force is the same for both cases;

$a / 2 = \beta c \quad \Rightarrow \therefore a = 2 \beta c \Rightarrow \beta_1 c = 2 \beta c \Rightarrow \boxed{\beta_1 = 2 \beta.}$....(4-15)

Substitute in Equ. (4-12) results:

$\alpha f'_c \, b \, c = \gamma f'_c \, \beta_1 \, c \, b \quad \Rightarrow \boxed{\gamma = \alpha / \beta_1 = 0.85 \text{(constant)}}$...........(4-16)

Also, from experimental results, the values of β_1 as follows:

For $f'_c \leq 30$ MPa,.............⇒ $\beta_1 = 0.85$

$$c_b = \frac{600}{600+f_y} d \text{ ....(4-20)}$$

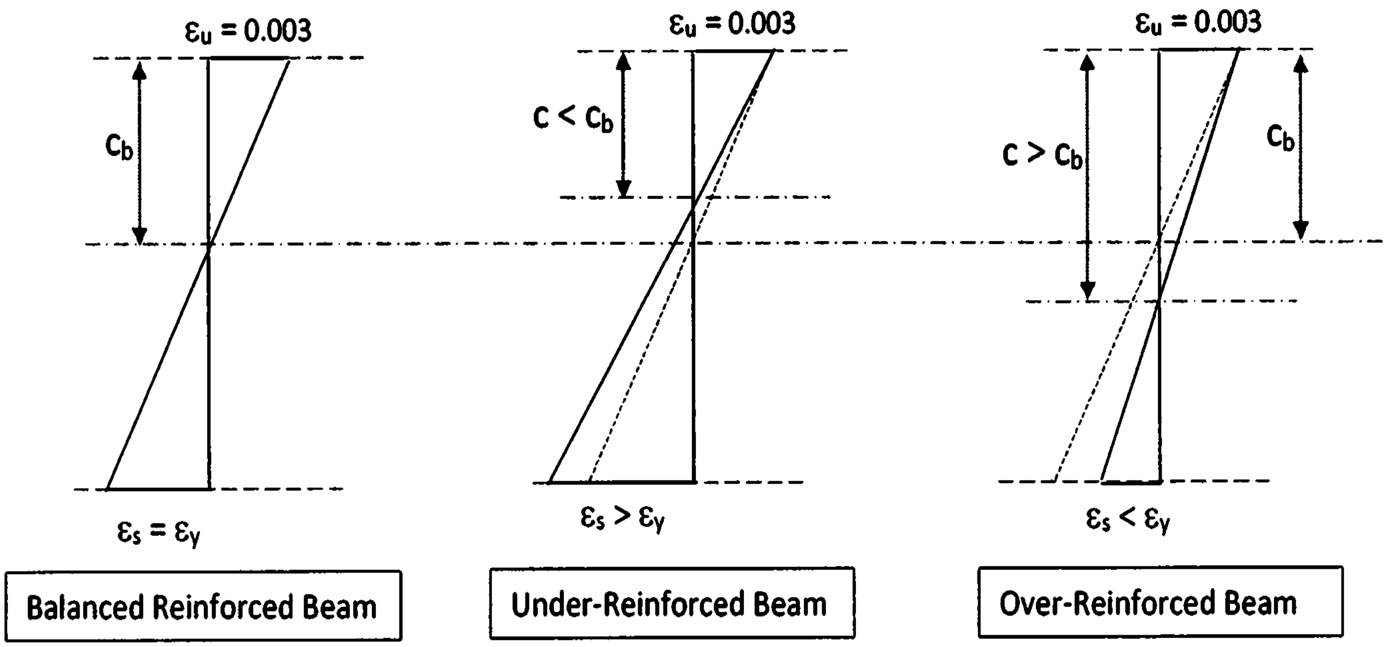
Substitute in Equ. (4-19) results:

$$\boxed{\rho_b = 0.85\beta_1 \frac{f'_c}{f_y} \cdot \frac{600}{600+f_y}} \text{ ....(4-21)}$$

<u>4.4.2 Under-Reinforced Concrete Beams</u>

Beams in which the steel reinforcement is less than those causing the balanced failure, i.e., $\rho = A_s / bd < \rho_b$, here......... $c < c_b$ and $\varepsilon_s > \varepsilon_y$

This failure occurs when the steel reinforcement reaches f_y first. After that the neutral axis lower and the concrete strains increases until reaches 0.003 followed by crushing of concrete as shown in the figure below. This kind of failure is called "***Tension Failure***".



Strain distributions for different types of reinforced beams.

In order to find the ultimate design strength, ϕM_n, the nominal design moment, M_n calculated using Equs. (4-22) to (4-24) is multiplied by the strength reduction factor, ϕ, calculated based on the value of steel strain, ε_t.

1- Over-Reinforced Beams:

When $\rho > \rho_b$, the beams will be over-reinforced and the steel stress, f_s, will equal unknown. The equilibrium equation will be:

$C = T, \Rightarrow 0.85 f'_c \, ab = A_s \, f_s, \Rightarrow 0.85 f'_c \, \beta_1 \, bc$ ....(4-25)

In the above equation (4-25), there are two unknowns (c and f_s), so, it is important to develop another equation relating c and f_s. From strain distribution:

$$\frac{\varepsilon_u}{c} = \frac{\varepsilon_s}{d-c}, \rightarrow \therefore \; \varepsilon_s = \frac{d-c}{c} \varepsilon_u$$

$f_s = \varepsilon_s E_s = \frac{d-c}{c} \varepsilon_u E_s = 600 \frac{d-c}{c}$, where $\varepsilon_u = 0.003$ and $E_s = 200{,}000$ MPa.

Substituting f_s in equilibrium equation (4-25) to get 2nd order equation in terms of c. Then solve for c and then find $a = \beta_1 c$ and $f_s = 600 \frac{d-c}{c}$. Then the design moment strength (M_n) is calculated using Equs. (4-22) to (4-24) in condition that f_y to be replaced by f_s.

In this case, an equation to calculate the value of c can be derived in terms of:

$c = K_u \, d$, K_u can be calculated as follows:

$C = T, \Rightarrow 0.85 f'_c \, ab = A_s \, f_s, \Rightarrow 0.85 f'_c \, \beta_1 \, bc = \rho bd \, f_s = 0.85 f'_c \, \beta_1 \, bK_u \, d$

$$\varepsilon_s = \frac{d-c}{c} \varepsilon_u = \frac{d-K_u d}{K_u d} \varepsilon_u = \frac{1-K_u}{K_u} \varepsilon_u$$

$f_s = \varepsilon_s E_s = \frac{1-K_u}{K_u} \varepsilon_u E_s = 600 \frac{1-K_u}{K_u}$, where $\varepsilon_u = 0.003$ and $E_s = 200{,}000$ MPa.

4.4.3 Over-Reinforced Concrete Beams

Beams in which the steel reinforcement is greater than those causing the balanced failure, i.e., $\rho = A_s / bd > \rho_b$, here......... $c > c_b$ and $\varepsilon_s < \varepsilon_y$.

Normally, the failure occurs by crushing of concrete as it reaches ultimate strain (ε_u = 0.003) before the steel reaching the yield strength, f_y. This kind of failure is known as "***Compression Failure***". This failure is happened suddenly without previous warnings. Thus, the ACI-code does not allow this kind of failure to happen by designing the beams to be under-reinforced always.

4.5 Practical Consideration in Beams Design

1- *Beam Selfweight Estimation*: If the section size is assumed for analysis purposes, it will be used then for selfweight estimation. For design purposes, if there is no information about the section size, it will be assumed based on experience and used to calculate the beam selfweight. This assumption has to be checked after getting design results.

2- *Beam Section Aspect Ratio*: If there are no architectural requirements or other restrictions, the favorite economical aspect ratio (effective depth/width) is between 2 to 3.

3- *Concrete Cover*: Concrete cover must be provided to protect steel reinforcement from the surrounding exposure, fire and to provide enough bond between concrete and steel. The minimum concrete cover based on ACI-Code (7.7.1) are:

(a): Concrete in cast against & permanently exposed to soil..........75 mm

(b): Concrete exposed to earth or weather:

- Wall panel, slabs & Joists...25 mm
- Other members...40 mm

<u>3- Minimum Steel Ratio</u>: In addition to tension and compression failure, there is another type of failure happened in under-reinforced beams. If the beam bending moment capacity is less than the cracking moment, a sudden beam failure will occur without any advanced warnings.

To avoid the above kind of failure, the ACI-Code (10.5) specify minimum steel ratio to be:

$$\rho_{min} = \frac{As_{min}}{b_w d} = \frac{0.25\sqrt{fc}\, b_w d}{fy} \cdot \frac{1}{b_w d} = \boxed{\frac{\sqrt{fc}}{4\, fy} \geq \frac{1.4}{fy}} \quad \text{.......................(4-19)}$$

Where:

b_w = the web width for T-section and equals to **b** for rectangular section.

Generally, to ensure tension failure rather than compression or cracking failure, the reinforcement ratio must be:

$\rho_{min} < \rho < \rho_{max}$ ....(4-20)

Based on the ACI-Code, the above limitation can be neglected if the provided steel reinforcement is greater than the required amount by one-third and this will results to less reinforcement for large sections.

The minimum steel ratio derived above is not applicable to structural slabs and footings. The minimum steel ratio for them will be the temperature and shrinkage one.

4.6 ACI-Code Provisions for Under-Reinforced Beams

There are three main provisions:

1- **Maximum Steel Ratio:** This ratio can be calculated by two methods:

(a): Direct Method: by adopting steel ratio less than ρ_b that results in a net tensile strain in extreme tension steel at normal strength of 0.00376. The limit of 0.004 is slightly more conservative (ACI-Code 10.3.5).

$$\rho_{max} = 0.75\ \rho_b \quad \text{......(4-10)}$$

(b): Indirect Method: by limiting the minimum strain in extreme tension steel, (ε_t). The extreme distance designated as (d_t). From the strain distribution:

$$\varepsilon_t = \varepsilon_u \frac{d_t - c}{c} \quad \text{......(4-11)}$$

If the steel reinforcement lies in one layer, $d_t = d$. For more than one layer, $d_t > d$. In this derivation, it is assumed that $d_t \approx d$.

To ensure tension failure, ACI-Code (10.3.5) states that for members with factored axial compressive load < 0.1 f'_c A_g, ε_t at nominal strength shall not be less than 0.004.

$$\varepsilon_t \geq 0.004$$

To develop equation for ρ_{max} that cause steel strain of 0.004 using the same principle for derivation of ρ_b.

$$\rho = \frac{0.85\beta_1 c f'_c}{f_y d} \quad \text{......(4-12)}$$

From strain distribution and by substituting ε_t instead of ε_s

$$c = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t} d \quad \text{......(4-13)}$$

Substitute c in equ (4-13) in to equ. (4-12):

$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t} \quad \text{(4-14)}$$

Substitute the minimum strain value (ε_t = 0.004) results:

$$\boxed{\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + 0.004}} \quad \text{(4-15)}$$

The least of equs. (4-10) & (4-15) to be the control.

2- **Strength Reduction Factor (ϕ) Calculation:** This factor depends on the type of failure. In tension-controlled members, ϕ = 0.9, while for compression-controlled members, ϕ = 0.75 for members with spiral reinforcement and ϕ = 0.65 for other reinforced members (ACI- Code 9.3.2).

(a) Tension – Controlled Members

Members with net tensile strain in the extreme tension steel, $\varepsilon_t \geq 0.005$. In this case, the strength reduction factor, ϕ = 0.9. The steel ratio that cause steel strain of 0.005 can be calculated the same way as ρ_{max} and will be denoted as ρ_t:

$$\rho_t = 0.85\beta_1 \frac{f_c'}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + 0.005} \quad \text{(4-16)}$$

(b) Compression – Controlled Members

Members with net tensile strain in the extreme tension steel, $\varepsilon_t \leq 0.002$. In this case, the strength reduction factor, ϕ = 0.75 for members with spiral reinforcement and ϕ = 0.65 for other reinforced members. ε_t can be calculated by finding the compression zone depth, a, and then the neutral axis depth, c using equ. (4-11).

4.7 Analysis & Design of Singly Reinforced Concrete Rectangular Beams

4.7.1 Analysis of Singly Reinforced Concrete Rectangular Beams

The analysis is to determine the nominal or ultimate moment strength as both the section size of the beam together with the material properties are known or given. The analysis can be carried out through evaluating the steel ratio, ρ, and compare it with the balanced steel ratio, ρ_b, and then classify the beams in to two kinds based on steel amount:

1- Balanced or Under-Reinforced Beams:

When $\rho \leq \rho_b$, the beams will be either under-reinforced or balanced reinforced as mentioned before. Here, the steel stress, f_s, will equal to yielding strength, f_y. The analysis can be done by calculating the depth of the Whitney stress block, a, from equilibrium of forces:

$$C = T, \Rightarrow 0.85 f'_c\, ab = A_s f_y, \Rightarrow \boxed{a = \frac{A_s f_y}{0.85 f'_c\, b}} \quad \text{.............................(4-21)}$$

The nominal moment strength can be calculated by taking moment about compression force center:

$$\boxed{M_n = A_s f_y \left(d - \frac{a}{2}\right)} \quad \text{..(4-22)}$$

Or by taking moment about the tension force center:

$$\boxed{M_n = 0.85 f'_c\, ab \left(d - \frac{a}{2}\right)} \quad \text{...(4-23)}$$

The nominal moment strength can be calculated also in terms of the actual steel ratio (ρ) as:

$$a = \frac{A_s f_y}{0.85 f'_c\, b} = \frac{\rho\, bd f_y}{0.85 f'_c\, b} = \frac{\rho\, d f_y}{0.85 f'_c}, M_n = A_s f_y \left(d - \frac{a}{2}\right) = \rho bd f_y \left(d - \frac{\rho\, d f_y}{(2)0.85 f'_c}\right)$$

$$\boxed{\therefore M_n = \rho b d^2 f_y \left(1 - 0.59 \rho \frac{f_y}{f'_c}\right).} \quad \text{...(4-24)}$$

$\rho bd\, f_s = 0.85 f'_c\, \beta_1\, bK_u\, d \Rightarrow \rho f_s = 0.85 f'_c\, \beta_1\, K_u \Rightarrow \rho\, 600 \frac{1-K_u}{K_u} = 0.85 f'_c\, \beta_1\, K_u$

$K_u = \frac{600}{0.85 f'_c\, \beta_1} . \rho \frac{1-K_u}{K_u}$, let $m = \frac{600}{0.85 f'_c\, \beta_1}$, $K_u = m . \rho \frac{1-K_u}{K_u}$, $K_u^2 = \rho m\,(1 - K_u)$

$$K_u = \sqrt{\left(\frac{\rho m}{2}\right)^2 + \rho m} - \frac{\rho m}{2} \quad \text{..(4-26)}$$

4.7.2 Design of Singly Reinforced Concrete Rectangular Beams

The design is to determine the appropriate section size and the required reinforcement amount as both the applied service load together with the material properties are known or given.

CASE # 01: Section size or part of it is unknown

1- Calculate the design bending moments (M_u) from structural analysis after including the beam selfweight by assuming the section size.

2- Calculate both the maximum steel ratio (ρ_{max}) and the minimum steel ratio (ρ_{min}).

3- Select the required steel ratio (ρ) such: $\rho_{min} < \rho < \rho_{max}$. It is recommended to select $\rho \leq \rho_t$ to get the maximum strength reduction factor ($\phi = 0.9$) allowed.

4- Determine the section dimensions (bd^2) using: $M_n = \rho b d^2 f_y \left(1 - 0.59 \rho \frac{f_y}{f_c}\right)$.

5- Based on experience, assume either b or d to find the other dimension.

6- Calculate the required steel ratio ($As = \rho\, bd$) and the number of appropriate size steel bars and spacing according to ACI-Code provisions.

7- Find the total beam depth (h) as follows:

h = d + 40 + (assumed stirrup size) + (assumed main bars size/2).......... If one layer.

h = d + 40 + (assumed stirrup size) + (assumed main bars size) + (spacing between main bars/2)............. If two layer.

CASE # 02: Section size is known

1. Calculate the design bending moments (M_u) from structural analysis after including the beam selfweight.
2. Calculate both the maximum steel ratio (ρ_{max}) and the minimum steel ratio (ρ_{min}).
3. The strength reduction factor (ϕ) will primarily assumed to be 0.9.
4. Determine the required steel ratio (ρ) as: $M_u = \emptyset M_n = \emptyset \rho b d^2 f_y \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$. This will give 2nd order equation in terms of ρ and can be solved as:

$$M_u = \emptyset \rho b d^2 f_y \left(1 - \frac{\rho f_y}{2*0.85 f_c'}\right) \div \phi b d^2 \Rightarrow \frac{M_u}{\emptyset b d^2} = \rho f_y \left(1 - \frac{\rho f_y}{2*0.85 f_c'}\right)$$

Let $\frac{M_u}{\emptyset b d^2} = R$ and $\frac{f_y}{0.85 f_c'} = m,$ then:

$$R = \rho f_y \left(\frac{2-\rho m}{2}\right) \rightarrow \therefore 2R = 2\rho f_y - \rho^2 f_y m$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{f_y}}\right) \quad \text{.......................................(4-27)}$$

5. The assumed value of strength reduction factor (ϕ) to be checked as:
   - If $\rho \leq \rho_t$ .... The assumed value is OK ($\phi = 0.9$).
   - If $\rho > \rho_t$ .... The value of ϕ will be less than 0.9. Therefore, the design moment strength (ϕM_n) will be less the design moment (M_u). In this case, the reinforcement has to be increased and calculate the value of ϕ form Equs. (4-17) and (4-18) and then calculate ϕM_n to make sure that ($\phi M_n \geq M_u$).

6. Calculate the required steel ratio (As = ρ bd) and the number of appropriate size steel bars and spacing according to ACI-Code provisions.

Example (4-1):

A reinforced concrete rectangular beam of width b = 250 mm and effective depth of d = 460 mm has a compressive strength f'_c = 20 MPa and reinforcement tensile strength f_y = 300 MPa. Calculate the ultimate design bending moment for:

(a) : Steel area $A_s = 2000\ mm^2$.

(b) : Steel area $A_s = 5160\ mm^2$.

(c) : the maximum design moment based on ACI-Code provisions.

Solution:

(a): Steel ratio $A_s = 2000\ mm^2$

$$\rho_b = 0.85\beta_1 \frac{f_c}{f_y} \cdot \frac{600}{600+f_y} = 0.85\,(0.85)\frac{20}{300} \cdot \frac{600}{600+300} = 0.032$$, $\beta_1 = 0.85$ since $f'_c \leq 30$ MPa.

$$\rho = \frac{A_s}{bd} = \frac{2000}{250\,(460)} = 0.0174 < \rho_b = 0.032 \Rightarrow$$ **the section is under-reinforced.**

$$\rho_t = 0.85\beta_1 \frac{f'_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + 0.005} = 0.85(0.85)\frac{20}{300} \cdot \frac{0.003}{0.003+0.005} = 0.018$$

Since $\rho = 0.0174 < \rho_t = 0.018$ ....$\Rightarrow$ **$\phi = 0.9$**

$$a = \frac{A_s f_y}{0.85 f'_c\, b} = \frac{2000\,(300)}{0.85\,(20)(250)} = 141 \text{ mm}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 2000\,(300)\left(460 - \frac{141}{2}\right) * 10^{-6} = 233.7 \text{ kN.m}$$

Also, M_u can be calculated using Equs. (4-23) and (4-24):

$$M_n = 0.85 f'_c\, ab \left(d - \frac{a}{2}\right) = 0.85(20)(141)(250)\left(460 - \frac{141}{2}\right) * 10^{-6} = 233.4 \text{ kN.m}$$

$$M_n = \rho b d^2 f_y \left(1 - 0.59\rho\frac{f_y}{f_c}\right) = 0.0174(250)(460)^2(300)(1 - 0.59(0.0174))\frac{300}{20} * 10^{-6}$$

$= 233.6$ kN.m

$M_u = \phi M_n = 0.9\,(233.7) =$ **210 kN.m** (c): the maximum design moment based on ACI-Code provisions.

The maximum design moment based on ACI-Code provision will be determined based on maximum steel ratio allowed by the code.

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + 0.004} = 0.85(0.85)\frac{20}{300} \cdot \frac{0.003}{0.003+0.004} = \boxed{0.0206}$$

A_s (max) = ρ_{max} bd = 0.0206 (250)(460) = 2369 mm^2

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2369\,(300)}{0.85\,(20)(250)} = \boxed{167 \text{ mm}}$$ c = a/β_1 = 167/0.85 = 196.5 mm

Since ρ = 0.0206 > ρ_t = 0.018 ....⇒ ε_t must be calculated using Equ. (4-11)

$$\varepsilon_t = \varepsilon_u \frac{d_t - c}{c} = 0.003 \frac{460-196.5}{196.5} = 0.0040$$

Since 0.002 < ε_t = 0.0040 < 0.005...⇒ ϕ must be calculated using Equ. (4-18)

ϕ = 0.483 + 83.3 ε_t = 0.483 + 83.3 (0.0040) = $\boxed{0.816}$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 2369\,(300)\left(460 - \frac{167}{2}\right) * 10^{-6} = 267.6 \text{ kN.m}$$

Also, M_u can be calculated using Equs. (4-23) and (4-24):

$$M_n = 0.85 f_c' ab \left(d - \frac{a}{2}\right) = 0.85(20)(167)(250)\left(460 - \frac{167}{2}\right) * 10^{-6} = 267.2 \text{ kN.m}$$

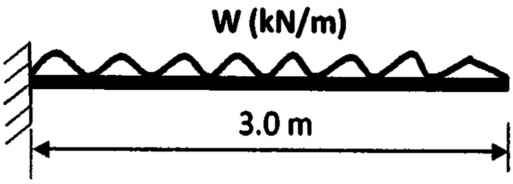
$$M_n = \rho b d^2 f_y \left(1 - 0.59\rho \frac{f_y}{f_c'}\right) = 0.0206(250)(460)^2(300)(1 - 0.59(0.0206))\frac{300}{20} * 10^{-6}$$

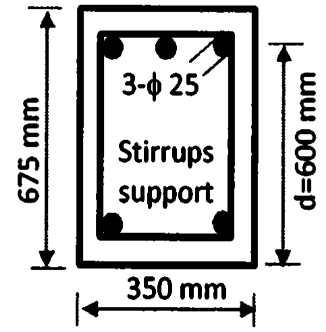
= 267.3 kN.m

M_u = ϕ M_n = 0.816 (267.6) = $\boxed{218.36 \text{ kN.m}}$

Example (4-2):

Check the adequacy of the given beam based on bending requirements if the service dead load (including beam selfweight) = 8 kN/m and the service live load = 18 kN/m. Note that the concrete compressive strength f'_c = 35 MPa and reinforcement tensile strength f_y = 300 MPa.





Solution:

Since f'_c = 35 MPa > 30 MPa, ⇒ β_1 can be calculated from Equ. (4-17) as:

$\beta_1 = 0.85 - 0.05\,(f'_c - 30)/7 = 0.85 - 0.05\,(35 - 30)/7 =$ **0.81**

$A_s = 3\,(\pi(25)^2/4) = 1473\ mm^2$

$$\rho = \frac{A_s}{bd} = \frac{1473}{350\,(600)} = 0.007$$

$$\rho_b = 0.85\beta_1 \frac{f_c}{f_y}.\frac{600}{600+f_y} = 0.85\,(0.81)\frac{35}{300}.\frac{600}{600+300} = 0.0536$$

Since $\rho = 0.007 < \rho_b = 0.0536$ ⇒ **the section is under-reinforced.**

$$\rho_t = 0.85\beta_1 \frac{f'_c}{f_y}.\frac{\varepsilon_u}{\varepsilon_u+0.005} = 0.85(0.81)\frac{35}{300}.\frac{0.003}{0.003+0.005} = 0.0\ 3$$

Since $\rho = 0.007 < \rho_t = 0.003$ ....⇒ **$\phi = 0.9$**

$$M_n = \rho b d^2 f_y \left(1 - 0.59\rho\frac{f_y}{f_c}\right) = 0.007(350)(600)^2(300)(1 - 0.59(0.007))\frac{300}{35} * 10^{-6}$$

= 255.2 kN.m

- Section capacity (ϕM_n) = 0.9 (255.2) = **229.7 kN.m**

$W_u = 1.2\,D + 1.6\,L = 1.2\,(8) + 1.6\,(18) = 38.4$ kN/m

- Design moment (M_u) = $W_u\,L^2/2 = 38.4\,(3)^2/2$ = **178.8 kN.m**

Since $M_u = 178.8 < \phi M_n = 229.7$ ⇒ The given section is adequate.

- $M_u = \emptyset \rho b d^2 f_y \left(1 - 0.59\rho \frac{f_y}{f_c}\right)$

$129.6\, x10^6 = 0.9\,(0.016) bd^2 (400) \left(1 - 0.59(0.016)\frac{400}{30}\right) = 5.035\ bd^2$

$\therefore bd^2 = 25.74 \times 10^6$

Select beam width to be 250 mm, $d^2 = 25.74 \times 10^6 / 250 = 102.96 \times 10^3$

d = 321 mm ⇒ say 320 mm

- Find the required steel area, $A_s = \rho\, bd = 0.016\,(250)(320) = 1280\ mm^2$

Use ϕ 25 mm diameter bars, (A_b= 491 mm^2)

No. of bars = 1280/491 = 2.61⇒ say 3

∴ use 3-ϕ 25 mm diameter bars

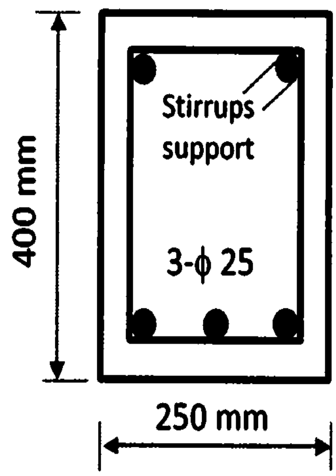
Check the adequacy of the calculated beam width for the number of steel bars.

Required width = 40+40+10+10+3 (25)+(3-1)(25) = 225 mm

Provided width = 250……OK ☺

Calculate the total beam depth (h) as: (assume one layer of reinforcement)

h = d + 40 + 10 + 12.5 = 320 + 62.5 = 383 mm ⇒ say 400 mm



Example (4-4):

Determine the reinforcement required for a beam of width b = 300 mm and total depth h = 700 mm under service dead load moment (including beam selfweight) = 100 kN.m and service live load moment = 150 kN.m. Note that the concrete compressive strength f'_c = 20 MPa and reinforcement tensile strength f_y = 400 MPa.

Solution:

- Calculate the ultimate design moment (M_u):

$M_u = 1.2\,M_d + 1.6\,M_L = 1.2\,(100) + 1.6\,(150) = \boxed{360\text{ kN.m}}$

- Calculate both ρ_{max} and ρ_{min}:

$$\rho_b = 0.85\beta_1\frac{f'_c}{f_y}\cdot\frac{600}{600+f_y} = 0.85\,(0.85)\frac{20}{400}\cdot\frac{600}{600+400} = 0.0217,\ \beta_1 = 0.85 \text{ since } f'_c \le 30\text{ MPa}.$$

$$\rho_{max} = 0.85\beta_1\frac{f'_c}{f_y}\cdot\frac{\varepsilon_u}{\varepsilon_u+0.004} = 0.85(0.85)\frac{20}{400}\cdot\frac{0.003}{0.003+0.004} = \boxed{0.0155\text{ (control)}}$$

$$\rho_{min} = \frac{\sqrt{f'_c}}{4\,f_y} \ge \frac{1.4}{f_y} = \frac{\sqrt{20}}{4\,(400)} \ge \frac{1.4}{400} = 0.00280 \ge \boxed{0.0035\text{ (control)}}$$

- The strength reduction factor (ϕ) will primarily assumed to be 0.9.
- Calculate steel ratio as :

d = 700-40-10-12.5 = 637.5 mm⇒ say d = 635 mm

$$R = \frac{M_u}{\phi b d^2} = \frac{360x10^6}{0.9(300)(635)^2} = 3.3, \qquad m = \frac{f_y}{0.85\,f'_c} = \frac{400}{0.85(20)} = 23.53$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{f_y}}\right) = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2(23.53)(3.3)}{400}}\right) = \boxed{0.0093}$$

Since $\rho_{min} = 0.0035 < \rho = 0.0093 < \rho_{max} = 0.0155$.....⇒ Design is OK ☺

$A_s = \rho\ bd = 0.0093\ (300)(635) = 1772\ mm^2$

- Re-check the assumed value of ϕ as :

$$\rho_t = 0.85\beta_1 \frac{f'_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + 0.005} = 0.85(0.85)\frac{20}{400} \cdot \frac{0.003}{0.003 + 0.005} = 0.0136$$

Select $\rho = 0.0093 < \rho_t = 0.0136$.....⇒ Assumption is OK ☺

- Find the number and arrangement of steel bars.

Use ϕ 25 mm diameter bars, ($A_s = 491\ mm^2$)

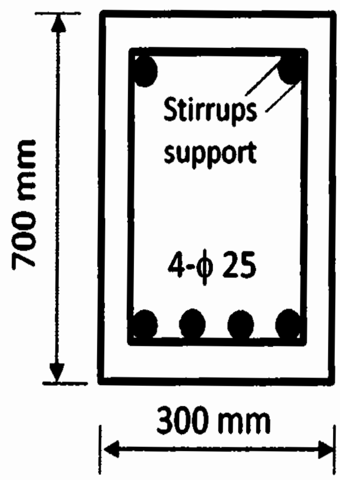
No. of bars = 1772/491 = 3.61⇒ say 4

∴ use 4-ϕ 25 mm diameter bars

Check the adequacy of the calculated beam width for the number of steel bars.

Required width = 40+40+10+10+4 (25)+(4-1)(25) = 275 mm

Provided width = 300......OK ☺



4.8 Analysis & Design of Singly Reinforced Concrete Rectangular Beams Using Tables

Sometimes, in order to make easy the analysis and design of singly reinforced concrete rectangular beams, special Tables can be used that represent the solutions of design and analysis equations. Those Tables can be constructed using Equ. (4-24) as follows:

$$M_n = \rho b d^2 f_y \left(1 - 0.59\rho \frac{f_y}{f_c}\right) \div f'_c\, bd^2$$

$$\frac{M_n}{f_c b d^2} = \rho \frac{f_y}{f_c} \left(1 - 0.59\rho \frac{f_y}{f_c}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4\text{-}28)$$

Let $\frac{M_n}{f_c b d^2} = k$ and $\rho \frac{f_y}{f_c} = \omega$

Equ. (4-28) becomes:

$k = \omega\,(1 - 0.59\,\omega)$ ....(4-29)

By selecting values for ω, the corresponding values of k can be calculated. This will results in a relation between the nominal strength and the steel ratio as listed in the Table below.

The first row and column represent the values of ω, while the remaining cells represent the values of k. Knowing that the value of ω consists of three digits composed of the values in the 1st column plus those in the 1st row.

Tables have been prepared for $\rho \le \rho$ only (tension failure) and cannot be used for balanced or compression failures.

ω	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
	$k = M_n / f'_c bd^2 = M_u / \phi f'_c bd^2$									
0	0	0.0010	0.0020	0.0030	0.0040	0.0050	0.0060	0.0070	0.0080	0.0090
0.01	0.0099	0.0109	0.0119	0.0129	.0139	0.0149	0.0159	0.0168	0.0178	0.0188
0.02	0.0197	0.0207	0.0217	0.0226	0.0236	0.0246	0.0256	0.0266	0.0275	0.0285
0.03	0.0295	0.0304	0.0314	0.0324	0.0333	0.0343	0.0352	0.0362	0.0372	0.0381
0.04	0.0391	0.0400	0.0410	0.0420	0.0429	0.0438	0.0448	0.0457	0.0467	0.0476
0.05	0.0485	0.0495	0.0504	0.0513	0.0523	0.0532	0.0541	0.0551	0.0560	0.0569
0.06	0.0579	0.0588	0.0597	0.0607	0.0616	0.0625	0.0634	0.0643	0.0653	0.0662
0.07	0.0671	0.0680	0.0689	0.0699	0.0708	0.0717	0.0726	0.0735	0.0744	0.0753
0.08	0.0762	0.0771	0.0780	0.0789	0.0798	0.0807	0.0816	0.0825	0.0834	0.0843
0.09	0.0852	0.0861	0.0870	0.0879	0.0888	0.0897	0.0906	0.0915	0.0923	0.0932
0.10	0.0941	0.0950	0.0959	0.0967	0.0976	0.0985	0.0994	0.1002	0.1011	0.1020
0.11	0.1029	0.1037	0.1046	0.1055	0.1063	0.1072	0.1081	0.1089	0.1098	0.1106
0.12	0.1115	0.1124	0.1133	0.1141	0.1149	0.1158	0.1166	0.1175	0.1183	0.1192
0.13	0.1200	0.1209	0.1217	0.1226	0.1234	0.1243	0.1251	0.1259	0.1268	0.1276
0.14	0.1284	0.1293	0.1301	0.1309	0.1318	0.1326	0.1334	0.1342	0.1351	0.1359
0.15	0.1367	0.1375	0.1384	0.1392	0.1400	0.1408	0.1416	0.1425	0.1433	0.1441
0.16	0.1449	0.1457	0.1465	0.1473	0.1481	0.1489	0.1497	0.1506	0.1514	0.1522
0.17	0.1529	0.1537	0.1545	0.1553	0.1561	0.1569	0.1577	0.1585	0.1593	0.1601
0.18	0.1609	0.1617	0.1624	0.1632	0.1640	0.1648	0.1656	0.1664	0.1671	0.1679
0.19	0.1687	0.1695	0.1703	0.1710	0.1718	0.1726	0.1733	0.1741	0.1749	0.1756
0.20	0.1764	0.1772	0.1779	0.1787	0.1794	0.1802	0.1810	0.1817	0.1825	0.1832
0.21	0.1840	0.1847	0.1855	0.1862	0.1870	0.1877	0.1885	0.1892	0.1900	0.1907
0.22	0.1914	0.1922	0.1929	0.1937	0.1944	0.1951	0.1959	0.1966	0.1973	0.1981
0.23	0.1988	0.1995	0.2002	0.2010	0.2017	0.2024	0.2031	0.2039	0.2046	0.2053
0.24	0.2060	0.2067	0.2075	0.2082	0.2089	0.2094	0.2103	0.2110	0.2117	0.2124
0.25	0.2131	0.2138	0.2145	0.2152	0.2159	0.2166	0.2173	0.2180	0.2187	0.2194
0.26	0.2201	0.2208	0.2215	0.2222	0.2229	0.2236	0.2243	0.2249	0.2256	0.2263
0.27	0.2270	0.2277	0.2284	0.2290	0.2297	0.2304	0.2311	0.2317	0.2324	0.2331
0.28	0.2337	0.2344	0.2351	0.2357	0.2364	0.2371	0.2377	0.2384	0.2391	0.2397
0.29	0.2404	0.2410	0.2417	0.2423	0.2430	0.2437	0.2443	0.2450	0.2456	0.2463
0.30	0.2469	0.2475	0.2482	0.2488	0.2495	0.2501	0.2508	0.2514	0.2520	0.2527
0.31	0.2533	0.2539	0.2546	0.2552	0.2558	0.2565	0.2571	0.2577	0.2583	0.2590
0.32	0.2596	0.2602	0.2608	0.2614	0.2621	0.2627	0.2633	0.2639	0.2645	0.2651
0.33	0.2657	0.2664	0.2670	0.2676	0.2682	0.2688	0.2694	0.2700	0.2706	0.2712
0.34	0.2718	0.2724	0.2730	0.2736	0.2742	0.2748	0.2754	0.2760	0.2766	0.2771
0.35	0.2777	0.2783	0.2789	0.2795	0.2801	0.2807	0.2812	0.2818	0.2824	0.2830
0.36	0.2835	0.2841	0.2847	0.2853	0.2858	0.2864	0.2870	0.2875	0.2881	0.2887
0.37	0.2892	0.2898	0.2904	0.2909	0.2915	0.2920	0.2926	0.2931	0.2937	0.2943
0.38	0.2948	0.2954	0.2959	0.2965	0.2970	0.2975	0.2981	0.2986	0.2992	0.2997
0.39	0.3003	0.3008	0.3013	0.3019	0.3024	0.3029	0.3035	0.3040	0.3045	0.3051
0.40	0.3056	0.3061	0.3066	0.3072	0.3077	0.3082	0.3087	0.3093	0.3098	0.3103

Example (4-5):

Repeat Example (4-1) when the provided steel reinforcement = 2000 mm^2 using tables.

Solution:

After calculating the value of steel ratio: $\rho = 0.0174$

$\omega = \rho \frac{f_y}{\acute{f}_c} = 0.0174 \, \frac{300}{20} = 0.261 = 0.26 + 0.001$

Using the table, the corresponding value of k = 0.2208

$\frac{M_n}{\acute{f}_c b d^2} = k \, , M_n = k \acute{f}_c b d^2 = 0.2208\,(20)\,(250)(460)^2 \times 10^6 = 233.6$ kN.m

The rest can be calculated as in Example (4-1).

Example (4-6):

Repeat Example (4-3) using tables.

Solution:

After calculating the value of steel ratio: $\rho = 0.016$ and the design moment: $M_u = 129.6$ kN.m

$\omega = \rho \frac{f_y}{\acute{f}_c} = 0.016 \, \frac{400}{30} = 0.213 = 0.21 + 0.003$

Using the table, the corresponding value of k = 0.1862

$\frac{M_n}{\acute{f}_c b d^2} = k \, , b d^2 = \frac{M_n}{k \acute{f}_c} = \frac{129.6 x 10^6 / 0.9}{0.1862\,(30)} = 25.78 \times 10^6$ mm^3

The rest can be calculated as in Example (4-3).

Example (4-8):

Repeat Example (4-4) using tables.

Solution:

After calculating the value of the design moment: M_u = 360 kN.m

$k = \frac{M_n}{f'_c b d^2} = \frac{360x10^6/0.9}{20(300)(635)^2}$ = 0.1653, the nearest value is 0.1656

Using the table, the corresponding value of ω = 0.186

$\omega = \rho \frac{f_y}{f'_c} \rightarrow \rho = \frac{\omega f'_c}{f_y} = \frac{0.186(20)}{400}$ = 0.0093

The rest can be calculated as in Example (4-4).

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قسم
الهندسة المدنية
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Al-Mansour University College

Civil Engineering Department
3rd. Stage

Reinforced Concrete Design Lectures

2022– 2023

Lec.3

خرسانة مسلحة

Flexurul Analysis and Design of Brams

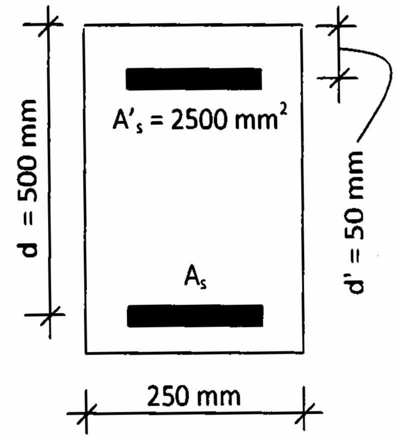
Lec. Basher Faisal

Example (06-1):

For the given doubly reinforced concrete beam section, calculate the nominal bending strength (M_n) if:

(a): $A_s = 4000\ mm^2$ (b): $A_s = 5000\ mm^2$ (c): $A_s = 8000\ mm^2$

Consider $A_s' = 2500\ mm^2$, $f'_c = 30\ N/mm^2$ and $f_y = 350\ N/mm^2$.



Solution:

(a): $A_s = 4000\ mm^2$

$$\rho_b = 0.85\beta_1 \frac{f_c'}{f_y}\left(\frac{600}{600+f_y}\right) = 0.85(0.85)\frac{30}{350}\left(\frac{600}{600+350}\right) = 0.039,\quad \rho' = \frac{A_s'}{bd} = \frac{2500}{250(500)} = 0.02$$

$$\rho_{cy}' = 0.85\beta_1 \frac{f_c'}{f_y}\left(\frac{d'}{d}\right)\left(\frac{600}{600-f_y}\right) + \rho' = 0.85(0.85)\frac{30}{350}\left(\frac{50}{500}\right)\left(\frac{600}{600-350}\right) + 0.02 = 0.0349$$

$$\rho = \frac{A_s}{bd} = \frac{4000}{250(500)} = 0.032$$

Since $\rho < \rho_{cy}'$ → the compression steel does not yield ($f_s' < f_y$)

- Check yielding of tension steel through calculating ρ_b' and compare it with ρ as:

$$f_s' = 600 - (600 + f_y)\frac{d'}{d} = 600 - (600 + 350)\frac{50}{500} = 505\ \text{Mpa} > f_y \rightarrow f_s' = f_y$$

$$\rho_b' = \rho_b + \rho'\frac{f_s'}{f_y} = 0.039 + 0.02\left(\frac{350}{350}\right) = 0.059$$

Since $\rho < \rho_b'$ → yielding of tension steel ($f_s = f_y$)

- Calculate the depth of the neutral axis "c" as:

$$k_1 = \frac{A_s f_y - 600A_s'}{0.85\beta_1 f_c' b} = \frac{4000(350) - 600(2500)}{0.85(0.85)(30)(250)} = -18.45$$

$$k_2 = \frac{600A_s' d'}{0.85\beta_1 f_c' b} = \frac{600(2500)(50)}{0.85(0.85)(30)(250)} = 13840.83$$

$$c = \frac{k_1 + \sqrt{k_1^2 + 4k_2}}{2} = \frac{(-18.45) + \sqrt{(-18.45)^2 + 4(13840.83)}}{2} = 108.8\ \text{mm}$$

$$a = \beta_1 c = 0.85\,(108.8) = 92.5\ \text{mm},\quad f_s' = 600\left(\frac{c-d'}{c}\right) = 600\left(\frac{108.8-50}{108.8}\right) = 324.3\ \text{Mpa}$$

Lecture # 06: Analysis and Design of Doubly Reinforced Concrete Rectangular Beams

6.1 Introduction

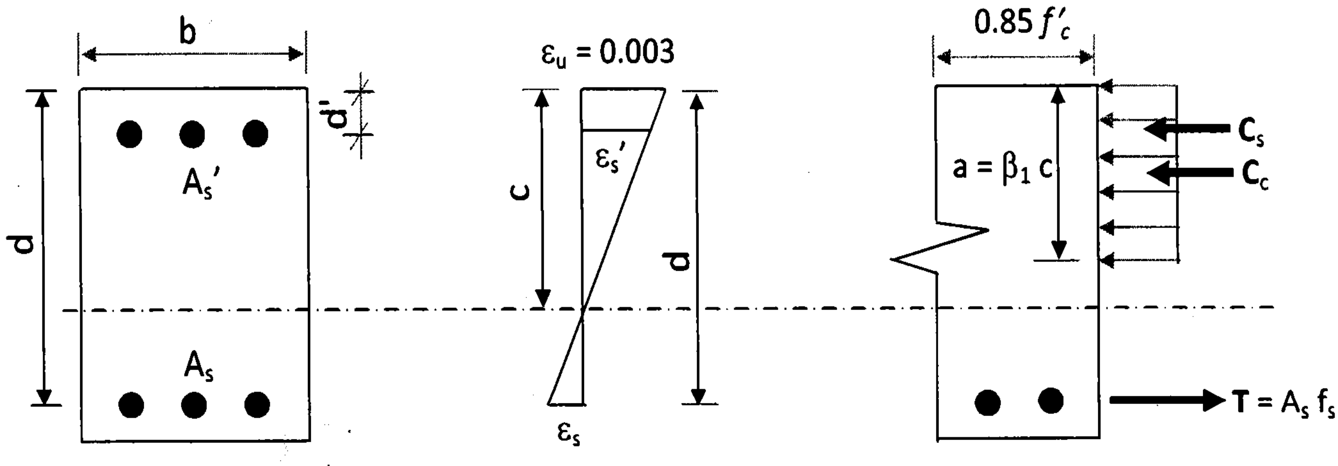
Design of singly reinforced concrete is based on the fact that the failure will happened in tension and this can be done through providing steel ratio less than or equal the maximum steel ratio ($\rho \leq \rho_{max.}$).

Due to architectural reasons, sometimes the beam size becomes limited to certain dimensions. In this case, the beam section becomes insufficient to carry the applied moment and the addition of extra tension steel will violate the above condition ($\rho \leq \rho_{max.}$) and results in compression failure ($\varepsilon_s < \varepsilon_y$) and this is prohibited by the ACI-Code and it should be avoided.

In order to overcome the above, compression steel will be provided that develop a force equivalent to that results from the extra tension steel, and consequently maintain the location of the neutral axis (c) to ensure tensile failure. In this case, the beam will be doubly reinforced. The figure below illustrates both stress and strain distributions for doubly reinforced beam.

Sometimes, compression reinforcement may be added for:

1- Reducing the creep deflection
2- Stirrup support, &
3- Resisting the tensile stresses results from conversion of moment's directions.



Doubly reinforced section ***Strain Distribution*** ***Stress Distribution***

Equilibrium of the 1[st] part:

$C_s = T_1 \rightarrow 0.85f_c' a\, b = A_{s1} f_y \rightarrow$ $\boxed{\therefore\ a = \frac{A_{s1} \cdot f_y}{0.85 f_c' b}}$ ......(6-5)

In order to apply the above equations, the tension steel stress should be checked & compared with f_y. In this case, the balanced steel ratio (ρ_b') can be calculated assuming that the compression steel stress reaches f_y as:

From equilibrium of forces:

$T = C \rightarrow A_s f_y = 0.85 f_c' a\, b + A_s' f_y \rightarrow \rho_b' b d f_y = 0.85 f_c' \beta_1 c_b b + \rho' b d f_y \div b\, d f_y$

$\rho_b' = 0.85 B_1 \frac{f_c'}{f_y} \cdot \frac{c_b}{d} + \rho'$..... from strain distribution, $\rightarrow \frac{c_b}{d} = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$, use $\varepsilon_u = 0.003$ & $\varepsilon_y = f_y/E_s$

$\rho_b' = 0.85\beta_1 \frac{f_c'}{f_y} \cdot \left(\frac{600}{600+f_y}\right) + \rho' \rightarrow$ $\boxed{\rho_b' = \rho_b + \rho'}$ ......(6-6)

To ensure tension failure, the ACI-Code requires that the strain of the far steel bar to be less than 0.004. The maximum tension steel ratio (ρ_{max}') that cause this strain can be evaluated as above by replacing ε_y by 0.004 as:

$\rho_{max}' = 0.85\beta_1 \frac{f_c'}{f_y} \cdot \left(\frac{\varepsilon_u}{\varepsilon_u + 0.004}\right) + \rho' \rightarrow$ $\boxed{\rho_{max}' = \rho_{max} + \rho'}$ .....(6-7)

6.2.2 Yielding of Tension Steel Only.

The main factor that affects yielding the compression steel is the amount of tension steel. Any increase in tension steel results in increasing of neutral axis depth "c" and this will increase the compression steel stress and tends it to yield.

In order to check the yielding of compression steel, an equation to find the tension steel amount that cause yielding of compression steel can be derived as:

From strain distribution:

$\frac{c}{d'} = \frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} \rightarrow \therefore\ c = \left(\frac{600}{600 - f_y}\right) d'$ ......(6-8)

From equilibrium of forces:

$T = C \rightarrow A_s f_y = 0.85 f_c' a\, b + A_s' f_y \div b\, d f_y \rightarrow \rho_{cy}' = 0.85\beta_1 c \frac{f_c'}{f_y d} + \rho'$

Substitute $c = \left(\frac{600}{600-f_y}\right) d'$ results in $\boxed{\rho_{cy}' = 0.85\beta_1\left(\frac{600}{600-f_y}\right)\frac{f_c'}{f_y}\frac{d'}{d} + \rho'}$ ....(6-9)

- If $\rho \geq \rho_{cy}'$ → the compression steel reaches f_y..... analysis of 6.2.1 is OK
- If $\rho < \rho_{cy}'$ → the compression steel does not reach f_y and in this case, the balanced tension steel ratio which was derived based on the fact that the compression steel yields in balanced condition should be re-derived.

The yielding of compression steel at tension failure ensure its yielding in balanced failure because the neutral axis depth "c" is high. While, if the compression steel does not yields at tension failure, this make it's yielding in balanced failure uncertain and due to this, it is necessary to check its yielding in balanced stage and derive the balanced steel ratio in case of $\varepsilon_s' < \varepsilon_y$ as:

From strain distribution:

$$\frac{\varepsilon_s'}{c_b - d'} = \frac{\varepsilon_u}{c_b} \rightarrow \varepsilon_s' = \left(\frac{c_b - d'}{c_b}\right)\varepsilon_u \,,\ f_s' = E_s\varepsilon_s' = E_s\varepsilon_u\left(\frac{c_b - d'}{c_b}\right) = E_s\varepsilon_u\left(1 - \frac{d'}{c_b}\right)$$

Substitute for c_b from $\frac{c_b}{d} = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$ results in:

$$f_s' = E_s\varepsilon_u\left(1 - \frac{\varepsilon_u + \varepsilon_y}{\varepsilon_u}.\frac{d'}{d}\right) = E_s\left[\varepsilon_u - (\varepsilon_u + \varepsilon_y).\frac{d'}{d}\right] \leq f_y$$

$$f_s' = 600 - (600 + f_y)\frac{d'}{d} \leq f_y \quad ......(6\text{-}10)$$

From equilibrium of forces:

$T = C \rightarrow A_s f_y = 0.85 f_c' a\, b + A_s' f_s'$ ... substitute for c_b and $\div\ b d f_y$

$\rho_b' = 0.85\beta_1 \frac{f_c'}{f_y}\left(\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}\right) + \rho' \frac{f_s'}{f_y}$, use $\varepsilon_u = 0.003$ & $\varepsilon_y = f_y/E_s$

$f_s = 600\left(\frac{d-c}{c}\right) = 600\left(\frac{660-404}{404}\right) = 380$ N/mm^2 $< f_y = 400$ N/mm^2 …… OK ☺

$$M_n = M_{n1} + M_{n2} = 0.85f_c'(b - b_w)h_f\left(d - \frac{h_f}{2}\right) + 0.85f_c'\,b_w a(d - \frac{a}{2})$$

$$M_n = 0.85(20)(720 - 250)150\left(660 - \frac{150}{2}\right) + 0.85(20)250(343)(660 - \frac{343}{2}) \times 10^{-6}$$

$M_n = 1413$ kN.m ☺

9.3 Design of T-Beams

The design includes evaluating the section size and reinforcement details. For T-sections, normally both the beams spacing and slab thickness are given, therefore, both the web dimensions and reinforcement details are required. When designing the T-section for bending resistance, the size of the web will be generally small because of the large width of the compression area and hence the large reinforcement will be provided due to small effective depth. This also require more shear reinforcement and the deflection will be relatively high because of the less dimensions.

The web dimensions normally selected based on:

1- Less usage of steel reinforcement.
2- Keeping less shear stress in web.
3- To make the section appropriate to carry negative moment at supports as the section becomes rectangular with size of b_w d.

In order to ensure tension failure, the ACI-Code requires the T-section steel ratio (ρ_w) to be less than the maximum steel ration ($\rho_{w(max)}$) as:

$$\rho_w \leq \rho_{w(max)} = \rho_{max} + \rho_f$$

The analysis will be conducted through writing both equilibrium and compatibility equations as in previous derivation with substituting f_s and f_s' in terms of "c" as:

From equilibrium of forces:

$T = C \rightarrow A_s f_s = 0.85 f_c' a\, b + A_s' f_s'$, $f_s' = 600\left(\frac{c-d'}{c}\right)$, $f_s = 600\left(\frac{d-c}{c}\right)$

$$A_s\, 600\left(\frac{d-c}{c}\right) = 0.85 \beta_1 f_c' c b + A_s' 600\left(\frac{c-d'}{c}\right) \ldots \text{(6-15)}$$

Then calculate "c" and $f_s' = 600\left(\frac{c-d'}{c}\right)$

If $f_s' \le f_y \rightarrow$ the solution is OK

If $f_s' > f_y \rightarrow$ The above equation should be re-solved by consider $f_s' = f_y$. Then calculate "c" and $\rightarrow a = \beta_1 c$. f_s and f_s' to be calculated from the above equation and then find the nominal strength M_n as:

$$\boxed{M_n = 0.85 f_c' a\, b\, (d - \tfrac{a}{2}) + A_s' f_s'\, (d - d')} \ldots\ldots \text{(6-16)}$$

Summary:

1- Calculate ρ, ρ_{cy}'... then calculate ρ_b' from equ. (6-6) if $\rho \ge \rho_{cy}'$ or from equ. (6-11) if $\rho < \rho_{cy}'$.

2- If $\rho \le \rho_b'$ & $\rho \ge \rho_{cy}'$... the analysis will be through equs. (6-1) and (6-2).

3- If $\rho \le \rho_b'$ & $\rho < \rho_{cy}'$... calculate "c" from equ. (6-13) and calculate M_n from equ. (6-14).

4- If $\rho > \rho_b'$.... Calculate "c" by solving equ. (6-15) and find f_s and f_s' and use equ. (6-16) to calculate the nominal strength "M_n".

$M_n = 0.85 f_c' a\, b\, (d - \frac{a}{2}) + A_s' f_s'\, (d - d')$

$M_n = (0.85\,(30)(92.5)(250)(500 - \frac{92.5}{2}) + 2500(324.3)(500\text{-}50)) \times 10^{-6} =$ **632.4 kN.m**

(b): $A_s = 5000\ mm^2$

$\rho_b = 0.039,\ \ \rho' = 0.02,\ \ \rho_{cy}' = 0.0349$ (from branch "a")

$\rho = \frac{A_s}{bd} = \frac{5000}{250(500)} = 0.04$

Since $\rho > \rho_{cy}'$ → yielding of compression steel ($f_s' = f_y$)

- Check yielding of tension steel through calculating ρ_b' and compare it with ρ as:

$\rho_b' = \rho_b + \rho' = 0.039 + 0.02 = 0.059$

Since $\rho < \rho_b'$ → yielding of tension steel ($f_s = f_y$)

$a = \frac{(A_s - A_s').f_y}{0.85 f_c' b} = \frac{(5000-2500)350}{0.85(30)(250)} = 137.3$ mm, $A_{s1} = A_s' = 2500\ mm^2$

$M_n = 0.85 f_c' a\, b\, (d - \frac{a}{2}) + A_s' f_y\, (d - d')$

$M_n = (0.85\,(30)(137.3)(250)(500 - \frac{137.3}{2}) + 2500(350)(500\text{-}50)) \times 10^{-6} =$ **771.3 kN.m**

(c): $A_s = 8000\ mm^2$

$\rho_b = 0.039,\ \ \rho' = 0.02,\ \ \rho_{cy}' = 0.0349$ (from branch "a")

$\rho = \frac{A_s}{bd} = \frac{8000}{250(500)} = 0.064$

Since $\rho > \rho_{cy}'$ → yielding of compression steel ($f_s' = f_y$)

- Check yielding of tension steel through calculating ρ_b' and compare it with ρ as:

$\rho_b' = 0.059$ (from branch "b")

Since $\rho > \rho_b'$ → tension steel does not yield ($f_s < f_y$)

From equilibrium of forces:

$T = C \rightarrow A_s f_s = 0.85 f_c' \beta_1 c b + A_s' f_y \rightarrow$

$A_s (\frac{d-c}{c}) 600 = 0.85(30)(0.85)c(250) + (2500)(350) \rightarrow c = 323\ mm$

$a = \beta_1 c = 0.85(323) = 274.6\ mm$

$f_s = 600\,(\frac{d-c}{c}) = 600\,(\frac{500-323}{323}) = 328.8$ MPa $< f_y = 350$ MPa.....OK

$M_n = 0.85 f_c' a\, b\, (d - \frac{a}{2}) + A_s' f_y\, (d - d')$

$M_n = (0.85\,(30)(274.6)(250)(500 - \frac{274.6}{2}) + 2500(350)(500\text{-}50)) \times 10^{-6} = \boxed{1028.7\ kN.m}$

6.3 Ultimate Design Strength Calculation

In order to calculate the ultimate design strength (ϕM_n), the value of the strength reduction factor (ϕ) must be calculated. The steel ratio (ρ_t') that cause steel strain $\varepsilon_t = 0.005$ can be calculated as:

(a) Both tension & compression steel yield:

$$\rho_t' = \rho_t + \rho' = 0.85\beta_1 \frac{f_c'}{f_y}\left(\frac{\varepsilon_u}{\varepsilon_u + 0.005}\right) + \frac{A_s'}{bd}$$

(a) Tension steel yields only:

$$\rho_t' = \rho_t + \rho' \frac{f_s'}{f_y} = 0.85\beta_1 \frac{f_c'}{f_y}\left(\frac{\varepsilon_u}{\varepsilon_u + 0.005}\right) + \frac{A_s'}{bd}, \text{ where:}$$

$$f_s' = E_s\left[\varepsilon_u - (\varepsilon_u + 0.005)\frac{d'}{d}\right] \le f_y$$

Or ε_t can be calculated and adopted to find out the value of ϕ instead of ρ_t'.

6.4 Design of Doubly Reinforced Concrete Rectangular Beams

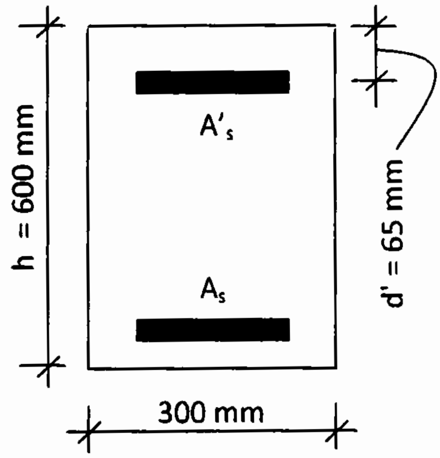
When designing singly reinforced concrete rectangular beams, the allowable steel ration will by equal to ρ_{max} in order to ensure tension failure. Sometimes, due to architectural or service reasons, the section will be limited and insufficient to resist the applied moment. In this case, it is recommended to add compression steel and corresponding tension steel in order to keep the neutral axis for the case of ρ_{max} and this will ensure tension failure.

Design Procedure:

1- Calculate the design bending moment (M_u).

Example (06-2):

Calculate the amount and details of steel reinforcement for a beam of size b= 300 mm and h = 600 mm under a service dead moment (including moment results from selfweight) of 200 kN.m and service live moment of 180 kN.m. Consider $f'_c = 30\ N/mm^2$ and $f_y = 400\ N/mm^2$.



Solution:

d = h – 100 = 600 – 100 = 500 mm

$M_u = 1.2\ M_d + 1.6\ M_L = 1.2(200)+1.6\ (180) =$ **528 kN.m**

$\rho_{max.} = 0.85\beta_1 \frac{f'_c}{f_y}\left(\frac{\varepsilon_u}{\varepsilon_u+0.004}\right) =$ **0.0232**

$m = \frac{f_y}{0.85 f'_c} = \frac{400}{0.85(30)} = 15.7\ ,\ R = \frac{M_u}{\emptyset b d^2} = \frac{528\times10^6}{0.9(300)(500)^2} = 7.82$

$\rho = \frac{1}{m}\left[1-\sqrt{1-\frac{2mR}{f_y}}\right] = \frac{1}{15.7}\left[1-\sqrt{1-\frac{2(15.7)(7.82)}{400}}\right] =$ **0.0241**

Since $\rho > \rho_{max}$ ..→the section to be designed as doubly reinforced.

$A_{S1} = A_{S\ max.} = \rho_{max}\ (b)(d) = 0.0232\ (300)(500) =$ **3480 mm²**

$a = \frac{A_{s1}\cdot f_y}{0.85 f'_c b} = \frac{3480(400)}{0.85(30)(300)} =$ **182 mm** $,\ c = \frac{a}{\beta_1} = \frac{182}{0.85} = 214\ mm^2$

$M_{u1} = \emptyset A_{s1} f_y \left(d-\frac{a}{2}\right) = 0.816(3480)(400)(500-\frac{182}{2})\times10^{-6} =$ **464.6 kN.m**

$M_{u2} = M_u - M_{u1} \rightarrow 528 - 464.6 =$ **63.4 kN.m**

2- Calculate $\rho_{max.} = 0.85\beta_1 \frac{f_c'}{f_y}\left(\frac{\varepsilon_u}{\varepsilon_u+0.004}\right)$

3- Calculate $\rho = \frac{1}{m}\left[1 - \sqrt{1 - \frac{2mR}{f_y}}\right], m = \frac{f_y}{0.85f_c'}, R = \frac{M_u}{\phi b d^2}$

4- If $\rho \le \rho_{max}$. → design as single reinforced concrete section.

If $\rho > \rho_{max}$. → design as doubly reinforced concrete section.

5- Calculate the max. design moment caused by As_{max} as M_{u1}. Consider that $A_{s1} = As_{max} = \rho_{max}.bd$ and $\phi = 0.816$.

Calculate $a = \frac{A_{s1}.f_y}{0.85f_c' b}$ and the moment resisted by part of the tension steel and the concrete compression force as $M_{u1} = \phi M_{n1} = \phi A_{s1} f_y (d - \frac{a}{2})$

6- Calculate moment resisted by the rest of tension steel and the compression steel as:

$M_u = M_{u1} + M_{u2} \rightarrow M_{u2} = M_u - M_{u1}$

7- Calculate the compression steel stress as: $f_s' = 600\left(\frac{c-d'}{c}\right),\ c = \frac{a}{\beta_1}$

8- Calculate the compression steel area as: $A_s' = \frac{M_{u2}}{\phi f_s'(d-d')}$

9- Calculate the tension steel part corresponding to the compression steel as:

$A_{s2} f_y = A_s' f_s' \rightarrow A_{s2} = \frac{A_s' f_s'}{f_y}$

10- Calculate the total tension steel area as: $A_s = A_{s1} + A_{s2}$

$f_s' = 600\left(\frac{c-d'}{c}\right) = 600\left(\frac{214-65}{214}\right) = 418\ \text{MPa} > f_y \rightarrow \therefore\ f_s' = f_y$

$A_s' = \frac{M_{u2}}{\emptyset f_s'(d-d')} = \frac{63.4\times10^6}{0.816(400)(500-65)} = \boxed{447\ \text{mm}^2}$

$A_{s2} = \frac{A_s' f_s'}{f_y} = \frac{447(400)}{400} = 447\ \text{mm}^2$

$A_s = A_{s1} + A_{s2} \rightarrow A_s = 3480+447 = \boxed{3927\ \text{mm}^2}$

Tension steel (bottom steel)

Consider ϕ25 mm diameter bar, $A_b = 491\ \text{mm}^2$

No. of bars = A_s / A_b = 3927/491 = 7.99..say n = 8

Check if two layers is ok,

$b_{min} = 40(2) + 10(2) + 4(25) + 3(25) = 275\ \text{mm} < b_{avl.}$ →OK

Use 8-ϕ25 mm in two layers bottom

Compression steel (top steel)

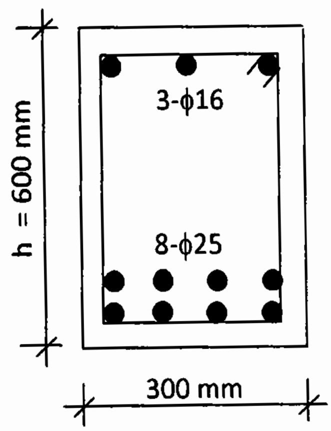
Consider ϕ16 mm diameter bar, $A_b = 201\ \text{mm}^2$

No. of bars = A_s' / A_b = 447/201 = 2.2..say n = 3

Check if one layers is ok,

$b_{min} = 40(2) + 10(2) + 3(16) + 2(16) = 180\ \text{mm} < b_{avl.}$ →OK

Use 3-ϕ16 mm in one layer top



Lecture # 09: Analysis and Design of T-Beams

9.1 Introduction

Casting of reinforced concrete beams together with slabs results in monolithic structure and with the help of stirrups, part of the slab will work with the beam to carry the compression. Therefore, part of the slab will be considered through the analysis and design of rectangular beam. In this case, the beam is called "***T-Beam***". The slab part is called "*flange*" while the bottom rectangular part is called "*web*".

The bending stress in the flange is irregular and it is maximum above the web directly and decreases gradually with the distance from the web. Therefore the ACI-Code specified certain width for the flange called "*effective flange width*" as follows:

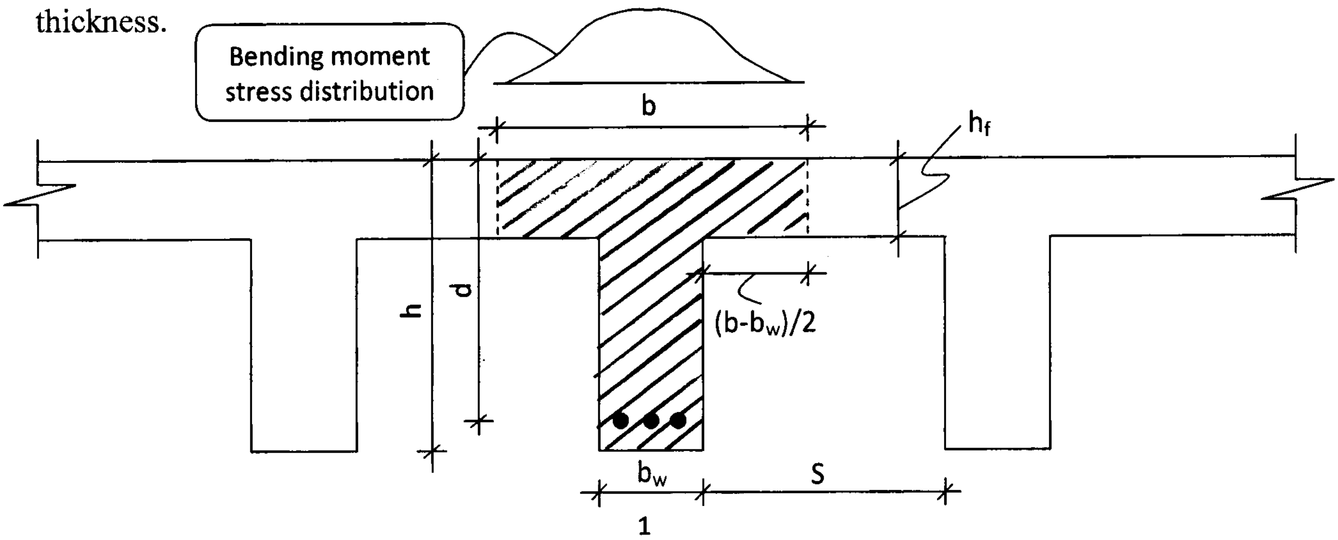
1- For symmetrical beams, i.e., beams connected with slabs from both sides, *the effective flange width*, ***b*** will be not more than:

(a): $b \leq L/4$ .... (9-1)

(b): $(b - b_w)/2 \leq S/2 \rightarrow \therefore b \leq S+b_w$ ....(9-2)

(c): $(b - b_w)/2 \leq 8\ h_f \rightarrow \therefore b \leq 16\ h_f + b_w$ ....(9-3)

Where: L = beam span, b_w = web width, S = clear spacing of beams & h_f = flange thickness.

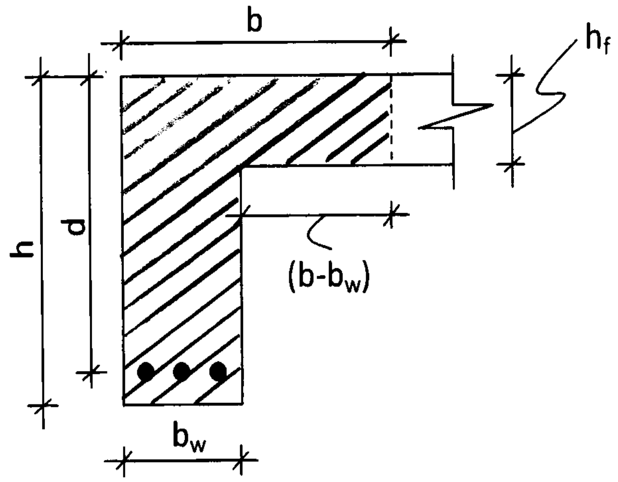


2- Edge beams, i.e., beams connected with slab from one side only (normally called L-beams), *the effective flange width*, ***b*** will be not more than:

(a): $(b - b_w)/2 \leq L/12 \rightarrow b \leq L/12 + b_w$ ....(9-4)

(b): $(b - b_w)/2 \leq 6\ h_f \rightarrow b \leq 6\ h_f + b_w$ ....(9-5)

(c): $(b - b_w)/2 \leq S/2 \rightarrow b \leq b_w + S/2$ ....(9-6)

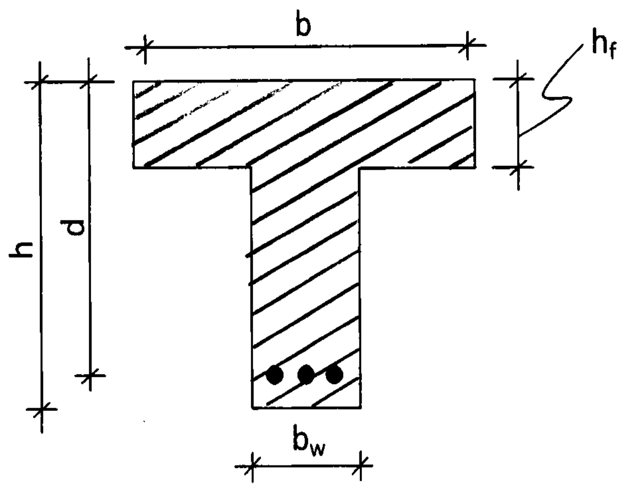


3- Isolated beams in which the flange is used to provide extra compression area. In this case, the *flange thickness*, h_f shall be not less:

(a): $h_f \geq b_w/2$ ....(9-7)

While, *the effective flange width*, ***b*** will be not more than:

(b) $b \leq 4\ b_w$ ....(9-8)



If a > h_f , *T- section analysis* will be required as follows:

- Assume all steel reinforcement yields, ($f_s = f_y$) and divide the steel reinforcement into two parts:

  1- A_{sf} = steel reinforcement to balance the flange compression.

  2- A_{sw} = steel reinforcement to balance the web compression.

The steel reinforcement, A_{sf}, can be determined from equilibrium of forces as:

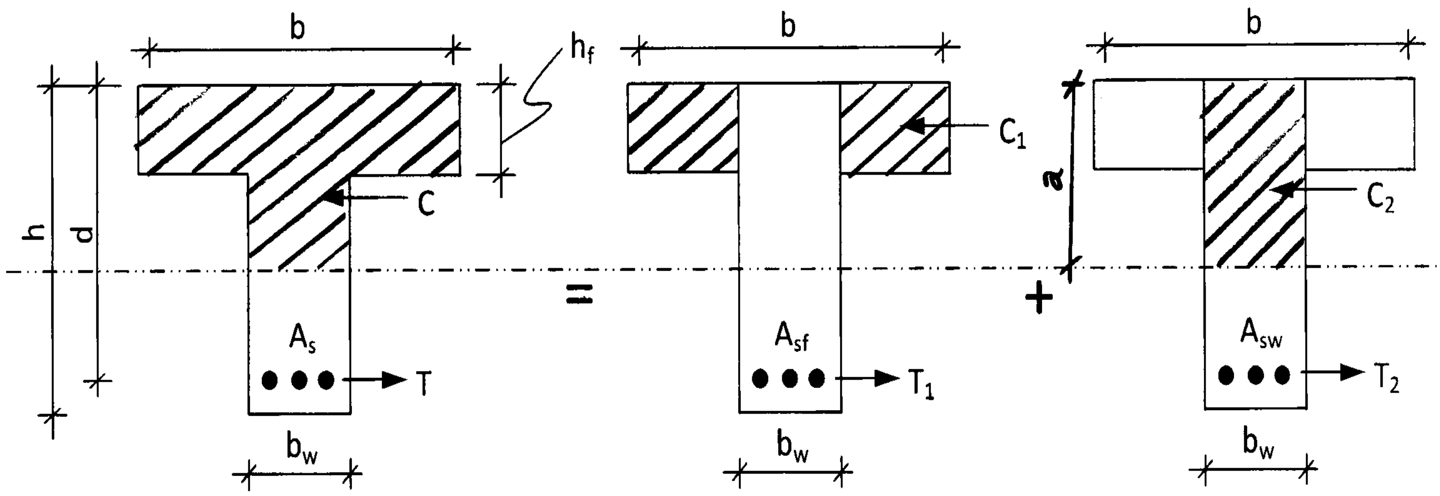
$T_1 = C_1$

$A_{sf} \cdot f_y = 0.85 f'_c (b - b_w) h_f$

$$A_{sf} = \frac{0.85 f'_c (b-b_w) h_f}{f_y} \quad \text{...........(9-11)}$$

The nominal strength developed by the tension force of steel (A_{sf}) and the flange compression force is:

$$M_{n1} = A_{sf} f_y \left(d - \frac{h_f}{2}\right) \text{ or } M_{n1} = 0.85 f'_c (b - b_w) h_f \left(d - \frac{h_f}{2}\right) \quad \text{................(9-12)}$$



$T = A_s f_s$

$C = C_1 + C_2$

$T = C$

$M_n = M_{n1} + M_{n2}$

$T_1 = A_{sf} f_s$

$C_1 = 0.85 f'_c\ h_f (b - b_w)$

$T_1 = C_1$

$M_{n1} = C_1 \left(d - \frac{h_f}{2}\right)$

$M_{n1} = T_1 \left(d - \frac{h_f}{2}\right)$

$T_2 = A_{sw} f_s$

$C_2 = 0.85 f'_c\ a\ b_w$

$T_2 = C_2$

$M_{n2} = C_2 \left(d - \frac{a}{2}\right)$

$M_{n2} = T_2 \left(d - \frac{a}{2}\right)$

While, the tension force of the rest of steel (A_{sw}) will be counteract by the web compression force.

The depth of the web compression area can be calculated from equilibrium of forces as:

$A_{sw} = A_s - A_{sf}$

$T_2 = C_2$

$A_{sw} f_y = 0.85 f'_c\ a\ b_w$

$$a = \frac{A_{sw} \cdot f_y}{0.85 f'_c\ b_w} \quad \text{.........................(9-13)}$$

The nominal strength developed by the tension force of steel (A_{sw}) and the web compression force is:

$$M_{n2} = A_{sw}\ f_y\ (d - \tfrac{a}{2}) \quad \text{or} \quad M_{n2} = 0.85 f'_c\ a\ b_w\ (d - \tfrac{a}{2}) \quad \text{.................(9-14)}$$

The total nominal strength of the section is:

$$M_n = M_{n1} + M_{n2} = A_{sf}\ f_y \left(d - \frac{h_f}{2}\right) + A_{sw}\ f_y\ (d - \tfrac{a}{2}) \quad \text{..........(9-15)}$$

Or from:

$$M_n = M_{n1} + M_{n2} = 0.85 f'_c (b - b_w) h_f \left(d - \frac{h_f}{2}\right) + 0.85 f'_c\ b_w a (d - \tfrac{a}{2}) \quad \text{.......(9-16)}$$

9.2.1 Check of Yielding of Steel Reinforcement

The previous analysis has been conducted based on the assumption that the steel reinforcement reaching its yielding value (f_y). In order to check this assumption, the steel ratio as a percentage of the web area ($b_w d$) has to be compared with the balanced steel ratio in which the concrete strain reaches its ultimate value ($\varepsilon_u = 0.003$) at the same time with yielding of steel reinforcement ($f_s = f_y$). The balanced steel ration is also taken as a percentage of the web area and can be derived as:

$T = C_1 + C_2 \rightarrow A_s f_y = 0.85 f'_c (b - b_w) h_f + 0.85 f'_c\ \beta_1 c\ b_w$ and

$T_1 = C_1 \rightarrow A_{sf} f_y = 0.85 f'_c (b - b_w) h_f$

➢ If $\rho_w \leq \rho_{w(max)}$ → the design is correct ☺.

➢ If $\rho_w > \rho_{w(max)}$ → the section to be enlarged of doubly reinforced to be adopted.

$$\rho_{min} = \frac{\sqrt{f_c'}}{4f_y} \geq \frac{1.4}{f_y}$$

➢ If $\rho_w > \rho_{min}$ → the design is OK ☺.

➢ If $\rho_w < \rho_{min}$ → ρ_{min} to be adopted as $\rho_w \geq \rho_{min}$ should be restricted.

In case of rectangular section (b×d), ρ_w to be calculated based on ($b_w d$) and compared with ρ_{min}.

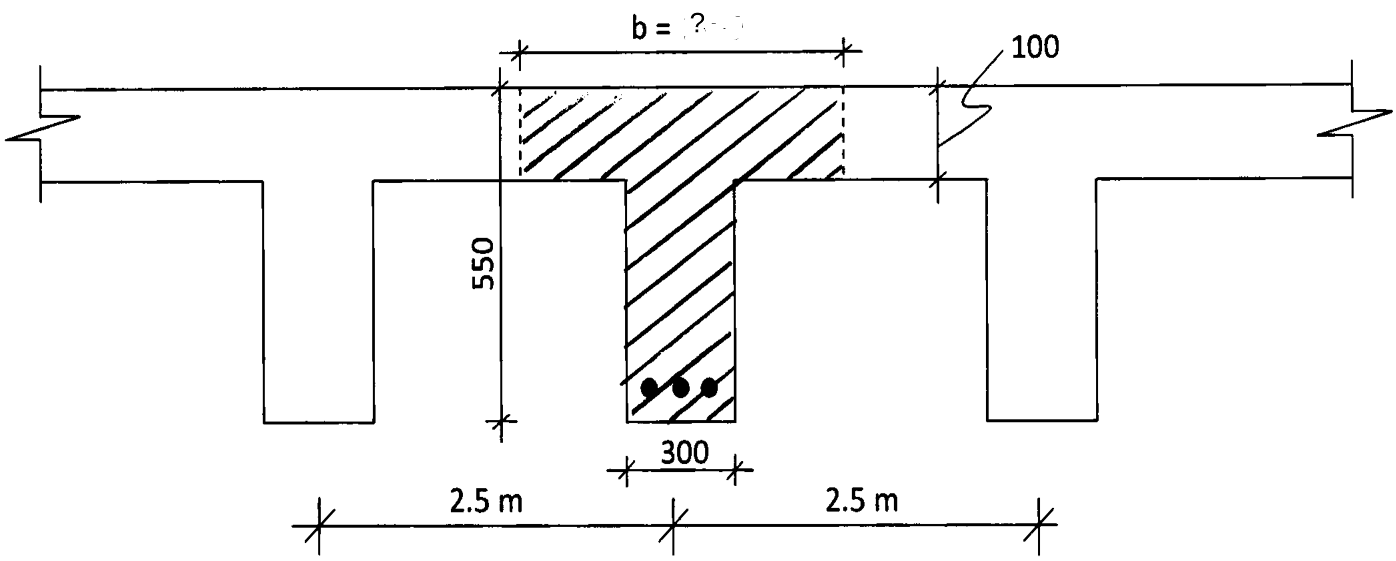
10-Check the assumed value of ϕ by calculating ρ_{wt} and compared with ρ_w.

➢ If $\rho_w \leq \rho_{wt} \rightarrow \phi = 0.9$

➢ If $\rho_w > \rho_{wt} \rightarrow \phi$ to be calculated based on the value of ε_t.

Example (9-2):

The floor shown in the figure below consists of concrete slab of 100 mm supported on cast-in-place beams of span 6 m with center-to-center spacing of 2.5m. If the service dead moment (including selfweight moment) equals to 84 kN.m and the service live moment equals to 163 kN.m, calculate the steel reinforcement required. Consider that $f'_c = 30$ N/mm^2 and $f_y = 400$ N/mm^2.



$A_s f_y = 0.85 f_c' \beta_1 c\, b_w + A_{sf} f_y$

From strains distribution:

$\frac{\varepsilon_u}{\varepsilon_y+\varepsilon_u} = \frac{c_b}{d} \rightarrow \therefore c_b = \frac{\varepsilon_u}{\varepsilon_y+\varepsilon_u} d$, substitute in equilibrium equ. and divide by ($b_w\, d f_y$):

$\frac{A_s}{b_w d} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_y+\varepsilon_u} + \frac{A_{sf}}{b_w d}$, let $\frac{A_s}{b_w d} = \rho_w$ and $\frac{A_{sf}}{b_w d} = \rho_f$ then:

$$\rho_{wb} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_y+\varepsilon_u} + \rho_f$$

Substitute $\varepsilon_u = 0.003$, $\varepsilon_y = f_y / E_s$ and $E_s = 200{,}000$ N/mm^2 results in:

$$\boxed{\rho_{wb} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{600}{600+f_y} + \rho_f = \rho_b + \rho_f} \quad \text{.......................(9-17)}$$

Where:

ρ_{wb} = balanced steel ratio for T-Beam.

ρ_b = balanced steel ratio for the rectangular part.

ρ_f = steel ratio balanced to flange compression force.

ρ_w = steel ratio for T-section, $\rho_w = (A_s / b_w d)$

All the above ratios will be based on the web area ($b_w d$).

If $\rho_w \le \rho_{wb} \rightarrow f_s = f_y \rightarrow$ the above analysis is correct.

If $\rho_w > \rho_{wb} \rightarrow f_s < f_y \rightarrow$ the above analysis cannot be adopted because both **a** and f_s are unknowns.

For the analysis, f_s has to be written in terms of **c** as:

From strain distributions:

$$\frac{\varepsilon_s}{d-c} = \frac{\varepsilon_u}{c} \rightarrow \therefore \varepsilon_s = \frac{d-c}{c} \varepsilon_u$$

$$f_s = E_s \varepsilon_s = E_s \frac{d-c}{c} \varepsilon_u \rightarrow \boxed{f_s = 600\left(\frac{d-c}{c}\right)} \quad \text{..........................(9-18)}$$

After substitution in equilibrium equation: $A_s f_s = 0.85 f_c' (b - b_w) h_f + 0.85 f_c'\, a\, b_w$

Results in:

$$A_s 600\frac{d-c}{c} = 0.85f_c' \beta_1 c\, b_w + 0.85f_c'(b - b_w)h_f \text{ ....(9-19)}$$

Solving for "*c*" and then "*a*" can be calculated as $a = \beta_1 c$

The nominal bending strength will be:

$$\boxed{M_n = 0.85f_c'(b - b_w)h_f\left(d - \frac{h_f}{2}\right) + 0.85f_c' b_w a(d - \frac{a}{2})} \text{ ....(9-20)}$$

In order to ensure tension failure, the ACI-Code requires the strain of the far steel bar to be not less than 0.004. Maximum tension steel ratio for T-section can be derived same manner as the derivation of ρ_{wb} by replacing ε_y by 0.004 results in:

$$\boxed{\rho_{w(\max)} = 0.85\beta_1 \frac{f_c'}{f_y}\frac{\varepsilon_u}{\varepsilon_u + 0.004} + \rho_f = \rho_{max} + \rho_f} \text{ ....(9-21)}$$

Where:

$\rho_{w\,(max)}$ = maximum steel ratio for T-Beam.

ρ_{max} = maximum steel ratio for the rectangular part.

ρ_f = steel ratio balanced to flange compression force.

9.2.2 Strength Reduction factor Calculation (ϕ)

Based on ACI-Code requirements, strength reduction factor (ϕ) is calculated same as for singly reinforced rectangular sections and can be derived as $\varepsilon_t = 0.005$ ($\phi = 0.9$):

$$\rho_{wt} = 0.85\beta_1 \frac{f_c'}{f_y}\frac{\varepsilon_u}{\varepsilon_u + 0.005} + \rho_f = \rho_t + \rho_f \text{ ....(9-21)}$$

Where:

$\rho_{w\,t}$ = steel ratio that cause strain of 0.005 for T-Beam.

ρ_t = maximum steel ratio for the rectangular part.

ρ_f = steel ratio balanced to flange compression force.

Example (9-1):

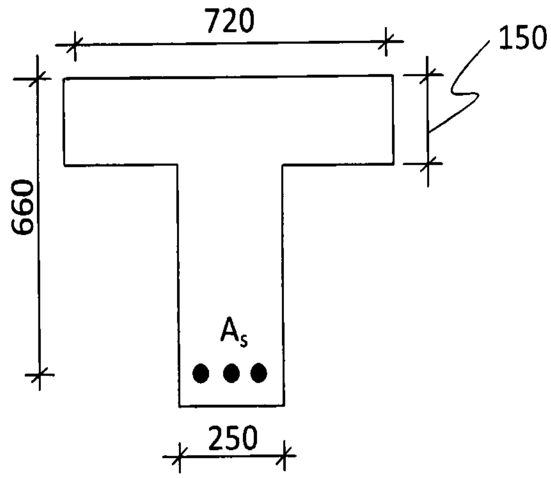
Calculate the nominal bending strength for the isolated T-section given below if the steel reinforcement provided was:

(a) : $A_s = 3000\ mm^2$.

(b) : $A_s = 4800\ mm^2$.

(c) : $A_s = 7000\ mm^2$

Consider that $f'_c = 20\ N/mm^2$ and $f_y = 400\ N/mm^2$.



Solution:

- Check AC-Code requirements for isolated T-section:

$h_f = 150$ mm $\geq b_w/2 = 250/2 = 125$ mm ....OK ☺

$b = 720$ mm $\leq 4\ b_w = 4\ (250) = 1000$ mm ....OK ☺

(a): $A_s = 3000\ mm^2$

Calculate the depth of the equivalent rectangular stress block as:

$a = \frac{A_s f_y}{0.85 f'_c\, b} = \frac{3000\,(400)}{0.85\,(20)(720)} = 98$ mm $< h_f = 150$ mm → rectangular section with size of $(b \times d) = (720 \times 660)$ mm

$$\rho_b = 0.85\beta_1 \frac{f'_c}{f_y} . \frac{600}{600+f_y} = 0.85\,(0.85)\frac{20}{400} . \frac{600}{600+400} = 0.0217$$

$\rho = \frac{A_s}{bd} = \frac{3000}{720\,(660)} = 0.0063 < \rho_b = 0.0217 \Rightarrow$ the section is under-reinforced.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 3000\,(400)\left(660 - \frac{98}{2}\right) * 10^{-6} = \boxed{733.2\ kN.m}$$ ☺

(b): $A_s = 4800\ mm^2$

Calculate the depth of the equivalent rectangular stress block as:

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{4800\,(400)}{0.85\,(20)(720)} = 157\text{ mm} > h_f = 150\text{ mm} \rightarrow \text{T-section analysis}$$

$$A_{sf} = \frac{0.85 f_c'(b-b_w)h_f}{f_y} = \frac{0.85(20)(720-250)150}{400} = 2996\text{ mm}^2$$

From branch (a), $\rho_b = 0.0217$, $\rho_f = \frac{A_{sf}}{b_w d} = \frac{2996}{250(660)} = 0.0182$

$\rho_{wb} = \rho_b + \rho_f = 0.0217 + 0.0182 = 0.04$

$\rho_w = \frac{A_s}{b_w d} = \frac{4800}{250\,(660)} = 0.029 < \rho_{wb} = 0.04$ .... under-reinforced section ($f_s = f_y$)

$A_{sw} = A_s - A_{sf} = 4800 - 2996 = 1804\text{ mm}^2$

$$a = \frac{A_{sw} f_y}{0.85 f_c' b_w} = \frac{1804\,(400)}{0.85\,(20)(250)} = 169.8\text{ mm}$$

$$M_n = M_{n1} + M_{n2} = A_{sf}\, f_y \left(d - \frac{h_f}{2}\right) + A_{sw}\, f_y \left(d - \frac{a}{2}\right)$$

$$M_n = 2996(400)\left(660 - \frac{150}{2}\right) + 1804(400)\left(660 - \frac{169.8}{2}\right) \times 10^{-6} = \boxed{1116\text{ kN.m}}$$ ☺

(b): $A_s = 7000\ mm^2$

Since the provided steel is greater than what provided in branch (b)→ a > 150 mm

From branch (b), $\rho_{wb} = 0.04$, $\rho_w = \frac{A_s}{b_w d} = \frac{7000}{250\,(660)} = 0.042 > \rho_{wb}$

∴ Over-reinforced section ($f_s < f_y$), using equ. (9-19):

$$A_s 600 \frac{d-c}{c} = 0.85 f_c' \beta_1 c\, b_w + 0.85 f_c'(b - b_w)h_f$$

$$(7000)600 \frac{660-c}{c} = 0.85(20)(250)(0.85)c + 0.85(20)(720 - 250)150$$

Solving for c = 404 mm, $a = \beta_1 c = 0.85\,(404) = 343$ mm

Solution:

- Calculate the effective flange width "b" (interior T-beam).

$$B \le \begin{cases} L/4 = 6000/4 = \boxed{1500\text{ mm}}\ \textit{control} \\ S+b_w = 2500-300+300 = 2500\text{ mm} \\ 16\,h_f + b_w = 16(100)+300 = 1900\text{ mm} \end{cases}$$

- Calculate the design bending moment M_u.

$M_u = 1.2\,M_D + 1.6\,M_L \rightarrow 1.2\,(84) + 1.6\,(163) = \boxed{361.6\text{ kN.m}}$

- Assume $f_s = f_y$ and $\phi = 0.9$, Let $a = h_f = 100$ mm

Assume 2 layers of reinforcement, $d = h - 100 = 550 - 100 = 450$ mm

$M_{uf} = \phi\,[c\,(d - \frac{h_f}{2})] \rightarrow 0.9\,[0.85\,f_c'\,b\,h_f\,(d - \frac{h_f}{2})]$

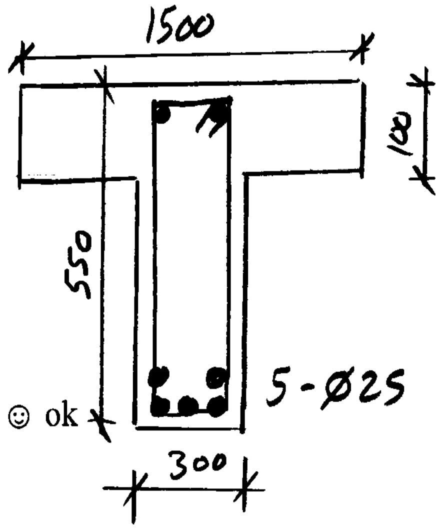
$= 0.9\,[0.85\times30\times1500\times100\,(450 - \frac{100}{2})\times10^{-6} = \boxed{1377\text{ kN.m}}$

Since $M_{uf} > M_u \rightarrow$ section to be designed as rectangular with $(b\times d) = (1500\times450)$ mm

- Calculate the required steel reinforcement

$R = \frac{M_u}{\phi b d^2} = \frac{361.6\times10^6}{0.9(1500)(450^2)} = 1.323$ and $m = \frac{f_y}{0.85\,f_c'} = \frac{400}{0.85\,(30)} = 15.68$

$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{f_y}}\right) = \frac{1}{15.68}\left(1 - \sqrt{1 - \frac{2(15.68)(1.323)}{400}}\right) = 0.003404$



$A_s = \rho\times b\times d = 0.003404\times1500\times450 = \boxed{2298\text{ mm}^2}$

$\rho_{max} = 0.85\beta_1\frac{f_c'}{f_y}\left(\frac{\varepsilon_u}{\varepsilon_u+0.004}\right) = 0.85(0.85)\frac{30}{400}\left(\frac{0.003}{0.003+0.004}\right) = 0.0232$

Since $\rho < \rho_{max} \rightarrow f_s = f_y$ ☺

$\rho_{min} = \frac{1.4}{f_y} = \frac{1.4}{400} = 0.0035$, $\rho_w = \frac{A_s}{b_w d} = \frac{2298}{300(450)} = 0.01702 > \rho_{min}$ ….. ☺ ok

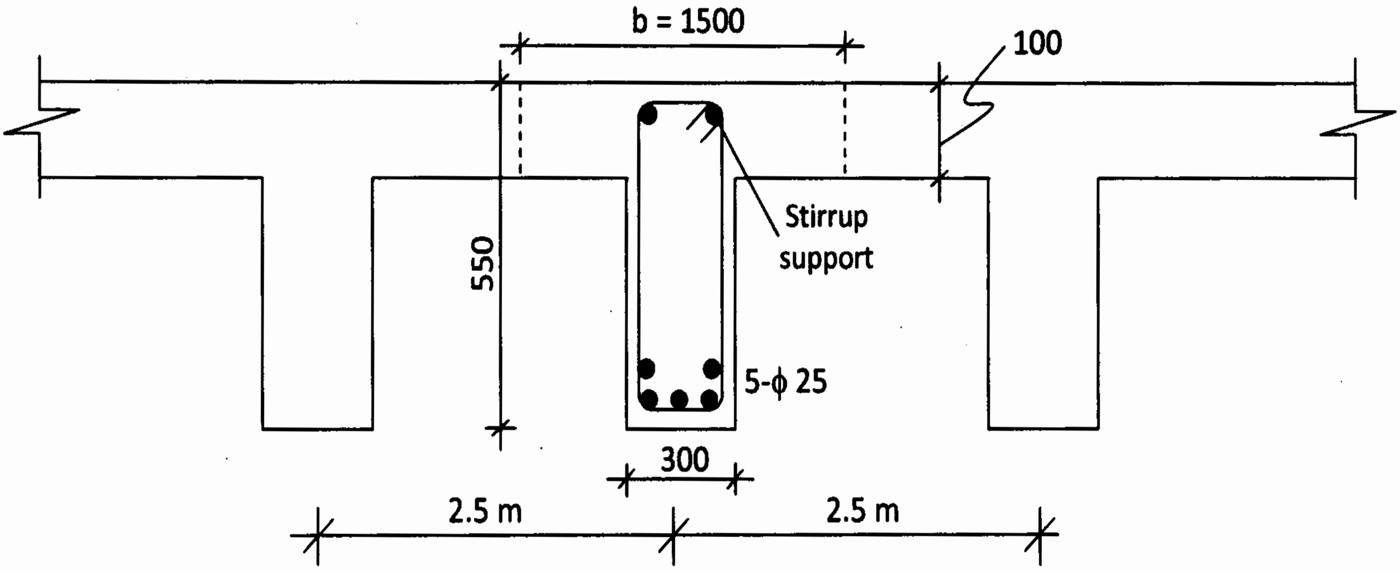
- Check the assumed value of ϕ

- $\rho_t = 0.85\beta_1\frac{f_c'}{f_y}\left(\frac{\varepsilon_u}{\varepsilon_u+0.005}\right) = 0.85(0.85)\frac{30}{400}\left(\frac{0.003}{0.003+0.005}\right) = 0.0203$

Since $\rho = 0.003404 < \rho_t$ ……. $\phi = 0.9$ …. ☺ ok

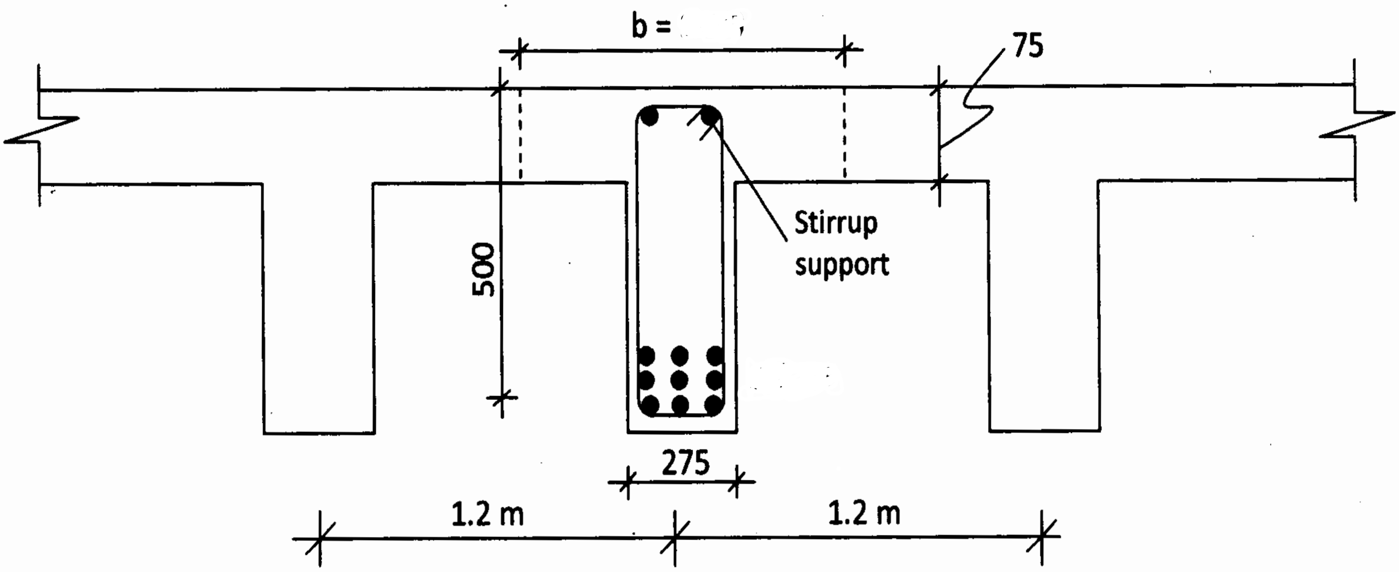
Use ϕ 25 mm diameter steel bar, $A_b = 491\ mm^2$

$$n = \frac{A_s}{A_b} = \frac{2298}{491} = 4.7 \rightarrow \text{Use } 5\text{-}\phi\ 25 \text{ mm in 2-layers}$$



Example (9-3):

The floor shown in the figure below consists of concrete slab of 75 mm supported on cast-in-place simply supported beams of span 7.2 m with center-to-center spacing of 1.2 m. If the total ultimate design moment including self weight equals to710 kN.m. Calculate the steel reinforcement required. Consider that $f'_c = 20\ N/mm^2$ and $f_y = 400\ N/mm^2$.



Solution:

- Calculate the effective flange width "b" (interior T-beam).

$$b \le \begin{cases} L/4 = 7200/4 = 1800 \text{ mm} \\ S+b_w = 1200 - 275 + 275 = \boxed{1200 \text{ mm}} \; \textbf{\textit{control}} \checkmark \\ 16\,h_f + b_w = 16(75) + 275 = 1475 \text{ mm} \end{cases}$$

- Assume $f_s = f_y$ and $\phi = 0.9$, Let $a = h_f = 75$ mm

$$M_{uf} = \phi\,[c\,(d - \frac{h_f}{2})] \rightarrow 0.9\,[0.85\,f_c'\,b\,h_f\,(d - \frac{h_f}{2})]$$

$$= 0.9\,[0.85 \times 20 \times 1200 \times 75\,(500 - \frac{75}{2}) \times 10^{-6} = \boxed{636.9 \text{ kN.m}}$$

Since $M_{uf} < M_u \rightarrow$ section to be designed as T-section.

- From equilibrium of forces, calculate the steel area that counteract the flange compression as:

$$A_{sf} = \frac{0.85 f_c'(b - b_w)h_f}{f_y} = \frac{0.85 \times 20(1200 - 275)75}{400} = 2948 \text{ mm}^2$$

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- Calculate the design moment results from both the compression flange and its equivalent steel area as:

$$M_{u1} = \phi A_{sf}\,f_y\left(d - \frac{h_f}{2}\right) = 0.9(2948)\,(400)\,\left(500 - \frac{75}{2}\right) \times 10^{-6} = 490.8 \text{ kN.m}$$

- Calculate the moment developed by the steel area that to be added to counteract the web compression as:

$$M_{u2} = M_u - M_{u1} \rightarrow M_{u2} = 710 - 490.8 = 219.2 \text{ kN.m}$$

- Calculate the steel area required to balance the web compression as:

$$m = \frac{f_y}{0.85\,f_c'} = \frac{400}{0.85\,(20)} = 23.53, \quad R = \frac{M_{u2}}{\phi b_w d^2} = \frac{219.2 \times 10^6}{0.9\,(275)500^2} = 3.54$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{f_y}}\right) = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2(23.53)3.54}{400}}\right) = 0.01$$

$A_{sw} = \rho \times b_w \times d = 0.01 \times 275 \times 500 = 1375\ mm^2$

- Calculate the total steel area as:

  $A_s = A_{sf} + A_{sw} \rightarrow 2948 + 1375 = 4323\ mm^2$

- Check the calculated steel ratio to be within the ACI-Code limits as:

$$\rho_w = \frac{A_s}{b_w d} = \frac{4323}{(275)(500)} = 0.0314,\ \ \rho_f = \frac{A_{sf}}{b_w d} = \frac{2948}{(275)(500)} = 0.0214$$

$$\rho_{max} = 0.85\beta_1 \frac{f_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + 0.004} = 0.85\ (0.85) \frac{20}{400} \cdot \frac{0.003}{0.003+0.004} = 0.0155$$

$\rho_{w(max)} = \rho_{max} + \rho_f \rightarrow 0.0155 + 0.0214 = 0.0369$

Since $\rho_w < \rho_{w(max)}$ → the design is correct ☺.

$$\rho_{min} = \frac{\sqrt{f_c'}}{4 f_y} \geq \frac{1.4}{f_y} = \frac{\sqrt{20}}{4(4000} \geq \frac{1.4}{400} = 0.00279 \geq \boxed{0.0035}\ \surd$$

Since $\rho_w > \rho_{min}$ → the design is OK ☺.

- Check the assumed value of ϕ by calculating ρ_{wt} and compared with ρ_w.

$$\rho_t = 0.85\beta_1 \frac{f_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + 0.005} = 0.85\ (0.85) \frac{20}{400} \cdot \frac{0.003}{0.003+0.005} = 0.0135$$

$\rho_{wt} = \rho_t + \rho_f \rightarrow 0.0135 + 0.0214 = 0.035$

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Since $\rho_w < \rho_{wt} \rightarrow \phi = 0.9$ OK ☺√

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Or

$$a = \frac{A_{sw} f_y}{0.85 f_c' b} = \frac{1375\ (400)}{0.85\ (20)(275)} = \ 118\ mm,\ c = a/\beta_1 = 118/0.85 = 139\ mm$$

$$\varepsilon_t = \varepsilon_u \left(\frac{d-c}{c}\right) = 0.003 \left(\frac{500-139}{139}\right) = 0.0078 > 0.005 \rightarrow \phi = 0.9\ \text{ OK ☺√}$$

Use ϕ 25 mm diameter steel bar, $A_b = 491\ mm^2$

$$n = \frac{A_s}{A_b} = \frac{4323}{491} = 8.8 \rightarrow \text{Use 9-}\phi\text{ 25 mm in 3-layers}$$

Draw the results here