A Comparison of the Estimation Methods for the Non-Linear Parameter of First Order Moving Average Model

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Abstract

A lot of researchers face a difficulty in estimating parameter of moving average model of non-linear parameter. For that reason, we can’t use the traditional methods of estimation.

This research aims to compare four methods in estimating parameter of first order moving average model by using mean absolute percentage error as a criterion comparison with consideration (sample sizes, different initial value parameter, and different distributions of random errors of the sample),

The second method has shown to be the best among other methods of sample sizes, except where the initial value parameter of the model is 0.13 and here the third method can be the best for geometric distribution and for all sample sizes.

Keywords: Moving Average, Gaussian, Non-Gaussian, Non-linear, Least Square, Newton-Raphson, Iterative Process, Maximum Likelihood, Simulation.

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1. Introduction:

The phenomena are exposed to continuous of changes variables and analyzing time series as statistical tool used to study the phenomena development in relation to time, time series is a number of statistical observations to describe the change of the phenomena in relation to time.

We can analyze time series by knowing the phenomenal development in relation to time, and predicting parameter through the following periods to design and make futuristic programs.

the researchers (Box & Jenkins[1] :1968-1070), Suggested time series models and put mathematical formula to build them, Among researches dealt with different methods of estimating parameter moving average model, In 1996, Al-khudhairy[2] , suggested a method of estimating parameter of moving average model based on Newton-Raphson procedure, In 2001, Al-khudhairy[3] , used iterative process method to estimate the parameter of Non-Gaussian seasonal first order moving average model by simulation, In 2011, the researchers, Mohammed Al-khudhairy[4] & Wadhah S. I., studied Non-Gaussian second order moving average, In 2016, Mohammed [5] has suggested a method of estimating the parameter of Non-Gaussian moving average.

This research aims at comparing four methods to estimate the parameter of first order moving averages by using MAPE criterion to compare different sample sizes different initial values of parameter, assuming that the random errors of the model followed different distributions by using simulation.

2. Contribution/Originality

Researchers face difficulty in estimating the parameters of a moving average model and in specifying the appropriate method for this estimation due to the nonlinear nature of the model. The present study examines the possible methods to suggest one based on Newton-Raphson procedure. These methods will be compared in terms of sample size, initial values of the parameter, the positive/negative reference of these values, and the type of distribution random errors of the model. Simulation procedure is used.

3. Estimation of model's parameter [2,3,6] :

The general formula of first order moving average is written in this way: $X_t = a_t - \theta_1 a_{t-1}$, many researchers face difficulties in estimating the parameter of moving average model of non-linear parameter, for that reason, use the traditional methods of estimation.

a- Exact Maximum Likelihood Method (EML) [7,8] :

The concept of this method is to find the parameter of function value in its maximum, it is possible to derive the value as shown here:

$$L = (2\pi\sigma_a^2)^{-n/2}\left|M^{(0,1)}\right|^{1/2}.\, e^{-S(\theta)/2\sigma_a^2} \quad \text{... (1)}$$

$$M^{(0,1)} = (\gamma_0^{-1}) = (\sigma_a^2/(1-\theta_1^2))^{-1}.\,\sigma_a^2 = (1-\theta_1^2) \quad \text{... (2)}$$

$$S(\theta) = \sum_{t=-\infty}^{n}(a_t|X_t,\theta_1)^2 = \sum_{t=-\infty}^{n}(a_t)^2 \quad \text{... (3)}$$

The estimated value of the random error variance is calculated according to the following formula:

$\hat{\sigma}_a^2 = \frac{S(\hat{\theta})}{n} = (\sum_{t=-\infty}^{n}(a_t)^2)/n$ Then

$$\ln L = -\frac{n}{2}\ln(2\pi\sigma_a^2) + \frac{1}{2}\ln((1-\theta_1^2)) - \left(\sum_{t=-\infty}^{n}(a_t)^2\right)/2\sigma_a^2 \quad \text{... (4)}$$

Since that can be written at in the form of the following equation:

$$a_t = (1-\theta_1 B)^{-1}X_t = X_t/(1-\theta_1 B) \quad \text{... (5)}$$

It can be traced to its origin $(1/(1-\theta_1 B))$ amount being rounded to the following infinite Geometric Sequential:

$$a_t = (1 + \theta_1 B + \theta_1^2 B^2 + \theta_1^3 B^3 + \cdots)X_t$$

$$\therefore a_t = \sum_{j=0}^{\infty}\theta_1^j X_{t-j} \quad \text{... (6)}$$

Compensating for the value of (a_t) in the equation (4), we get:

$$\ln L = -\frac{n}{2}\ln(2\pi\sigma_a^2) + \frac{1}{2}\ln\left((1-\theta_1^2)\right) - \left(\frac{1}{2\sigma_a^2}\right)\sum_{t=-\infty}^{n}\left(\sum_{j=0}^{\infty}\theta_1^j X_{t-j}\right)^2$$

$$\frac{\partial \ln L}{\partial \theta_1} = -\frac{\theta_1}{1-\theta_1^2} - \frac{\sum_{t=-\infty}^{n}\acute{S}(\theta)}{2\sigma_a^2} \quad \text{... (7)}$$

And $\acute{S}(\theta) = 2\sum_{j=0}^{\infty}\theta_1^j X_{t-j} * \sum_{j=1}^{\infty}\theta_1^{j-1}X_{t-j}$

If we equal derivative to zero, we get: $\hat{\theta}_1 = \frac{1}{\hat{\sigma}_a^2}\left(\hat{\theta}_1^2\sum_{t=-\infty}^{n}\acute{S}(\hat{\theta}_1) - \sum_{t=-\infty}^{n}\acute{S}(\hat{\theta}_1)\right)$, The equation cannot be solved unless we use the replicating method by assuming an initial value to the parameter value (θ_1) to get the new estimating value (we can use ordinary least square estimator) as initial value, Then:

$$\hat{\theta}_1 = \frac{1}{\hat{\sigma}_a^2}\left(\hat{\theta}_{1,0}^2\sum_{t=-\infty}^{n}\acute{S}(\hat{\theta}_{1,0}) - \sum_{t=-\infty}^{n}\acute{S}(\hat{\theta}_{1,0})\right) \quad \text{... (8)}$$

b- Non-Linear Least Square Method (NLS) [4,9] :

We used linearization procedure which we can summarization by simplifying the above model as the first two nominal of Taylor series, in order to make it linear around primary values of parameter, we use least square estimations of parameter. This process is repeated until estimation values being stabilized.

To applicant Taylor series, we derive it in relation to (θ_1) parameter and find it in $(\theta_{1,0})$ primary value.

$\therefore\ x_{1,t} = \frac{-\partial a_t}{\partial \theta_1} |_{\theta_1=\theta_{1,0}} = \frac{-BX_t}{(1-\theta_{1,0}B)^2}$ Where $x_i = X_i - \bar{X}$ are the deviation values and not original values.

$x_{1,t}(1-\theta_{1,0}B)^2 = -BX_t$, And then $x_{1,t}\left(1-2\theta_{1,0}B+\theta_{1,0}^2B^2\right) = -BX_t$

$$x_{1,t} = 2\theta_{1,0}x_{1,t-1} - \theta_{1,0}^2 x_{1,t-2} - x_{t-1} \quad ,t = 1,2,\dots,n \qquad \dots(9)$$

Assuming that the previous values a's, X's equal to Non-conditional expected value and equal to zero:

$X_j] = 0\ ,j = 0,1,\dots$ And $[a_j] = 0\ ,j = 0,1,\dots$, $(a_{t,0})$ value, we can use it as the following:

$a_t = a_{t,0} - \sum_{i=1}^{q}(\theta_1-\theta_{1,0})\,x_{i,t}$ Where $[a_{t,0}] = [a_t/x_t,\theta_{1,0}]$

And $[a_{t,0}] = \sum_{i=1}^{q}(\theta_1-\theta_{1,0})\,x_{i,t} + (a_t)$

$$\therefore\ a_{t,0} \doteq (\theta_1-\theta_{1,0})x_{1,t} + a_{t,0} \qquad \dots(10)$$

The equation (10) is a linear regression and to extract (θ_1) to apply the ordinary least square method according to the following:

$$\theta_1 \doteq \theta_{1,0} + \frac{\sum_{t=0}^{n} a_{t,0}x_{i,t}}{\sum_{t=0}^{n} x_{1,t}^2} \qquad \dots(11)$$

Later, we use (θ_1) value by extruding new primary value is stabilize the process is until we repeated (θ_1) value.

c- Newton-Raphson Method [2,3,5] :

The researcher suggest this method by depending on non-linear procedure which is Newton-Raphson procedure to estimate the model parameter, the equation (5):

$$\sum_{t=1}^{n} a_t^2 = \sum_{t=1}^{n}[(1-\theta_1B)^{-1}X_t]^2 \qquad \dots(12)$$

$$\frac{\partial \sum_{t=1}^{n} a_t^2}{\partial \theta_{1,0}} = 2\sum_{t=1}^{n}\left[(1-\theta_{1,0}B)^{-1}X_t\right]\left[-1.(1-\theta_{1,0}B)^{-2}(-B)X_t\right] \qquad \dots(13)$$

Let $F(X) = \sum_{t=1}^{n}\frac{X_tBX_t}{(1-\theta_{1,0}B)^3} = 0 \qquad \dots(14)$

$\frac{\partial F(X)}{\partial \theta_{1,0}} = \sum_{t=1}^{n}\frac{-X_tBX_t(3)(1-\theta_{1,0}B)^2(-B)}{(1-\theta_{1,0}B)^6}$ then:

$$F'(X) = \sum_{t=1}^{n} \frac{X_{t-1}^2}{(1-\theta_{1,0}B)^4} = 0 \qquad ...(15)$$

$$\theta_1 = \theta_{1,0} - \left[\left(\sum_{t=1}^{n} \frac{X_t X_{t-1}}{(1-\theta_{1,0}B)^3}\right) / \left(\sum_{t=1}^{n} \frac{X_{t-1}^2}{(1-\theta_{1,0}B)^4}\right) \right] \qquad ...(16)$$

$$\theta_1 = 2\theta_{1,0} - \left(\sum_{t=1}^{n} X_t X_{t-1} / \sum_{t=1}^{n} X_{t-1}^2\right) \qquad ...(17)$$

($\theta_{1,0}$) is a primary value assumed by the researcher, estimating it as one of the estimating methods as moment method.

d- Iterative Process Method [4,6,8] :

To estimate (θ_1) parameter, we write it as follows:

$$\gamma_0 = (1+\theta_1^2)\sigma_a^2 \quad \text{and} \quad \gamma_1 = -\theta_1\sigma_a^2$$

We estimate ($\hat{\sigma}_a^2$) according to the following formula:

$$\hat{\sigma}_a^2 = C_0/(1-\theta_1^2) \qquad ...(18)$$

After substituting the initial value of the equation (18) that equal zero of (θ_1) parameter $\hat{\sigma}_a^2 = C_0$.

Then are estimate the (θ_1) parameter according to the following formula:

$$\theta_1 = -[C_1/\hat{\sigma}_a^2] \qquad ...(19)$$

Where: $C_k = (1/n)\sum_{t=1}^{n} X_t X_{t-k} \quad , k = 0,1$ ... (20)

The process must be repeated until the stabilization level of (θ_1) parameter value is reached.

Table (1): MAPE value's when a_t is distribution Binomial distribution.

	-0.6				-0.13			
n	ML	NL	NR	IP	ML	NL	NR	IP
30	0.863684	0.132223	1.97307	0.549401	0.1855	0.166119	0.79823	0.434963
75	0.90216	0.099175	1.99597	0.552559	0.190046	0.107434	0.84035	0.455873
125	0.891599	0.047057	1.99442	0.553887	0.194129	0.040428	0.86281	0.468646
200	0.903584	0.054102	2.00479	0.555615	0.196095	0.062261	0.87753	0.477131

	0.13				0.6			
n	ML	NL	NR	IP	ML	NL	NR	IP
30	0.19511	0.20773	0.156355	0.29675	0.82846	0.177561	1.54347	0.3004
75	0.198	0.09339	0.135404	0.33486	0.88581	0.096753	1.47068	0.24837
125	0.19352	0.08255	0.08606	0.30909	0.89708	0.082296	1.49071	0.27119
200	0.19252	0.05383	0.080582	0.30292	0.8921	0.072089	1.49128	0.26691

Table (2): MAPE value's when a_t is distribution Geometric distribution.

	-0.6				-0.13			
n	**ML**	**NL**	**NR**	**IP**	**ML**	**NL**	**NR**	**IP**
30	**1.71682**	**0.14415**	**1.88786**	**0.50735**	**1.08213**	**0.16614**	**0.74914**	**0.40225**
75	**1.79504**	**0.18648**	**1.92084**	**0.52258**	**1.14333**	**0.14573**	**0.77379**	**0.42184**
125	**1.80623**	**0.11628**	**1.93894**	**0.52920**	**1.15787**	**0.19347**	**0.76895**	**0.41152**
200	**1.84068**	**0.18594**	**1.96192**	**0.53868**	**1.16904**	**0.15829**	**0.78984**	**0.43007**

	0.13				0.6			
n	**ML**	**NL**	**NR**	**IP**	**ML**	**NL**	**NR**	**IP**
30	**0.72755**	**0.26666**	**0.19725**	**0.20067**	**0.14463**	**0.18708**	**1.52682**	**0.28296**
75	**0.78473**	**0.13606**	**0.12279**	**0.19747**	**0.14792**	**0.12782**	**1.52296**	**0.28666**
125	**0.79553**	**0.16243**	**0.05461**	**0.24542**	**0.16058**	**0.15756**	**1.52143**	**0.28947**
200	**0.80645**	**0.12294**	**0.04172**	**0.22141**	**0.15638**	**0.14672**	**1.52230**	**0.29076**

Table (3): MAPE value's when a_t is distribution Normal distribution.

	-0.6				-0.13			
n	**ML**	**NL**	**NR**	**IP**	**ML**	**NL**	**NR**	**IP**
30	**1.76402**	**0.12842**	**1.59016**	**0.33974**	**1.16298**	**0.18650**	**0.39907**	**0.21156**
75	**1.68566**	**0.09011**	**1.66176**	**0.38885**	**1.08482**	**0.08495**	**0.36607**	**0.11275**
125	**1.61819**	**0.09899**	**1.64250**	**0.37521**	**1.13440**	**0.04978**	**0.38083**	**0.11791**
200	**1.60531**	**0.07020**	**1.65451**	**0.38550**	**1.11761**	**0.06365**	**0.39615**	**0.13179**

	0.13				0.6			
n	**ML**	**NL**	**NR**	**IP**	**ML**	**NL**	**NR**	**IP**
30	**0.88418**	**0.15609**	**0.35864**	**0.17581**	**0.51622**	**0.19591**	**1.66909**	**0.38672**
75	**0.88231**	**0.05785**	**0.33912**	**0.08052**	**0.44044**	**0.05940**	**1.63276**	**0.37191**
125	**0.87200**	**0.06648**	**0.33744**	**0.07989**	**0.40162**	**0.10390**	**1.61429**	**0.35771**
200	**0.87073**	**0.06018**	**0.43944**	**0.17349**	**0.42630**	**0.03296**	**1.62554**	**0.36680**

Table (4): MAPE value's when a_t is distribution Cauchy distribution.

	-0.6				-0.13			
n	**ML**	**NL**	**NR**	**IP**	**ML**	**NL**	**NR**	**IP**
30	**1.60115**	**0.10556**	**1.63223**	**0.36345**	**1.14677**	**0.10920**	**0.38360**	**0.13943**
75	**1.58869**	**0.04422**	**1.61903**	**0.36268**	**1.13834**	**0.02130**	**0.37076**	**0.11799**
125	**1.60002**	**0.01790**	**1.63026**	**0.37016**	**1.13647**	**0.00958**	**0.39108**	**0.12886**
200	**1.59688**	**0.02034**	**1.64640**	**0.38038**	**1.13388**	**0.01668**	**0.39676**	**0.13384**

	0.13				0.6			
n	**ML**	**NL**	**NR**	**IP**	**ML**	**NL**	**NR**	**IP**
30	**0.88040**	**0.18862**	**0.25883**	**0.11172**	**0.45582**	**0.31534**	**1.60317**	**0.36549**
75	**0.88593**	**0.04361**	**0.40163**	**0.13742**	**0.39269**	**0.08666**	**1.58752**	**0.33607**
125	**0.87968**	**0.04309**	**0.41922**	**0.15439**	**0.42111**	**0.02330**	**1.64357**	**0.37839**
200	**0.87084**	**0.02033**	**0.38282**	**0.12070**	**0.41225**	**0.01875**	**1.64958**	**0.38251**

4- Discussion:

The researcher generate data under the proposition that the model's random errors distribute continuously and discretely. The parameter of the non-linear model was estimated by four different methods; one of them was proposed by the researchers based on Newton-Raphson procedure. These methods were compared by the average of absolute relative Errors, a standard can be regarded good if the distributions are different. Four sample sizes were selected (-6, -13, +6, +13), and the random errors distribute (Cauchy , normal, binomial, and geometric).Also, four simulation experiments were designed, and their results are shown in tables(1,2,3, and 4).For all sizes of the sample, the proposed method gave results relatively similar to the other three. Besides, non-linear least square method has proved to be the best when the initial value of the parameter is (0.13). Better results are identified when the random errors distribute continuously, not discreetly.

5- Conclusions:

1- The second method has shown to be the best among other methods of sample sizes, except where the initial parameter of the model is0.13 and here the third method can be the best for geometric distribution and for all sample sizes.
2- The second method is the best if the random errors are Gaussian distributed in comparison with other distributions, for all sample sizes and initial values of the model parameter.
3- Results show that 0.13 is the best initial value for the model parameter for all distributions, results, and sample sizes.
4- n terms of estimation, the continuous distributions are found to be better than discrete ones for all results, sample sizes, distributions, and initial values.
5- For all sizes of the samples, the proposed method gave results relatively similar to the other three.
6- The proposed method is found to be better than other methods of Estimation for all sample sizes in terms of the initial value of the parameter is 0.13 and the random errors distribute geometrically.
7- This paper helps other researchers develop the proposed method and Iterative process to estimate parameters of a non-linear moving average model of higher order.

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مقارنة لطرائق التقدير لمعلمة أنموذج الأوساط المتحركة غير الخطية من الدرجة الأولى

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المستخلص

كثير من الباحثين يواجهون صعوبة في تقدير معلمة أنموذج الأوساط المتحركة غير الخطية ولذاك السبب لا نستطيع استخدام الطرائق التقليدية للتقدير.

يهدف هذا البحث لمقارنة أربعة طرائق في تقدير المعلمة لأنموذج الأوساط المتحركة من الدرجة الأولى بأستخدام متوسط الخطأ النسبي المطلق كمعيار للمقارنة آخذين بنظر الاعتبار (حجوم العينات , قيم أولية مختلفة للمعلمة ولتوزيعات مختلفة للاخطاء العشوائية للأنموذج).

أظهرت الطريقة الثانية انها الأفضل من بين الطرائق الاخرى ولجميع احجام العينات , فيما عدا كون الاخطاء العشوائية للأنموذج تتوزع توزيعا هندسيا فأن الطريقة الثالثة والتي اقترحها الباحث في هذا البحث تكون الأفضل ولجميع احجام العينات.

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