Generalization of Double Integral Transform with Three Parameters and Its Applications

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Abstract: In this study, a mixture of the one parameter general Jafari transform and the two parameters generalization of ZZ transform were introduced, resulting in three parameters double integral transform. This generality in this mixture is important, and it was applied in the telegraph problem.

Keywords: Generalization of ZZ transform, General of integral transform, Partial differential equations, Telegraph equation, Double integral transform.

1. Introduction

Due to the importance of double transforms in partial differential equations and their systems, as well as in integro-partial differential equations, many researchers have presented double integral transforms such as the double Laplace, double SEE, double Aboodh, double Sumudu, double complex SEE, double complex EE, double complex Al-Zughair, double Emad-Falih, double Formable, and others [1-7].

As is widely recognized, there are transforms with one parameter, such as Laplace, SEE, Complex SEE, Al-Zughair, Al-Tememe, KAJ, Sumudu, Elzaki , Mayan, GFE, Emad-Falih, Emad –Sara and many others [8-21].

There are also transforms with two parameters, such as ZZ transform, Shehu transform, Maha transforms and others [22,32,26-27]. This paper provides a general overview of these single parameters and two parameters transforms constitutes a special case of double transform presented in this paper.

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-Double Laplace-Shehu transform [28], double Kamal –Shehu [29], Double Sumudu –Shehu [30], Double Sawi-Shehu [30] , Double Aboodh-Shehu [30], Double Laplace-Mayan [34] and others these double transforms are special cases of the generalization of double transform of three parameters.

2. Generalization of Double Integral Transform with Three Parameters

Definition 2.1[31,32]: The general transform (Jafari Transform) for the function f(τ) which is an integrable function, denoted to J(.) is Jafari transform for the above function is given by the form:

$$T\{f(\tau);\mathcal{S}\} = J(\mathcal{S}) = r(\mathcal{S}).\int_0^\infty f(\tau).e^{-w(\mathcal{S})\tau}\,d\tau.$$

While $\tau \geq 0; r(\mathcal{S}) \neq 0$ and $w(\mathcal{S})$ are real functions and its positive and $\mathcal{S}$ is a parameter.

Table 1: The general transform (Jafari Transform) for some famous functions [31,32]

No.	$f(\tau) = T^{-1}\{J(\mathcal{S})\}$	$J(\mathcal{S}) = T\{f(\tau);\mathcal{S}\}$
1	K ,where k is a constant	$\frac{kr_1(\mathcal{S})}{w_1(\mathcal{S})}$
2	τ	$\frac{r_1(\mathcal{S})}{[w_1(\mathcal{S})]^2}$
3	τ^n , $n > -1$	$\frac{\Gamma(n+1).r_1(\mathcal{S})}{w(\mathcal{S})^{n+1}}$
4	$\cos a\tau$	$\frac{w_1(\mathcal{S}).r_1(\mathcal{S})}{[w_1(\mathcal{S})]^2+a}$
5	$\sin b\tau$	$\frac{b\,r_1(\mathcal{S})}{[w_1(\mathcal{S})]^2+b^2}$
6	$e^{a\tau}$	$\frac{r_1(\mathcal{S})}{w_1(\mathcal{S})-a}$

Definition 2.2: The generalization of ZZ transform for the function $f(\tau)$ is defined by [33]:

$$H_g\{f(\tau)\} = r(v,u).\int_0^{\infty} e^{-w(v,u)\tau} f(\tau)d\tau = Z(v,u).$$

For all $\tau \geq 0$ and r, w are real functions of two parameters v and u.

Table 2: The generalization of ZZ transform for some basic functions [33]:

No.	$f(\tau) = H_g{}^{-1}\{Z(v,u)\}$	$Z(v,u) = H_g\{f(\tau); v,u\}$
1	K ,where k is a constant	$\frac{kw_2(v,u)}{r_2(v,u)}$; $with\, r_2(v,u) \neq 0$
2	$e^{\alpha\tau}$	$\frac{w_2(v,u)}{r_2(v,u)-\alpha}$ $, r_2(v,u) > \alpha$
3	$cos\, \alpha\tau$	$\frac{r_2(v,u)\, w_2(v,u)}{[r_2(v,u)]^2+\alpha^2}$
4	$sin\, \beta\tau$	$\frac{\beta w_2(v,u)}{[r_2(v,u)]^2+\beta^2}$
5	$sinh\, \alpha\tau$	$\frac{\alpha w_2(v,u)}{[r_2(v,u)]^2-\alpha^2}$ $, with\;\; r_2(v,u) > \alpha$
6	$cosh\, \beta\tau$	$\frac{r_2(v,u)w_2(v,u)}{[r_2(v,u)]^2-\beta^2}$ $, with \quad r_2(v,u) > \beta$

So the new general double integral transform for an integrable function $f(x,\tau)$ is given by:

$$S_g^2\{f(x,\tau)\} = r_1\,(\mathcal{S})\, r_2(v,u) \int_0^{\infty}\int_0^{\infty} e^{-[w_1(\mathcal{S})\tau + w_2(v,u)x]} \,. f(x,\tau) \,. d\tau \,. dx$$
$$= JZ(\mathcal{S},v,u)$$

Here $S_g^2\{.\}$ is symbolizing for the operator to the last general transform, while $r_1(\mathcal{S}) \neq 0, r_2(v,u) \neq 0$ and $w_1(\mathcal{S})$, $w_2(v,u)$ are real positive function.

Properties 2.3:

- Linearity Property:

$$JZ(\mathcal{S},v,u)\{\alpha f(x,\tau) + \beta g(x,\tau) = \alpha\, JZ[f(x,\tau)] + \beta\, JZ[g(x,\tau)]$$

- Shifting Property:

If $S_g^2\{x,\tau\} = JZ(\mathcal{S},v,u)$.

So

$$S_g^2\{e^{-(\alpha x+\beta\tau)}\, f(x,\tau)\}$$

$$= r_1(\mathcal{S})\,.\,r_2(v,u)\int_0^\infty\int_0^\infty e^{-w_1(\mathcal{S})\tau-\beta\tau}\,.\,e^{-w_2(v,u)x-\alpha x}\,.\,f(x,\tau)d\tau\,dx$$

$$= r_1(\mathcal{S})\,.\,r_2(v,u)\int_0^\infty\int_0^\infty e^{-[(w_1(\mathcal{S})+\beta)\tau+(w_2(v,u)+\alpha)x]}\,.\,f(x,\tau)\,d\tau\,dx$$

- Change of scale property:

$S_g^2\{x,\tau\} = JZ(\mathcal{S},v,u)$.

$S_g^2\{f(\alpha x,\beta\tau)\} = \frac{1}{\beta\alpha}JZ(\mathcal{S},v,u;\alpha,\beta)$.

Table 3: The generalization of double integral transform with three parameters for some elementary functions:

No.	$f(\tau) = {S_g^2}^{-1}\{JZ(\mathcal{S},v,u)\}$	$JZ(\mathcal{S},v,u) = S_g^2\{f(\tau);\mathcal{S},v,u\}$
1	1	$\frac{r_1(\mathcal{S})\,w_2(v,u)}{w_1(\mathcal{S})\,r_2(v,u)}$
2	$e^{\alpha x+\beta\tau}$	$\frac{r_1(\mathcal{S})\,w_2(v,u)}{([w_1(\mathcal{S})-\beta])[r_2(v,u)-\alpha]}$
3	$e^{i(\alpha x+\beta\tau)}$	$\frac{r_2(v,u)r_1(\mathcal{S})}{[w_2(v,u)-i\alpha]\,[w_1(\mathcal{S})-i\beta]}$
4	$\cos(\alpha x+\beta\tau)$	$\frac{r_1(\mathcal{S})\,r_2(v,u).(w_1(\mathcal{S})\,w_2(v,u)-\alpha\beta)}{[(w_2(v,u)^2+\alpha^2)][{w_1}^2(\mathcal{S})+\beta^2]}$
5	$\sin(\alpha x+\beta\tau)$	$\frac{r_1(\mathcal{S})\,r_2(v,u)[\alpha w_1(\mathcal{S})+\beta w_2(v,u)]}{({w_2}^2(v,u)+\alpha^2)\,[{w_1}^2(\mathcal{S})+\beta^2]}$

6	$sinh(\alpha x+\beta\tau)$	$\frac{(\alpha w_1(\mathcal{S})+\beta r_2(v,u))(r_1(\mathcal{S})w_2(v,u))}{({w_1}^2(\mathcal{S})-\beta^2)\ [{r_2}^2(v,u)-\alpha^2]}$
7	$\cosh(\alpha x+\beta\tau)$	$\frac{(w_1(\mathcal{S})r_2(v,u)+\alpha\beta)(r_1(\mathcal{S})w_2(v,u))}{({w_1}^2(\mathcal{S})-\beta^2)\ [{r_2}^2(v,u)-\alpha^2]}$
8	$x^n\tau^m\ , m>0, n>0$	$\frac{\Gamma(m+1)r_1(\mathcal{S}).\ n!w_2(v,u)}{{w_1}^{m+1}(\mathcal{S})\ (r_2(v,u))^{n+1}}$

3. Theorems And Proof

Theorem 3.1: If $f(x,\tau)$ is a function of two variables x,τ with $f(0,\tau)$ is given function so:

$$S_g^2\left[\frac{\partial f(x,\tau)}{\partial x}\right]=r_1(\mathcal{S}).r_2(v,u)\int_0^\infty\int_0^\infty\frac{\partial f(x,\tau)}{\partial x}.\ e^{-[w_1(\mathcal{S})\tau+w_2(v,u)x]}\,d\tau\,dx\,.$$

Proof:

$$S_g^2\left[\frac{\partial f(x,\tau)}{\partial x}\right]=r_1(\mathcal{S})\,.r_2(v,u)\int_0^\infty e^{-w_1(\mathcal{S})\tau}\left[\int_0^\infty\frac{\partial f(x,\tau)}{\partial x}.\ e^{-w_2(v,u)x}\,dx\right]d\tau$$

$$=r_1(\mathcal{S})\,.r_2(v,u)\int_0^\infty[-f(0,\tau)+w_2(v,u)\int_0^\infty f(x,\tau).\ e^{-w_2(v,u)x}\,dx\,]\,d\tau$$

$$=-\,r_2(v,u)\,T(0,u)+w_2(v,u)\,JZ(\mathcal{S},v,u).$$

Theorem 3.2: If $f(x,\tau)$ is a function of two variables x,τ with $f(0,\tau)$ is given function so:

$$S_g^2\left\{\frac{\partial^2 f(x,\tau)}{\partial x^2}\right\}=-\,r_2(v,u)\,\frac{\partial T(0,u)}{\partial x}-r_2(v,u)w_2(v,u)\,T(0,u)+{w_2}^2(v,u)\,JZ(\mathcal{S},v,u).$$

Proof:

$$S_g^2\left\{\frac{\partial^2 f(x,\tau)}{\partial x^2}\right\}=r_1(\mathcal{S})\,.r_2(v,u).\int_{x=0}^\infty\int_{\tau=0}^\infty\frac{\partial^2 f(x,\tau)}{\partial x^2}.\ e^{-[w_1(\mathcal{S}).\tau+w_2(v,u).x]}\,d\tau\,dx\,.$$

$$=r_1(\mathcal{S})\,.r_2(v,u).\int_0^\infty e^{-w_1(\mathcal{S}).\tau}\,[\int_0^\infty\frac{\partial^2 f(x,\tau)}{\partial x^2}.\ e^{-w_2(v,u).x}\,d\tau\,]\,dx$$

$$=-\,r_2(v,u)\,\frac{\partial T(0,u)}{\partial x}+w_2(v,u)S_g^2\left[\frac{\partial f(x,\tau)}{\partial x}\right]$$

$$=-\,r_2(v,u)\,\frac{\partial T(0,u)}{\partial x}+w_2(v,u)\,[-r_2(v,u)\,T(0,u)+w_2(v,u)\,JZ(\mathcal{S},v,u)]$$

$$= - r_2(v,u)\ \frac{\partial T(0,u)}{\partial x} - r_2(v,u) w_2(v,u)\ T(0,u) +\ {w_2}^2(v,u)\ JZ(\mathcal{S},v,u).$$

Theorem 3.3: If $f(x,\tau)$ is a function of two variables x,τ and $f(0,\tau)$ is given function so:

$$S_g^2\left\{\frac{\partial^2 f(x,\tau)}{\partial \tau^2}\right\} = -r_1(\mathcal{S})\frac{\partial\ T(v,0)}{\partial \tau} - \ w_1(\mathcal{S})\ .r_1(\mathcal{S})T(v,0) +\ w_1(\mathcal{S})^2\ JZ(\mathcal{S},v,u).$$

Proof:

$$S_g^2\left\{\frac{\partial^2 f(x,\tau)}{\partial \tau^2}\right\} = r_1(\mathcal{S}).r_2(v,u)\ \int_{x=0}^{\infty}\int_{\tau=0}^{\infty}\frac{\partial^2 f(x,\tau)}{\partial \tau^2}\ .\ e^{-[w_1(\mathcal{S}).\tau\,+\,w_2(v,u).x]}\ d\tau\ dx\ .$$

$$= r_1(\mathcal{S})\ .r_2(v,u)\int_0^{\infty} e^{-w_2(v,u)x}\left[\int_0^{\infty}\frac{\partial^2 f(x,\tau)}{\partial \tau^2}\ .\ e^{-w_1(\mathcal{S})\tau}\ d\tau\ \right] dx$$

$$= - r_1(\mathcal{S})\frac{\partial\ T(v,0)}{\partial \tau} +\ w_1(\mathcal{S}) S_g^2\left[\frac{\partial f(x,\tau)}{\partial \tau}\right]$$

$$= -\ r_1(\mathcal{S})\frac{\partial\ T(v,0)}{\partial \tau} +\ w_1(\mathcal{S})\ .[-r_1(\mathcal{S})T(v,0) +\ w_1(\mathcal{S})\ JZ(\mathcal{S},v,u)]$$

$$= - r_1(\mathcal{S})\frac{\partial\ T(v,0)}{\partial \tau} - \ w_1(\mathcal{S}).r_1(\mathcal{S})T(v,0) +\ w_1(\mathcal{S})^2\ JZ(\mathcal{S},v,u).$$

4. Application

Assuming that telegraph equation can be written in the form:

$f_{\tau\tau} + a\ f_{\tau} + bf\ = c^2 f_{xx}$,where a, b and c are constants

With (B.C) $\quad f(0,\tau) = g_1(\tau) \quad , \quad f_x(0,\tau) = h_1(\tau)$

With (I.C) $\quad f(x,0) = g_2(\tau) \quad , \quad f_x(x,0) = h_2(x).$

Taking the general double integral transform to both side of equation we have:

$$S_g^2[\, f_{\tau\tau} + af_\tau + bf = c^2\, f_{xx}]$$

$$c^2\{\, -r_2(v,u)\, \frac{\partial T(0,u)}{\partial x} -\; r_2(v,u)\,.w_2(v,u)T(0,u) + {w_2}^2(v,u)\, JZ(\mathcal{S},v,u)\}$$

$$= -r_1(\mathcal{S})\frac{\partial T(v,0)}{\partial \tau} -\; w_1(\mathcal{S})\,.r_1(\mathcal{S})T(v,0) + {w_1}^2(\mathcal{S})\, JZ(\mathcal{S},v,u)$$

$$+a\{-r_1(\mathcal{S})\, T(v,0) + w_1(\mathcal{S})\, JZ(\mathcal{S},v,u)\} + b\, JZ(\mathcal{S},v,u).$$

And

$$\frac{\partial T(0,u)}{\partial x} = H_1(u) \qquad , \qquad T(0,u) = G_1(u)$$

$$\frac{\partial T(v,0)}{\partial \tau} = H_2(v) \qquad , \qquad T(v,0) = G_2(v)$$

$$JZ(\mathcal{S},v,u)[c^2\, {w_2}^2(v,u) - {w_1}^2(\mathcal{S}) - aw_1(\mathcal{S}) - b]$$

$$= -r_1(\mathcal{S})\, H_2(v) - w_1(\mathcal{S})\, r_1(\mathcal{S})\, G_2(v) - a\, r_1(\mathcal{S})\, G_2(v) +$$
$$c^2 r_2(v,v)\, H_1(u) + c^2\, r_2(v,u)\, w_2(v,u)\, G_1(u)$$

$$JZ(\mathcal{S},v,u) = \frac{-\, r_1(\mathcal{S})H_2(v) - w_1(\mathcal{S}).r_1(\mathcal{S})\, G_2(v) - ar_1(\mathcal{S}).G_2(v)}{c^2 {w_2}^2(v,u) - {w_1}^2(\mathcal{S}) - aw_1(\mathcal{S}) - b} +$$

$$\frac{c^2.r_2(v,u)\, H_1(u) + c^2.r_2(v,u)\, w_2(v,u)G_1(u)}{c^2 {w_2}^2(v,u) - {w_1}^2(\mathcal{S}) - aw_1(\mathcal{S}) - b}.$$

$$JZ(\mathcal{S},v,u) = \ell(\mathcal{S},v,u).$$

Now taking the inverse of the generalization of double integral with three parameters transform to both side of equation we get a solution to the given equation.

5. Conclusion

This study addressed two main aspects:

The first aspect concerns the dual generality of single-parameter and two-parameter integral transforms. This dual generality encompasses various dual transforms

previously introduced-such as the double Shehu-Laplace transform, among others-as outlined in the introduction.
The second aspect lies in the potential to utilize these transforms in applications or problems involving partial differential equations. This is particularly relevant given the existence of numerous complex physical applications where the number of parameters plays a crucial role in simplifying the steps required to derive exact solutions.

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تعميم تحويل تكاملي مزدوج بثلاثة مَعْلَمَات وتطبيقاته

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المستخلص: قُدِّم في هذه الدراسة مزيجٌ يجمع بين تحويل "جعفري" (Jafari) العام ذي المَعْلَمة الواحدة وتعميم تحويل "ZZ" ذي المَعلَمتين، مما أدى إلى صياغة تحويل تكاملي مزدوج بثلاثة مَعْلَمات. وتكتسب هذه الصيغة العامة الناتجة عن الدمج أهميةً خاصة، حيث تم تطبيقها على مسألة التلغراف.

الكلمات المفتاحية: : تعميم تحويل ZZ، تعميم التحويل التكاملي، المعادلات التفاضلية الجزئية، معادلة التلغراف، التحويل التكاملي المزدوج

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