Nonparametric Wavelet-Based Regression Modeling of Oil Revenues and Inflation Incorporating a Hurst Threshold for Irregularly Spaced Economic Data

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Abstract: The wavelet shrinkage estimator is considered an attractive technique for estimating nonparametric regression functions due to its high efficiency. However, it suffers from the requirement of a dyadic sample size and the assumption that observations are equally spaced and regularly distributed. When estimation is performed without satisfying these conditions, the resulting estimates are inaccurate.

This study aims to model the relationship between oil revenues and the inflation rate using a nonparametric regression model based on wavelet transforms, employing the Hurst threshold as an adaptive mechanism that reflects the long-memory properties of economic data. The significance of the study stems from the limitations of traditional methods, particularly the universal threshold, in representing irregular time series characterized by temporal dependence, which leads to information loss due to over smoothing

The study relies on real monthly data for the Iraqi economy over the period from 1/1/2011 to 30/11/2022, comprising (228) observations, This provides a suitable application environment for analyzing the dynamics of a rentier economy that relies heavily on oil revenues. Wavelet smoothing was integrated with threshold adjustment based on the Hurst exponent, allowing the model to adapt to the temporal structure of the series.

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The results demonstrated a clear superiority of the Hurst threshold model compared to the general threshold in terms of estimation accuracy, as it achieved a noticeable reduction in both RMSE and MAE, in addition to its ability to preserve the structural characteristics of inflation, as evidenced by the graphical analysis. The findings also revealed that the relationship between oil revenues and inflation is nonlinear and complex, influenced by economic shocks and temporal accumulations.

The study concludes that incorporating long-memory properties within the wavelet regression framework provides a more realistic representation of economic relationships and enhances the efficiency of nonparametric models, particularly in volatile rentier economies such as the Iraqi economy.

Keywords: Wavelet Transform، Wavelet Regression، Kovac and Silverman Algorithm، Hurst Threshold، Thresholding Rule

1. Research Objective

This study aims to model the relationship between oil revenues and inflation using wavelet-based nonparametric regression, incorporating the Hurst threshold as a method to enhance the smoothing process and to select the optimal level of wavelet thresholds, within the context of economic data characterized by irregular time spacing.The study also seeks to provide a more accurate model for explaining the dynamic behavior of economic variables associated with the oil sector.

2. Research Problem

One of the most important assumptions required when estimating the nonparametric regression function using wavelet methods is the equality of time intervals between the variables (x_i), in addition to the dyadic sample size condition, i.e., $n = 2^J$ where ($J = 4,5,6 \ldots$). However, when one or both of these assumptions are violated, it becomes difficult to apply these methods without modification, and their application without proper treatment leads to inaccurate results.

3. Significance of the Study

The significance of this study lies in its provision of an advanced methodology for analyzing the relationship between oil revenues and inflation, a relationship that is central to decision-makers, particularly in rentier economies. The study provides an updated framework based on wavelets and the volatile properties of data, which contributes to improving the accuracy of economic forecasts. It also helps to fill a research gap represented by the scarcity of studies that employ wavelet-based nonparametric regression for irregularly spaced data with the integration of the Hurst threshold.

4.Wavelet Analysis

Wavelets are defined as small signals localized in time with specific properties, as they incorporate both the analysis and windowing characteristics. There are many types of wavelets, each distinguished by particular properties that are utilized in transformation processes, and they are usually denoted by (t) Ψ [29]

The use of the term "wavelet" dates back to 1909 by Alfred Haar, who discovered the simplest type of wavelets, which were later named after him. Subsequently, in 1930, a group of researchers worked on developing wavelet functions known as scaling functions (Scale). Wavelets were further developed by Y. Meyer in 1985, and Daubechies also introduced orthogonal wavelets with compact support in 1988 .[5]

However, the major breakthrough in the field of wavelets was achieved by Stephane Mallat in 1989, through the development of a fundamental algorithm for implementing the discrete wavelet transform using filter-based techniques.

The need that led to the emergence of wavelets, and the remarkable development they experienced, is that all previous transforms—prior to the Fourier Transform and the Short Fourier Transform (Short Fourier Transform)—suffered from significant limitations in handling non-stationary signals, whether in the time or frequency domain. Earlier transforms tended to focus on either time while neglecting frequency, or vice versa.

In many cases, the focus was primarily on the time domain of the signal, while the frequency aspect was neglected. However, this raises an important question: why do we need information about the signal in the frequency domain? The answer lies in the fact that, in order to properly understand patterns, much of the most important and distinguishing information of a signal is often found in its frequency representation.

Accordingly, two types of signals can be identified: stationary signals, which do not change over time, and whose frequency representation is constant—that is, all frequency components of the signal are present at all times, and there is no need to determine the time–frequency localization of the signal. This type is relatively easy to handle, and previous transforms such as the Fourier Transform and the Short Fourier Transform were effective in dealing with it.[4]

The second type, which previous transforms failed to handle effectively, is the non-stationary signal that varies over time; that is, its frequency changes with time, as the information itself evolves over time (each time interval has its own specific frequency).

All these features of wavelets and their transforms have made them, at present, an important tool used across a wide range of scientific fields. They have become a common tool shared by many disciplines, with each field adopting them for its own applications. In mathematics, they are used in functional analysis, ordinary and partial differential equations, and numerical analysis. Statisticians have also employed them in many applications, particularly in nonparametric regression models and time series. In the natural sciences, physicists have played a major role in their development. Furthermore, there are numerous applications in economics, and more recently, wavelets have been used in image compression—such as in digital fingerprint storage by the United States Federal Bureau of Investigation—as well as in earthquake prediction and signal detection in underwater acoustics[7]

5 .Wavelet Theory

The representation of a signal in the time domain provides precise information about time (or location) without offering any information about the frequencies contained within it. In contrast, the Fourier Transform converts the signal from the time domain to the frequency domain, thereby limiting the precision of all frequency components of the signal; however, it neglects the time at which these frequencies occur within the time domain.[13]

Therefore, the idea of the Short-Time Fourier Transform (SHFT) emerged to address this issue by using a window function to segment the original function, after which the Fourier Transform is computed for each segment. However, this method also suffered from the problem of selecting an appropriate window width for analysis due to the presence of the Heisenberg uncertainty principle, which indicates that the area of the time–repetition resolution box cannot be reduced below one-half.

As the window width increases, the method approaches the Fourier Transform, meaning higher accuracy in repetition information but less accuracy in time information. Conversely, when the window width decreases, the accuracy of time information increases at the expense of repetition accuracy. Since low repetitions occur over long time intervals—often extending over the entire duration of the signal—the precision of time becomes less important. In contrast, high repetitions occur over short time intervals, requiring precise localization in time. Therefore, it is preferable to use a window width that balances the required accuracy in both repetition and time, as illustrated in the following figure [11,17].

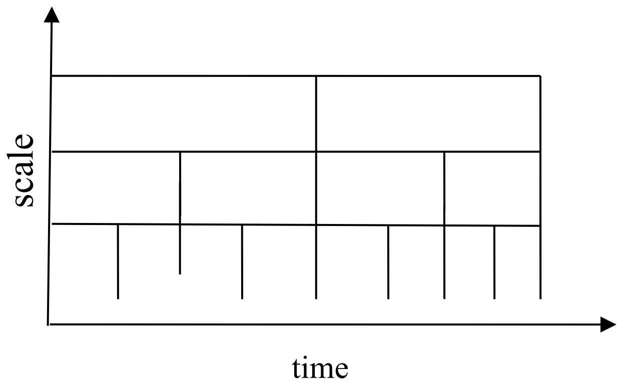


Figure (1) illustrates the use of a variable window in the wavelet transform. [14]

Therefore, the idea of the wavelet transform emerged, as it employs the scale parameter to determine the appropriate window width for the transformation.

To perform wavelet analysis on any signal, it is necessary to use a function called the Mother Wavelet. The term “wavelet” refers to a small wave, where the smallness is due to the limited width of the window in time (i.e., compact support). The term “mother” indicates that all functions with different support regions used in the transformation are derived from the main wavelet; thus, the mother wavelet serves as a prototype from which other window functions are generated.[21]

The mother wavelet is the function:

$$\psi \in L^2(R)$$

With zero average

$$\int_{-\infty}^{\infty} \boldsymbol{\psi(t)dt = 0} \qquad .... (1)$$

and with a finite wavelength, $\|\psi\| = 1$ localized in the neighborhood of **t=0.** Then, a time–frequency family can be generated from the mother function using the scale **S** and translation by the value **u** according to the following formula:

$$\boldsymbol{\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\left(\frac{t-u}{s}\right)} \qquad .... (2)$$

as the purpose of dividing by it $\sqrt{s}$ is to preserve the unit length of the resulting function $\|\boldsymbol{\psi_{u,s}(t)}\|$.

The wavelet transform of the function $f \in L^2(R)$ at scale S and time u is [18] :

$$Wf(u,s) = \langle f, \psi_{u,s} \rangle \qquad .... (3)$$

$$= \int_{-\infty}^{\infty} \boldsymbol{f(t)\frac{1}{\sqrt{s}}\psi^*\left(\frac{t-u}{s}\right)dt} \qquad .... (4)$$

6. Wavelet Bases

There are different types of wavelet bases, each characterized by specific properties that can be utilized in transformation processes. The most important and commonly used ones will be presented.

7. Haar Wavelet

It is the simplest form of orthogonal wavelets. It is a discrete wavelet and is also referred to as the (db1) wavelet. It belongs to the Daubechies wavelet family with one vanishing moment. It possesses a frequency function, which is expressed by the wavelet function as follows:[18]

$$\psi(t) = \begin{cases} 1 & for \quad 0 \le t < \frac{1}{2} \\ -1 & for \quad \frac{1}{2} \le t < 1 \\ 0 & \text{o.w} \end{cases} \qquad .... (5)$$

The scaling function is given as follows :

$$\emptyset\,(t) = \begin{cases} 1 & for \quad 0 \le t < 1 \\ 0 & o.w \end{cases} \qquad .... (6)$$

The wavelet function is characterized by consisting of two pulses positioned adjacent to each other on the time axis, with one being the inverse of the other, as illustrated in the figure.

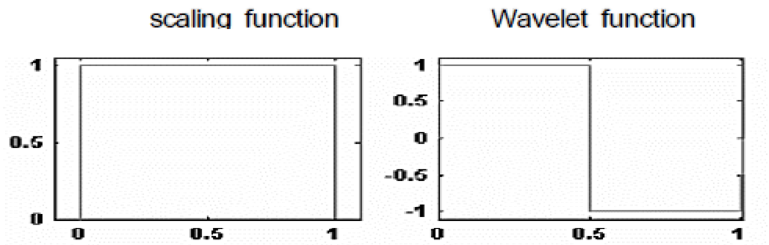


Figure (2) illustrates the scaling function and the wavelet function of the Haar wavelet (Haar).[6,20]

8. Extremal-Phase Wavelets (Daubechies)

The Daubechies wavelet family includes Biorthogonal properties, and is characterized by compact support, sufficient regularity, and asymmetry. The Haar basis is one of the Daubechies family wavelets in the case of 1 vanishing moment. It is the oldest, most well-known, and most suitable wavelet for educational purposes due to its simplicity. In addition, it highlights several wavelet properties, such as the oscillation property—where it rises and falls—which can be expressed mathematically, $\int_{-\infty}^{\infty} \psi(x)\,dx = 0$ ،as well as the compact support property, which is not shared by all wavelets.

It was discovered by the mathematician Alfred Haar in 1909. The Haar mother wavelet (Mother Wavelet Haar) is a mathematical function defined as follows [21]:

$$\psi(x) = \begin{cases} 1 & x \in \left[0, \frac{1}{2}\right), \\ -1 & x \in \left[\frac{1}{2}, 1\right), \\ 0 & otherwise. \end{cases} \qquad \text{..........} (7)$$

It is a function that is well localized in the time domain; however, it is clearly not continuous. In the frequency domain, it is weakly localized. It possesses a special role, as it is the only known wavelet that is symmetric and has compact support. It can be stated that if the function under study is smooth (with occasional jumps, for example), then it is appropriate to select a smooth wavelet with several vanishing moments. However, if the function under study is piecewise or piecewise constant, then it is appropriate to choose Haar wavelets. Figure (3) presents the Haar wavelet and some Daubechies wavelets.

Below, some examples of Daubechies mother wavelets with different degrees of smoothness are presented.

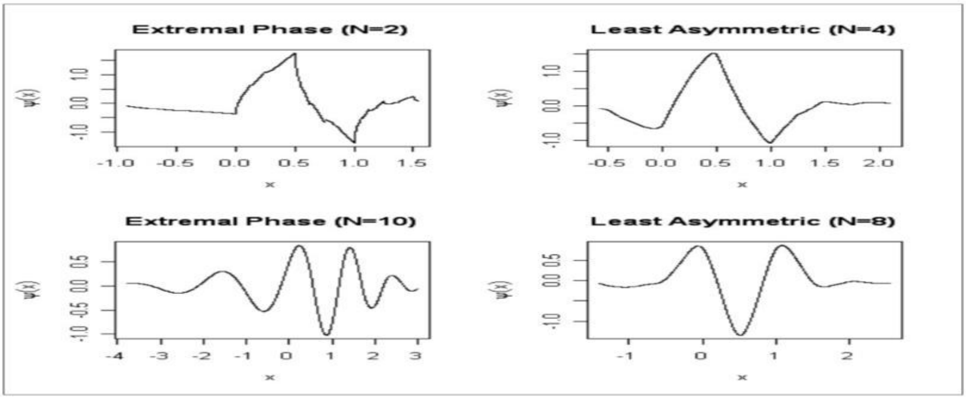


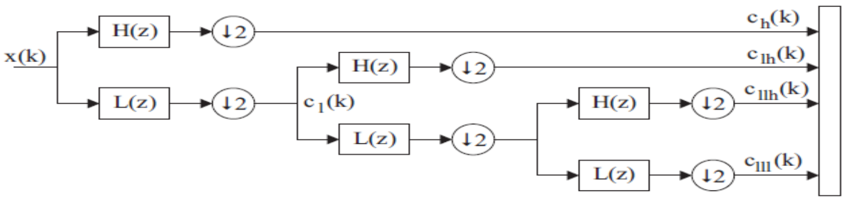
Figure (3) represents the graphical plots of the Daubechies mother wavelet [2] when N = 2,4,8,10.

9. Discrete Wavelet Transformation (DWT)

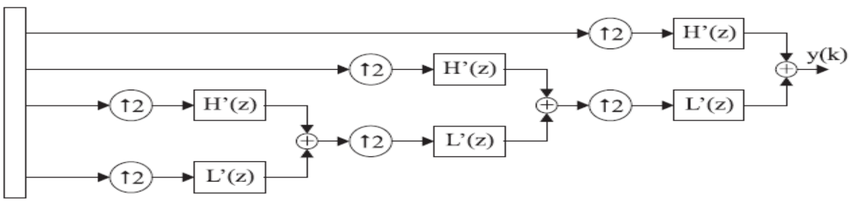
The discrete wavelet transform of a signal is considered an alternative to the time–frequency representation found in transforms such as the Discrete Fourier Transform and the Cosine Transform. It represents the next logical step in windowing techniques with variable window width, and thus can be regarded as an advancement over previous transforms such as the (STFT) and others.

In this approach, the window width is varied to obtain frequency information across different parts of the signal, leading to what are known as wavelets whose frequencies differ according to the bandwidth used. This is achieved by employing the wavelet transform through what is known as filter banks in order to perform multi-resolution (time–frequency) analysis, using special wavelet filters for both analysis and signal reconstruction.

A filter bank consists of a set of filters that decompose the signal into frequency bands. The discrete signal x(k) is input into the analysis bank and filtered using the filters H(Z) and L(Z), which in turn decompose the frequency content of the input signal into equal-width frequency bands. The filter L(Z) and the filter H(Z) represent low-pass and high-pass filters, respectively. The output of each filter contains half of the input frequency content, while maintaining the same number of samples. Consequently, the combined outputs of both filters contain the same total frequency content as the input, but with a doubled number of samples compared to the original signal.
Therefore, a process known as down sampling is applied by a factor of (2), denoted by (2↓), meaning that the number of samples is reduced by half. This operation is applied to the outputs of the filters in the analysis bank.
As for the reconstruction or recovery of the original signal, it is carried out using a synthesis filter bank. In the reconstruction stage, the signal is restored by increasing the number of samples through up sampling, denoted by (2↑). The signal then passes through the filters H(Z) and L(Z), which are constructed based on the input filters. Finally, the outputs of these filters are combined at this stage to obtain the reconstructed signal, denoted by Y(K).[3 ,4]



(a) analysis bank



(b) synthesis bank

Diagram (1) illustrates three-stage filtering, with the analysis filters at the top and the synthesis filters at the bottom.[19 ,18]

10. Wavelet Regression

Wavelet regression estimators are a method introduced by Donoho and Johnstone in 1991, and they are considered the initial spark in highly efficient nonparametric estimation techniques for information extraction. This approach has opened the door for many researchers to further develop it, and its development remains an open field for researchers across different disciplines.

One of the most important fundamental assumptions that must be satisfied is that the time intervals between the points over the interval are usually assumed to be equal, such that , and that the sample size is dyadic, i.e., in the form for each positive integer . The wavelet regression procedure can be summarized in the following steps:

1. Apply the discrete wavelet transform to the data to obtain the wavelet coefficients[29]

w = Wy

2. Modify the wavelet coefficients and compute the adjusted wavelet coefficients using thresholding functions, which will be explained in detail later.

3. Estimate the wavelet regression function by computing the inverse discrete wavelet transform (IDWT), where:[9]

$$\hat{f}(xi) = W^t w^* \qquad \text{..........} (8)$$

11.Thresholding Functions

One of the key properties of wavelet transforms is that they produce a sparse signal, where most of the wavelet coefficients are zero or close to zero, especially when left unmodified.

Noise is distributed uniformly across all coefficients; however, this noise is not sufficiently large to clearly distinguish between the wavelet coefficients of the original signal and the noisy coefficients. Hence, the fundamental importance of the thresholding process emerges, as the thresholding rule performs two essential tasks: first, it classifies the wavelet coefficients as either significant or insignificant; and second, it modifies the wavelet coefficient based on its classification.

Accordingly, thresholding functions operate specifically on the empirical coefficients wavelet , $\tilde{d}jk)$ (while the scaling function coefficients $(\tilde{C}j.k)$ remain unchanged. Although the most commonly used thresholding rules are the hard thresholding rule and the soft thresholding rule, researchers have proposed many other thresholding rules, each with its own advantages, which will be discussed in detail within this study.[22,27]

12. Hard Thresholding Method

It is a simple method carried out by setting to zero the elements whose absolute values are less than the threshold, and it is expressed mathematically as:

$$Thr_{\lambda}^{H}(y,\lambda) = \begin{cases} |y| \leq \lambda \; if \;\; 0 \\ |y| > \lambda if \;\; y \end{cases} \qquad \text{.......... (9)}$$

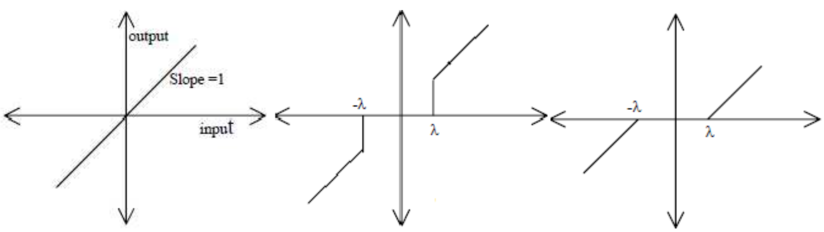
Where λ is the threshold value (Thresholding value).[27]

13. Soft Thresholding Method

It is an extension of the previous method and differs from it in that, after setting to zero the elements whose absolute values are less than the threshold, the nonzero elements are shifted toward zero. It is expressed mathematically by the following relation:

$$Thr_{\lambda}^{S}(y,\lambda) = \begin{cases} |y| \leq \lambda \qquad if \quad 0 \\ |y-\lambda| > \lambda \;\; if \quad sgn(y) \end{cases} \qquad \text{.......... (10)}$$

Where λ is the threshold value [2].



(a) (b) (c)

Figure (4) represents the linear (a), hard (b), and soft (c) thresholding functions [2]

14. Threshold Value

One of the most important steps in performing wavelet regression is the second step, which involves the optimal selection of the thresholding function. However, this selection entails a highly critical aspect, namely the threshold value. The appropriate choice of this value is decisive for the accuracy and efficiency of the estimation, as the threshold value determines which coefficients are retained and which are eliminated.

Accordingly, very small threshold values allow many coefficients to remain in the reconstruction process, which may result in an output signal that is still noisy and largely similar to the input signal, leading to inaccurate estimation. On the other hand, a large threshold value leads to the loss of a significant number of coefficients that are excluded from the reconstruction (estimation), resulting in an excessively smooth curve (over smoothing), which in turn destroys the details of the wavelet coefficients. Consequently, this leads to inefficient processing characterized by blurring and distortions.

Therefore, the appropriate selection of the threshold value plays a crucial role in the efficiency of the estimator, which has motivated many researchers to study this topic extensively, most notably Johnston and Donoho, as well as Silverman, Nason, and others. The following section presents some of the methods used for selecting the threshold value.[26,27]

15. Universal Thresholding Method

The universal thresholding method was introduced by (Donoho and Johnstone) and is given by the following expression :[2,37]

$$\lambda_{universal} = \sigma\sqrt{2\,log(n)} \qquad \text{.......... (11)}$$

n :the length of the signal.

σ :the standard deviation of the noise level.

The selection $\sqrt{2log}$intended to suppress noise with high probability.

16 .Hurst Threshold

Hurst-based Thresholding is considered an adaptive development of traditional thresholding methods in wavelet regression. It emerged in response to the

limitations of classical thresholds—particularly the universal threshold proposed by Donoho and Johnstone—in handling time series characterized by long-range dependence, a common feature in economic data. These traditional thresholds are typically constructed under assumptions of error independence and homogeneity, which may not reflect the true structure of economic time series, often leading to oversmoothing and the loss of important signal characteristics.
The conventional threshold is based on the following expression:

$\lambda=\sigma 2\log(n)$ .... (12)

where σ\sigmaσ represents an estimate of the noise standard deviation, and **n** is the sample size. Despite its simplicity and effectiveness in some applications, it does not account for the degree of temporal dependence inherent in the data.
From this perspective, the Hurst exponent **H** is introduced as an adaptive component that reflects the memory structure of the time series. Accordingly, the threshold is adjusted across wavelet decomposition levels rather than remaining constant. The Hurst threshold can be expressed as follows:

$\lambda j(H)=\lambda[1+\alpha(H-0.5)Jj]$ .... (13)

where H represents the base threshold, λ denotes the decomposition level, and j is the total number of levels, while J refers to a control parameter that determines the degree of influence exerted by λ the threshold.
The fundamental idea behind this formulation is that time series characterized by high values of (H>0.5H > 0.5H>0.5) reflect the presence of positive autocorrelation and long memory, which requires reducing the strength of the threshold in order to preserve the structural components of the signal.
In contrast, when the values are closer to randomness, a higher degree of smoothing can be applied without significant loss of information. Accordingly, the Hurst threshold provides a more precise balance between noise removal and the preservation of the dynamic properties of the series.

This approach represents a practical extension of the long-memory literature, as presented in the works of Beran (1994) and Taqqu et al. (1995), integrated within the wavelet smoothing framework developed by Donoho and Johnstone (1994, 1995). Its main advantages lie in improving estimation accuracy and reducing the problem of oversmoothing, particularly when applied to economic data characterized by irregularity and complex fluctuations, making it a suitable tool for modeling nonlinear economic relationships.

17. Wavelet Regression with Unequally Spaced Data

Wavelet regression is considered a highly promising approach in nonparametric regression. However, its application requires fundamental assumptions, namely that the sample size is dyadic (Dyadic), $n = 2^j$ of the form $x_i = \frac{i}{n}$, and that the data points are equally spaced in time.

The first condition, namely the dyadic sample size, is often satisfied either by increasing the sample size to reach the required form or by discarding some observations to achieve this structure. However, this procedure may affect the amount of information contained in the signal, or it may be difficult to obtain sufficient data to meet the required sample size.

The second condition poses even greater challenges. One of the simplest approaches to address the problem of unequal time spacing is interpolation. However, unfortunately, the wavelet representation of irregularly spaced data is not efficient, as the vanishing moments are generally no longer valid. As a result, the mean squared error tends to be relatively high.

This has led researchers to raise an important question: should one abandon a highly efficient estimation method for such types of data, or instead develop alternative methods capable of handling this issue efficiently In practice, researchers have chosen the latter, and numerous effective methods have been developed to address this problem. Some of these methods will be presented in this study.

Accordingly, researchers have increasingly posed a key question: in the absence of data that satisfy the assumptions of this transform, should they remain unable to address the problem and thus sacrifice a highly efficient estimation method, or should they seek appropriate solutions Indeed, researchers have adopted the second approach.[18]

18 .Kovac and Silverman Algorithm

Kovac and Silverman (2000) proposed an algorithm that involves constructing a new grid that is equally spaced in time, denoted by $(t_0, t_1, \ldots\ldots\ldots, t_{N-1})$, over the interval [0,1]. For certain values $N = 2^j$ of J ∈ N, the original data are embedded into this grid, where they proposed that:

$$tk = \frac{k+0.5}{N} \qquad k = 0,1,\ldots\ldots,N-1 \qquad \text{.......... (14)}$$

such that $N = 2^j$ for each $J = \min\{j \in z, 2^j > n\}$.
The original data are embedded into the grid to obtain new data that are equally spaced with a dyadic sample size 2^j, using the following expression.
To obtain the new values y_i^* on the new grid according to the following formula:

$$y_k^* = \begin{cases} y_0 & if \quad t_k \le x_0 \\ y_i + (t_k - x_i)\dfrac{y_{i+1} - y_i}{x_{i+1} - x_i} & if \quad x_i \le t_k \le x_{i+1} \\ y_{n-1} & if \quad t_k \ge x_{n-1} \end{cases} \qquad \text{.......... (15)}$$

where the original data and the embedded data can be expressed as vectors, such that the original data are given by $y_i = (y_0, y_1, \ldots \ldots, y_{n-1})$ and the embedded data by $(y_1, \ldots \ldots\ldots, y_{N-1})$. The linear transformation described in equation (2) can be written in matrix form as follows:

$$y^* = Ry \qquad \text{.......... (16)}$$

where is the matrix of the linear transformation into and ; each row in typically contains either a single (1) or two nonzero entries whose sum equals (1). The quantities $)y^*)$ و $(t_k$ (and represent the new data that satisfy the conditions of the discrete wavelet transform, and thus the wavelet coefficients can be obtained according to the following expression:

$$w = WY^* \qquad \text{.......... (17)}$$

where is an orthogonal matrix with vanishing moments, associated with Daubechies wavelets of extremal phase (Extremal phase wavelets Daubechies), which possess compact support and are sufficiently regular.
Given that the study model assumes that the noise term $)\in_i($ is independently and identically distributed, the covariance matrix $\sum y^*$ for the new data is given by the following expression:

$$\sum y^* = \sigma^2 RR^T \qquad \text{.......... (18)}$$

where σ^2represents the individual wavelet coefficient data, which can be accurately computed from the data using a fast algorithm (b_2^J), where

$b = \max(by, Nhv)$ and by

denotes the bandwidth (threshold value) for the matrix $\sum y$. Kovac and Silverman (K.S) employed a universal threshold value $\lambda = \sqrt{2\ln(n)}$.

Finally, the inverse wavelet transform is applied to obtain the final estimate of the regression function according to the following equation:[15]

$$\hat{\boldsymbol{f}}_{\boldsymbol{ksw}} = \boldsymbol{W}^{\boldsymbol{T}}\boldsymbol{wy}^{*} \qquad \text{..........} \ (19)$$

19. Kovac and Silverman Algorithm with Hurst Threshold

This method is similar to the previous method in handling data irregularity; however, it differs in that the Hurst threshold is used as an attempt to improve estimation through the enhanced smoothing it provides, as mentioned earlier. The threshold value is adjusted in accordance with the degree of autocorrelation in the data.

Instead of adopting a fixed threshold across all levels of analysis, the Hurst threshold is introduced as a balancing factor that affects the degree of smoothing, allowing the preservation of important signal structures in data with long memory, and reducing over-smoothing, which may lead to loss of information.

20. Real Data[4]

Oil wealth is considered one of the fundamental components in countries that possess such resources, and our country is regarded as the third largest country in terms of oil wealth after the Kingdom of Saudi Arabia and the United States of America, as oil constitutes approximately 95% of the country's budget revenues.

Oil financial resources play a significant role in the economies of developing countries, including Iraq, where they are relied upon as the main source for financing development programs. The Iraqi economy is classified among developing economies, despite falling within the category of oil-based economies that are characterized by a high level of financing capacity. Inflation is considered one of the abnormal or undesirable phenomena that affect most oil-based economies, including Iraq, due to its impact on overall economic and social sectors. Over recent decades, the Iraqi economy has experienced significant fluctuations in economic variables depending on the economic and political conditions it has undergone. However, a common feature in recent years has been a substantial increase in prices, where changes in oil prices have had a clear impact on price levels. Based on this relationship between oil revenues and inflation, this study seeks to emphasize this importance and integrate the theoretical aspect with the applied aspect.

21. Factors Affecting Inflation

There are many factors affecting inflation; however, the most important, as previously mentioned and based on consultation with economists, is oil revenues, as they represent the primary source of foreign currency inflows and financial resources for the country. Therefore, they will be adopted as an explanatory variable affecting inflation.

22. Oil Revenues

An increase in oil prices leads to an increase in oil revenues, which in turn leads to higher inflation rates and subsequently higher prices of goods and services, negatively affecting individuals, particularly those with fixed incomes. It also leads to higher production and distribution costs. In many cases, continuous price increases cause employees and professionals to demand higher wages in order to keep pace with this rise. Food prices are considered the most affected by increases in oil revenues due to the direct impact of rising production costs, as well as indirectly through a reduction in supply resulting from the substitution of food crops with fuel production.

23. Data Classification

The study variables consist of the average oil revenues in Iraq, obtained from the official website of the Organization of Arab Petroleum Exporting Countries (OAPEC) for the period from 31/1/2011 to 31/7/2022 as an explanatory variable (R). Inflation data were obtained from the official website of the Central Bank of Iraq (IN) for the same period as the dependent variable. As mentioned in the theoretical section, the importance of the study lies in selecting the best transformation methods for estimating the wavelet regression function, through which the maximum amount of information can be extracted from the signal under study (IN).

The study also holds economic importance, as the stability of oil prices in global markets is an explicit objective of monetary policy in Iraq. Understanding the impact of fluctuations in global oil prices on the general price level under changing financial and economic conditions helps relevant authorities to control the main determinants of inflation and to interpret their contribution to the consumer price index in both the short and long term, through constructing the best wavelet regression model using optimal transformation methods.

The data were tested using SPSS software version (26), and it was found that the relationship between the explanatory and dependent variables is nonlinear. Therefore, the parametric model is not effective in explaining the relationship between (oil prices) as an explanatory variable and the inflation index as a dependent variable. It is observed that the parametric model fails to explain the relationship in a more detailed and accurate manner

24. Model Construction

One of the most important stages in model construction is identifying the dependent variable (Variable Dependent) and the explanatory variable (Variable Exploratory) based on the nature and type of the data under study, which are explained as follows:

25. Dependent Variable IN (represents inflation):

Inflation is defined as the continuous increase in the general price level and is measured by changes in the Consumer Price Index (CPI), which reflects the change between two time periods (a base period and the subsequent period) in the prices of goods and services consumed by households, either directly or indirectly. Inflation affects the purchasing power of consumers, and high inflation rates lead to many negative effects manifested in the aggravation of economic and social problems. Therefore, most central banks worldwide adopt monetary policies aimed at preventing inflation by maintaining monetary stability, through enhancing external financial revenues, which contribute to strengthening the national currency and achieving financial stability.

26. Explanatory Variable R (represents oil revenues):

In general, oil revenues can be defined as the revenues or returns obtained by some oil-producing and exporting countries in the world in exchange for the production and export of a natural resource, namely oil, for which they receive monetary returns as part of the real value of this resource. Since this variable has a nonlinear relationship with the dependent variable, in addition to the difficulty of controlling it due to its sensitivity to various factors—some political and others related to the policies of exporting countries and surrounding global issues—the optimal choice is to construct a nonparametric model. As previously explained, the parametric model fails in this context. Accordingly, the applied part of the study includes one explanatory variable, which is oil revenues, since the adopted research model is a simple nonparametric regression model, as explained in (22).

27. Results Analysis

Before starting the estimation process, a parametric regression model was estimated, specifically using the OLS method. It was found that, despite the model being statistically significant, the coefficient of determination was very low, reaching only 6%. Therefore, a linearity test was conducted to examine the relationship between the explanatory and dependent variables. As shown in Figure (5) below, the relationship between the study variables is nonlinear. Accordingly, the nonparametric regression function was estimated using the best estimation methods presented in the theoretical section.

The results presented in Table (1), supported by comparative graphical representations between the Universal threshold and the Hurst threshold, in the context of estimating the nonparametric regression function for the relationship between oil revenues and inflation rate, reveal substantial differences in estimation efficiency and in the nature of representing the economic relationship. Statistically, the Hurst threshold method demonstrates clear superiority, achieving extremely low values for both RMSE and MAE compared to the Universal threshold, indicating a higher ability to reduce estimation errors and approximate actual values. This conclusion is visually reinforced by the graphs, where the Hurst curve retains a higher degree of structured fluctuation and reflects the true structure of the data, particularly in regions with sharp changes. In contrast, the Universal curve exhibits a high degree of smoothing, which leads to the loss of many important details, including peaks and sudden declines.

From a nonparametric regression perspective, the estimated function obtained using the Hurst threshold appears more capable of representing the nonlinear relationship between oil revenues and inflation, as it shows a flexible response to changes in revenue levels, particularly in low and medium ranges characterized by high inflation volatility. In contrast, the use of the Universal threshold leads to a flatter estimate, reflecting a limited ability to capture the true nonlinearity of the relationship. This can be explained by its implicit assumption of a simple noise structure that does not align with the nature of economic data.

Economically, these results indicate that the relationship between oil revenues and inflation is not a stable linear relationship, but rather is characterized by a degree of complexity and dynamism associated with long-memory properties and the cumulative effects of oil shocks. Sharp increases in oil revenues are often associated with uneven inflationary pressures over time, resulting from multiple channels such as government spending, money supply, and exchange rates. Here, the importance of the Hurst threshold becomes evident, as it accounts for the degree of temporal dependence in the series, enabling it to preserve these dynamic characteristics within the estimated function. Accordingly, the results support the hypothesis that incorporating long-memory information into the wavelet smoothing process improves the characterization of the economic relationship and provides the model with greater explanatory power compared to traditional methods that tend to oversimplify the relationship at the expense of accuracy.As shown in the table below.

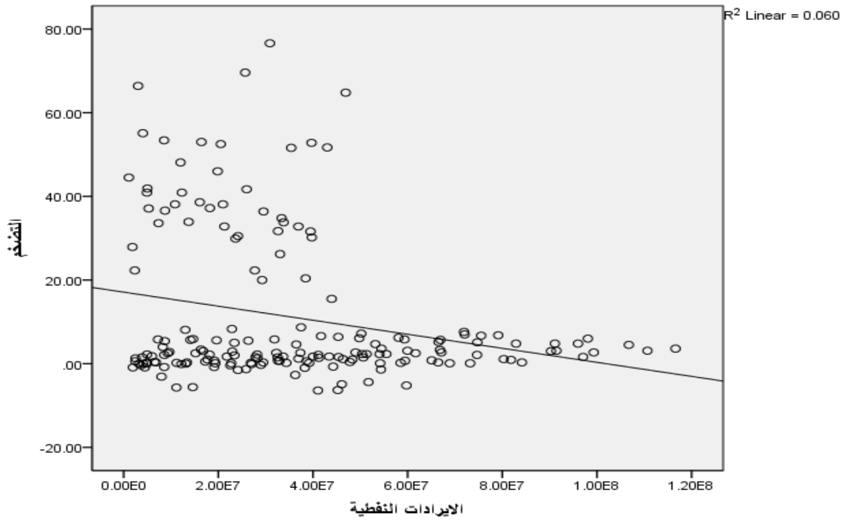
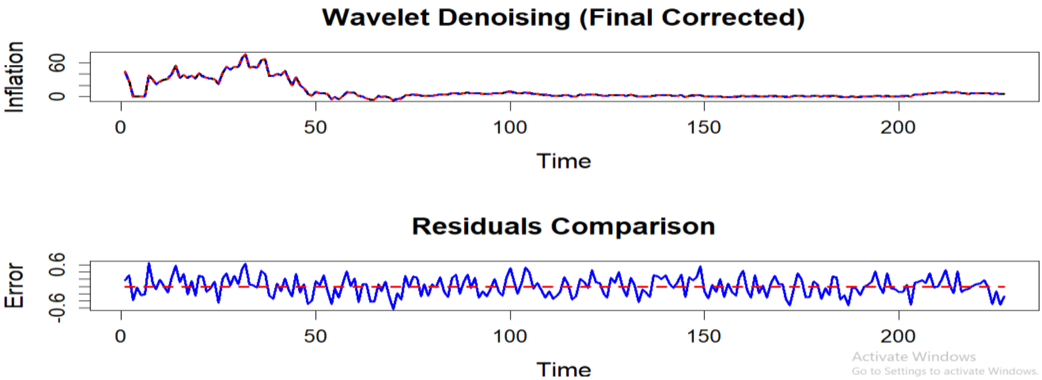
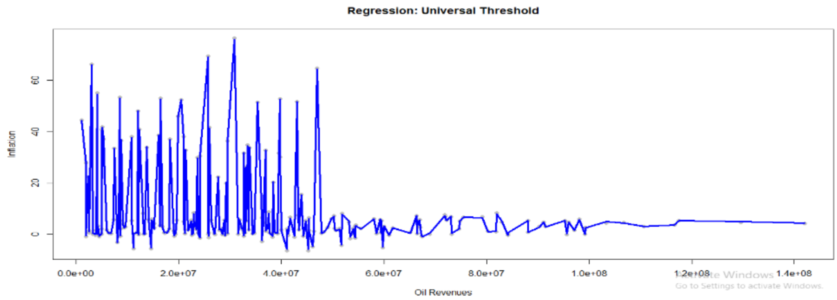


Figure (5) illustrates the nonlinear relationship between the explanatory variable and the dependent variable.

Table (1) presents the values of the Root Mean Squared Error (RMSE) and the Mean Absolute Error (MAE) for the inflation rate.

Estimation Method	Universal	Hurst
RMSE	0.260807	4.472636e-05
MAE	0.2158403	3.92209e-05





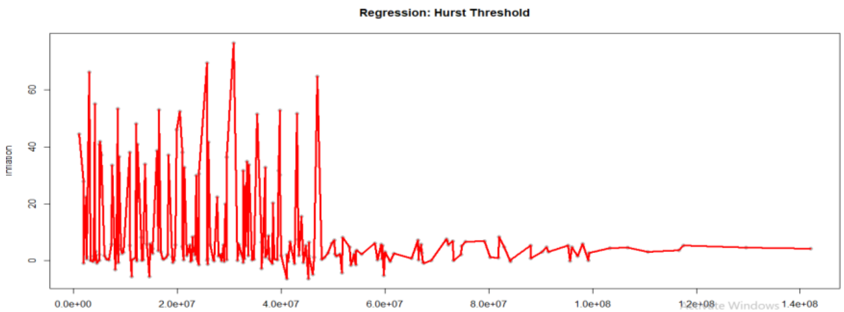


Figure (6) illustrates the actual and estimated data of the dependent variable (inflation) for the estimation methods presented above.

28. Conclusions:

The obtained results indicate that incorporating the Hurst threshold within the framework of wavelet-based nonparametric regression represents a substantial improvement in estimation efficiency compared to the conventional universal threshold. The model demonstrated a high capability in reducing estimation errors while preserving the dynamic characteristics of the series. This superiority can be attributed to the adaptive nature of the Hurst threshold, which allows the degree of smoothing to be aligned with the underlying temporal dependence structure in the data, in contrast to the universal threshold, which assumes independent noise and often leads to oversmoothing and loss of structural information.

The results also revealed that the relationship between oil revenues and the inflation rate is nonlinear and complex in nature, being significantly influenced by economic shocks and temporal accumulations. This is confirmed by the ability of the Hurst-based model to capture these variations more accurately. Accordingly, it can be concluded that the use of adaptive wavelet techniques based on long-memory properties not only improves estimation accuracy but also provides a more realistic representation of economic relationships, thereby enhancing the practical value of the model in economic policy analysis, particularly in rentier economies that rely heavily on oil revenues.

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نمذجة الإيرادات النفطية والتضخم باستخدام الانحدار اللامعلمي المويجي مع عتبة هيرست للبيانات الاقتصادية غير المنتظمة

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الملخص : يُعد مقدر الانكماش الموجي (Wavelet Shrinkage Estimator) من الأساليب الفعّالة في تقدير دوال الانحدار اللامعلمي لما يتمتع به من كفاءة عالية في التقدير. إلا أن تطبيقه التقليدي يواجه قيودًا تتمثل في اشتراط أن يكون حجم العينة ذا صيغة ثنائية (Dyadic Sample Size) وأن تكون المشاهدات متساوية التباعد ومنتظمة التوزيع. وعند عدم تحقق هذه الشروط، تنخفض دقة التقديرات المتحصل عليها بصورة ملحوظة.تهدف هذه الدراسة إلى نمذجة العلاقة بين الإيرادات النفطية ومعدل التضخم باستخدام نموذج انحدار لامعلمي قائم على تحويلات المويجات، مع دمج أسّ هيرست (Hurst Exponent) في عملية تحديد العتبة بوصفه آلية تكيفية تعكس خصائص الذاكرة طويلة الأمد للسلاسل الزمنية الاقتصادية. وتنبع أهمية هذه المنهجية من محدودية أساليب تحديد العتبة التقليدية، ولا سيما العتبة العامة(Universal Threshold) ، في

تمثيل السلاسل الزمنية غير المنتظمة التي تتسم بالاعتماد الزمني، الأمر الذي يؤدي إلى الإفراط في التنعيم وفقدان جزء من المعلومات البنيوية المهمة.اعتمدت الدراسة على بيانات شهرية للاقتصاد العراقي تغطي المدة من كانون الثاني/يناير 2011 إلى تشرين الثاني/نوفمبر 2022، وبعدد (228) مشاهدة، مما يوفر بيئة تطبيقية مناسبة لتحليل ديناميكيات الاقتصاد الريعي الذي يعتمد بدرجة كبيرة على الإيرادات النفطية. وقد جرى دمج أسلوب التنعيم بالمويجات مع تعديل العتبة بالاعتماد على أسّ هيرست، بما يمكّن نموذج الانحدار من التكيف مع البنية الزمنية وخصائص الاعتماد في البيانات.أظهرت النتائج التجريبية تفوق نموذج العتبة المعتمد على أسّ هيرست على نموذج العتبة العامة من حيث دقة التقدير، إذ حقق انخفاضًا ملحوظًا في قيم كل من متوسط الجذر التربيعي لمربع الخطأ (RMSE) ومتوسط الخطأ المطلق(MAE) ، فضلاً عن قدرته على الحفاظ بصورة أفضل على الخصائص البنيوية لسلسلة التضخم، وهو ما أكدته نتائج التحليل البياني. كما كشفت النتائج أن العلاقة بين الإيرادات النفطية ومعدل التضخم تتسم بعدم الخطية وبدرجة عالية من التعقيد، نتيجة التأثيرات التراكمية للصدمات الاقتصادية وخصائص الاعتماد الزمني طويل الأمد.وتخلص الدراسة إلى أن دمج خصائص الذاكرة طويلة الأمد ضمن إطار الانحدار بالمويجات يسهم في تقديم تمثيل أكثر واقعية للعلاقات الاقتصادية، ويعزز كفاءة نماذج الانحدار اللامعلمي، ولا سيما عند تحليل اقتصادات ريعية متقلبة مثل الاقتصاد العراقي.

الكلمات المفتاحية : تحويلات المويجات، الانحدار بالمويجات، خوارزمية كوفاتش وسيلفرمان، أسّ هيرست، العتبة المعتمدة على أسّ هيرست، قاعدة تحديد العتبة.

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